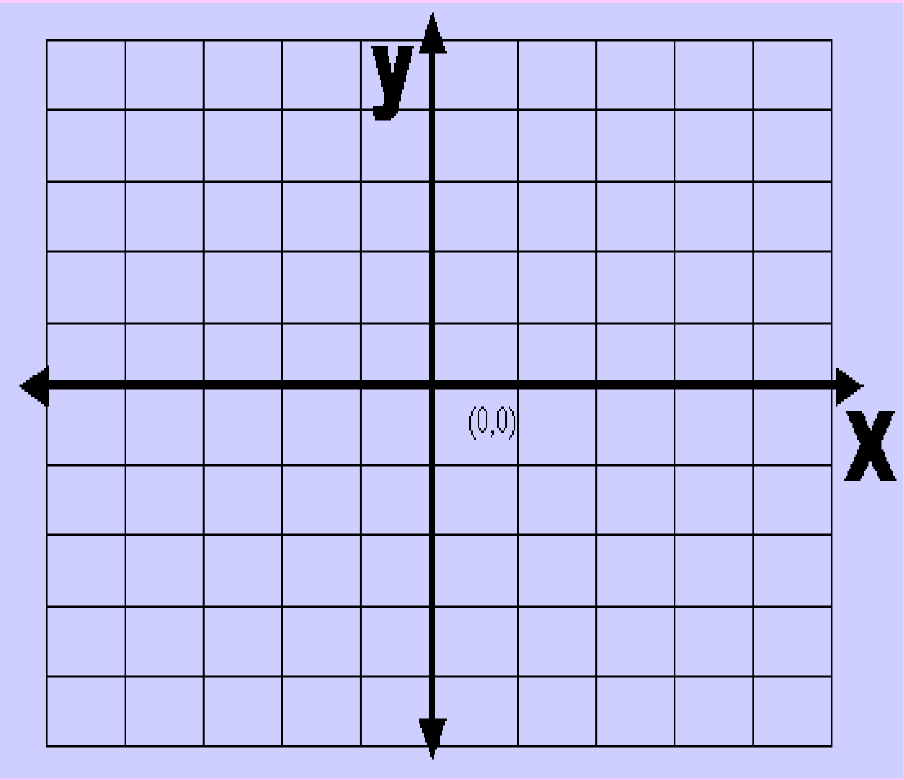
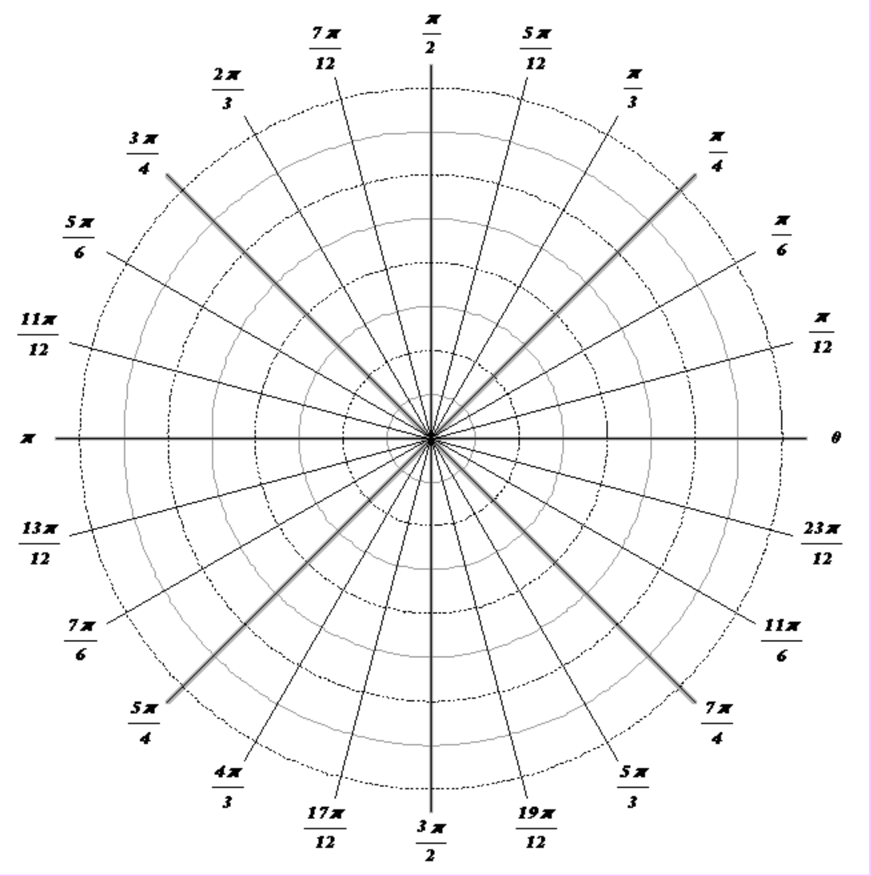
polar coordinates

$$(r,\ \theta)$$

You are familiar with plotting with a rectangular coordinate system.

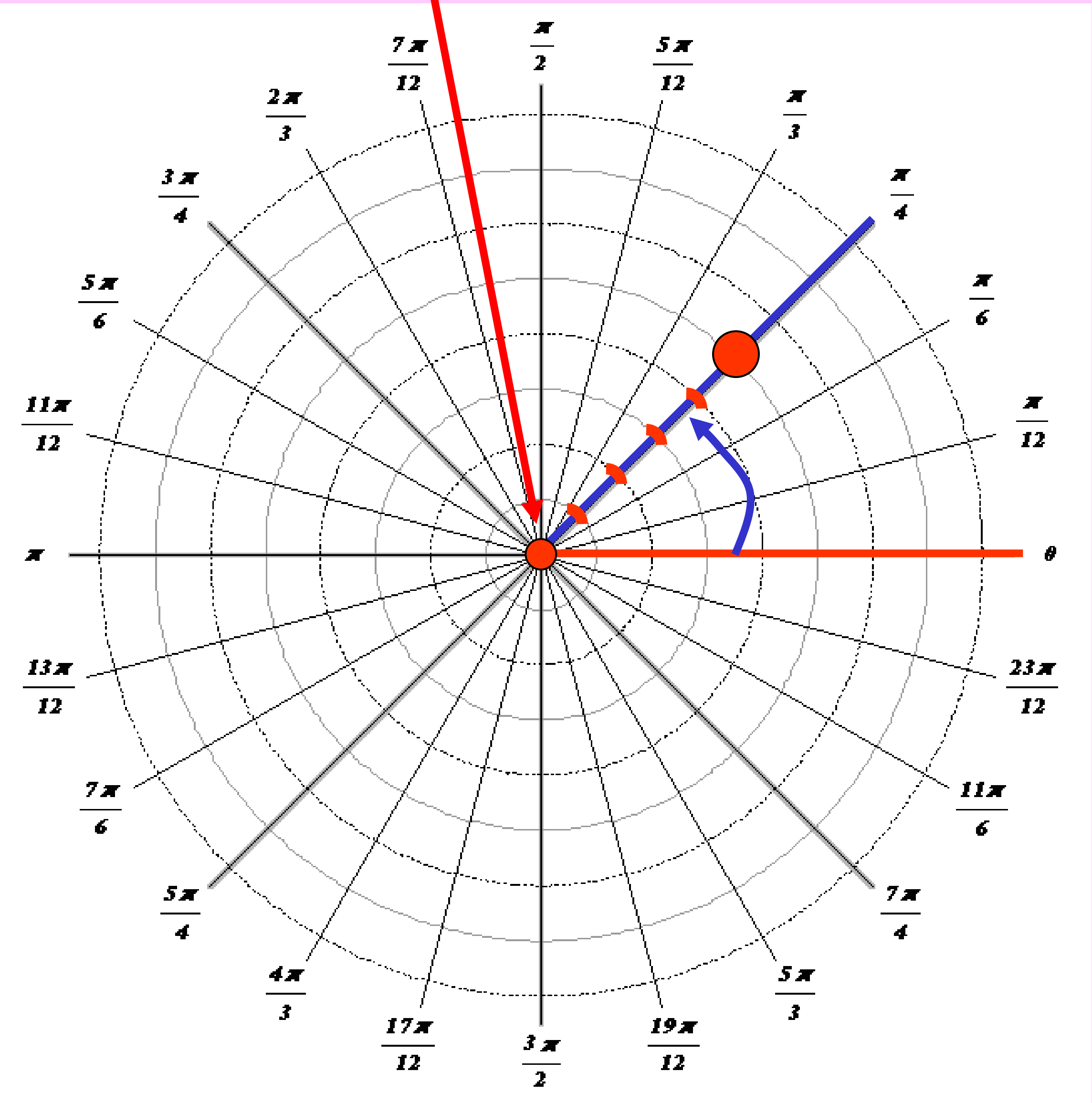
We are going to look at a new coordinate system called the polar coordinate system.





The center of the graph is called the pole.

Angles are measured from the positive *x* axis.



Points are represented by a radius and an angle

$$(r, \theta)$$

To plot the point

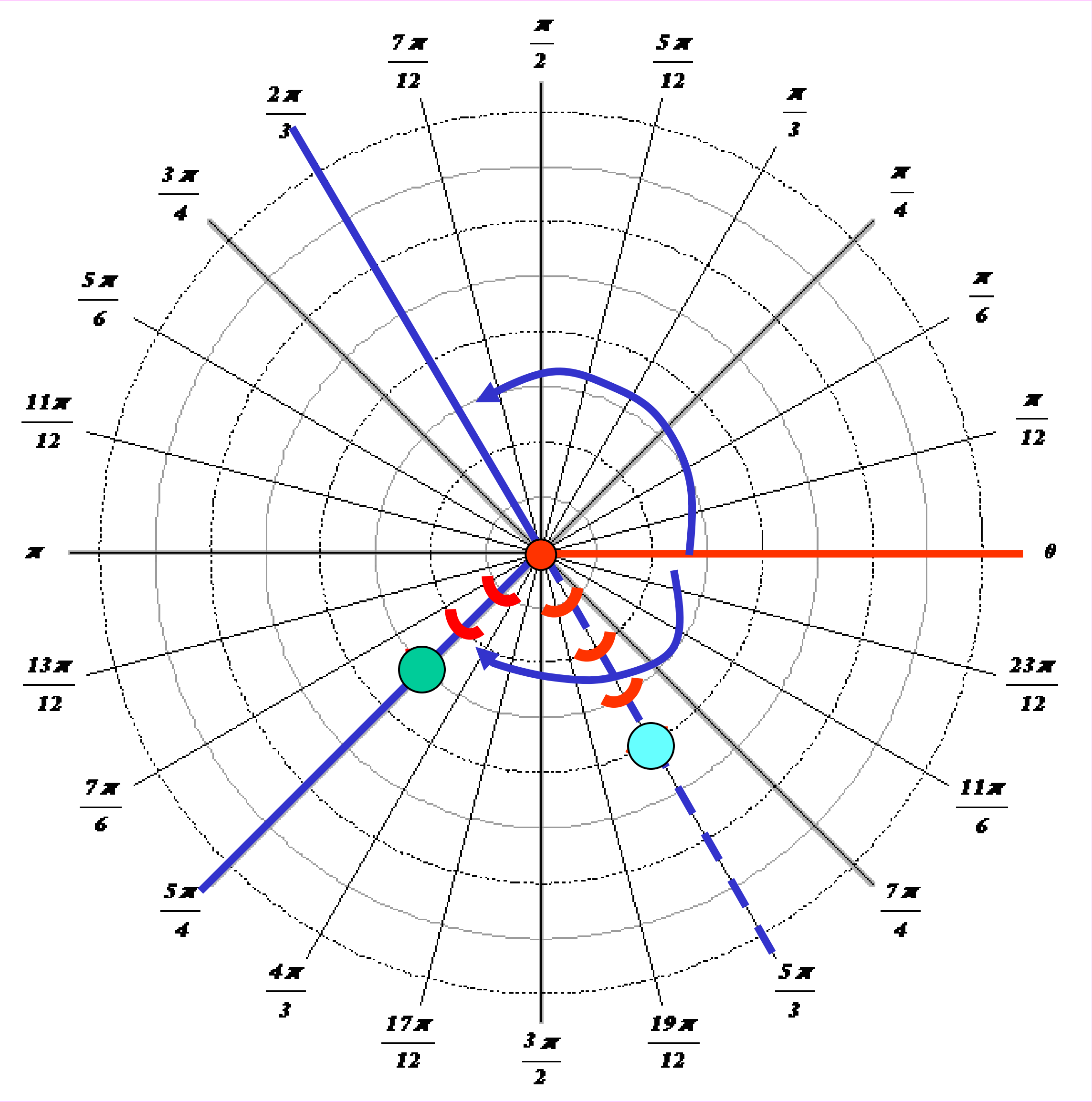
$$\left(5, \frac{\pi}{4}\right)$$

First find the angle

Then move out along the terminal side 5

A negative angle would be measured clockwise like usual.

$$\left(3, -\frac{3\pi}{4}\right)$$



To plot a point with a negative radius, find the terminal side of the angle but then measure from the pole in the negative direction of the terminal side.

$$\left(-4, \frac{2\pi}{3}\right)$$

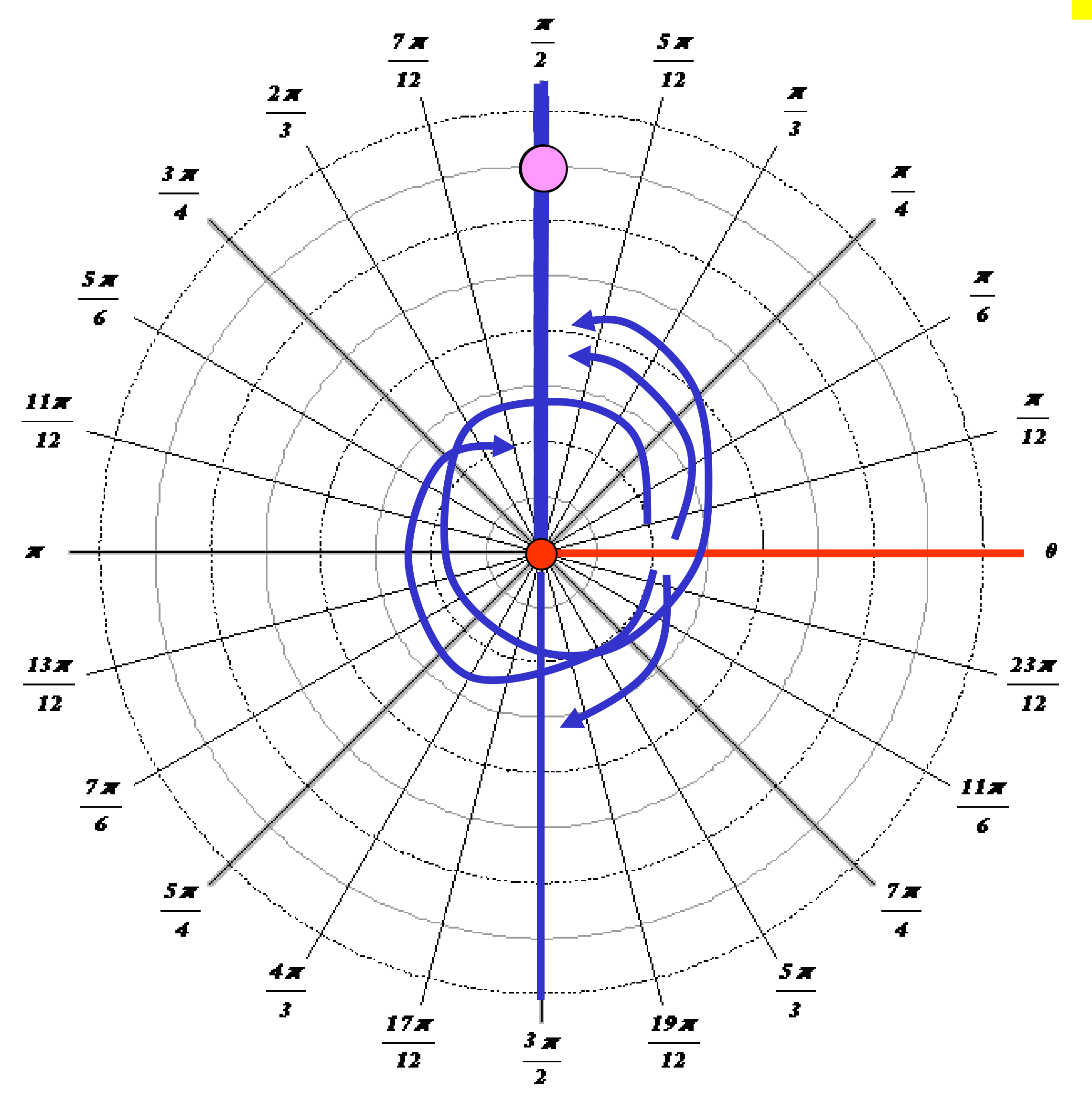
Let's plot the following points:

$$\left(7, \frac{\pi}{2}\right)$$

$$\left(-7, -\frac{\pi}{2}\right)$$

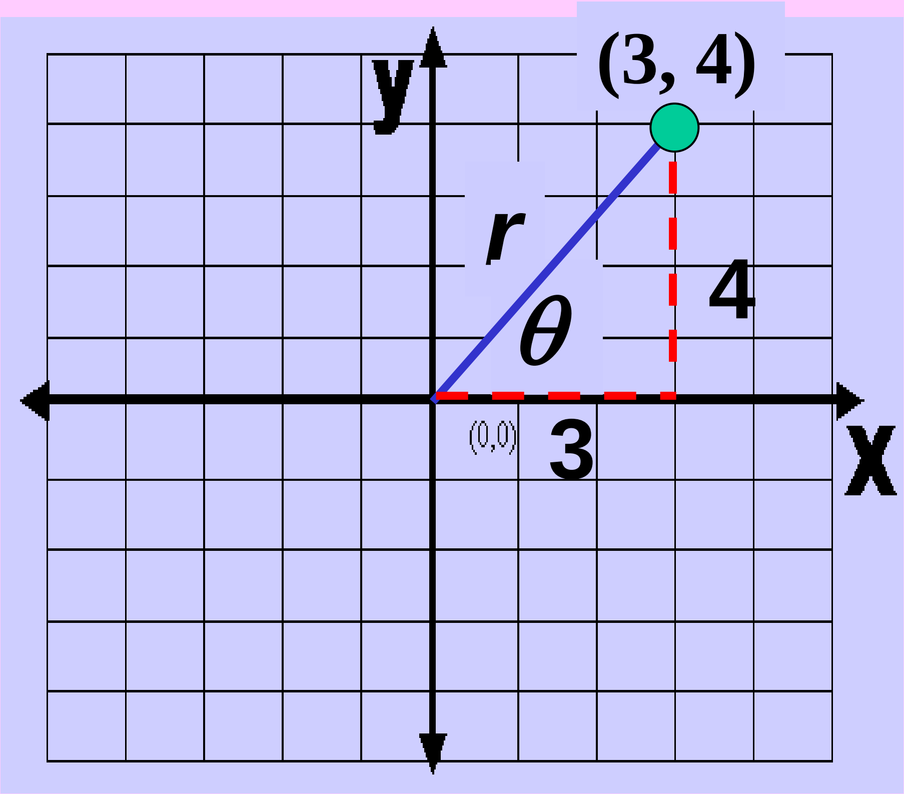
$$\left(7, \frac{5\pi}{2}\right)$$

$$\left(7, -\frac{3\pi}{2}\right)$$



Notice unlike in the rectangular coordinate system, there are many ways to list the same point.

Let's take a point in the rectangular coordinate system and convert it to the polar coordinate system.



Based on the trig you know can you see how to find *r* and *θ*?

$$3^2 + 4^2 = r^2$$

$$r = 5$$

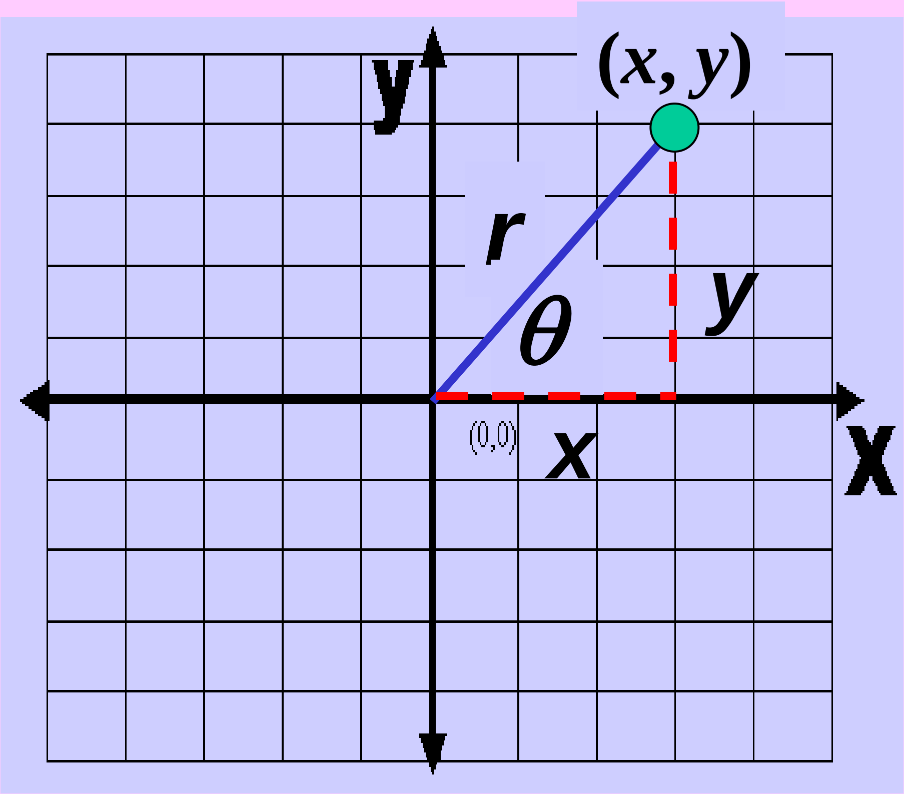
$$\tan\theta = \frac{4}{3}$$

We'll find θ **in radians**

$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 0.93$$

polar coordinates are: $(5, 0.93)$

Let's generalize this to find formulas for converting from rectangular to polar coordinates.



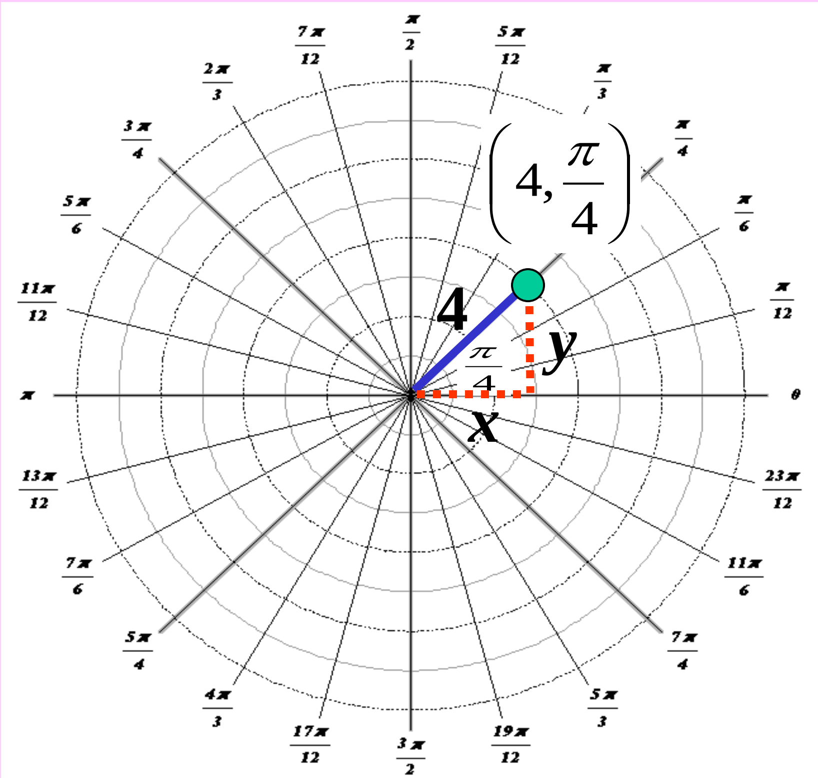
$$x^2 + y^2 = r^2$$

$$r = \sqrt{x^2 + y^2}$$

$$\tan\theta = \frac{y}{x}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Now let's go the other way, from polar to rectangular coordinates.



Based on the trig you know can you see how to find *x* and *y*?

$$\cos\frac{\pi}{4} = \frac{x}{4}$$

$$x = 4\left(\frac{\sqrt{2}}{2}\right) = 2\sqrt{2}$$

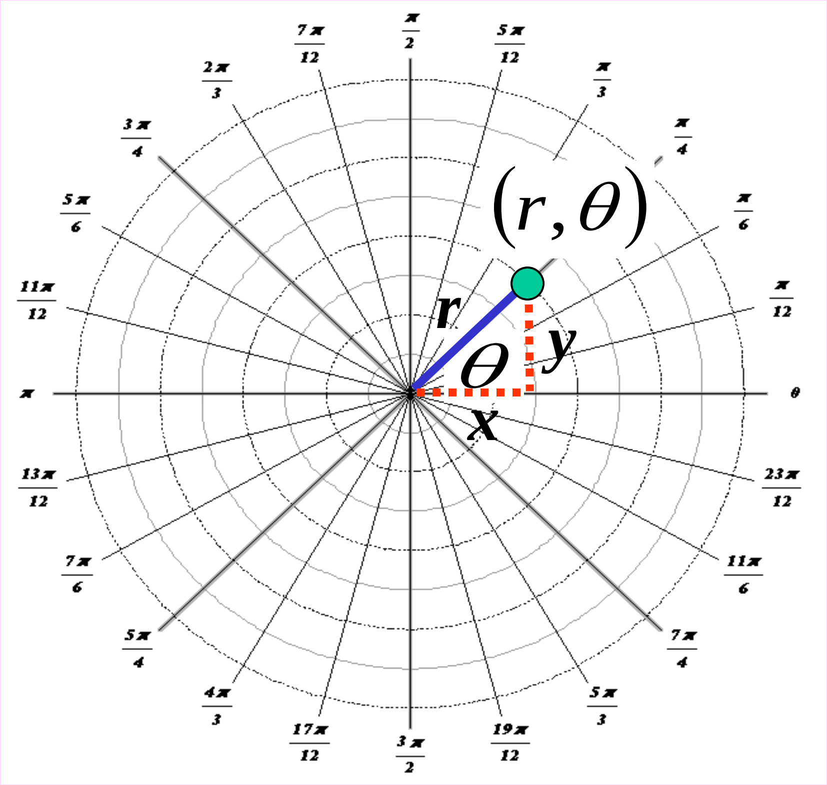
$$\sin\frac{\pi}{4} = \frac{y}{4}$$

$$y = 4\left(\frac{\sqrt{2}}{2}\right) = 2\sqrt{2}$$

rectangular coordinates are:

$$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

Let's generalize the conversion from polar to rectangular coordinates.



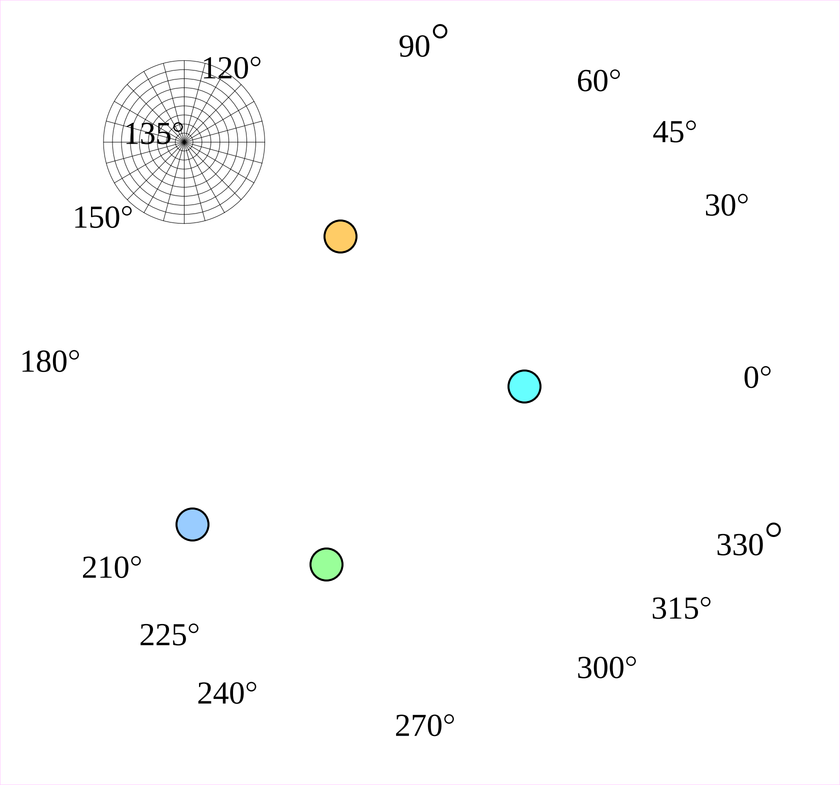
$$\cos\theta = \frac{x}{r}$$

$$x = r\cos\theta$$

$$\sin\theta = \frac{y}{r}$$

$$y = r\sin\theta$$

Polar coordinates can also be given with the angle in degrees.



(8, 210°)

(6, -120°)

(-5, 300°)

(-3, 540°)

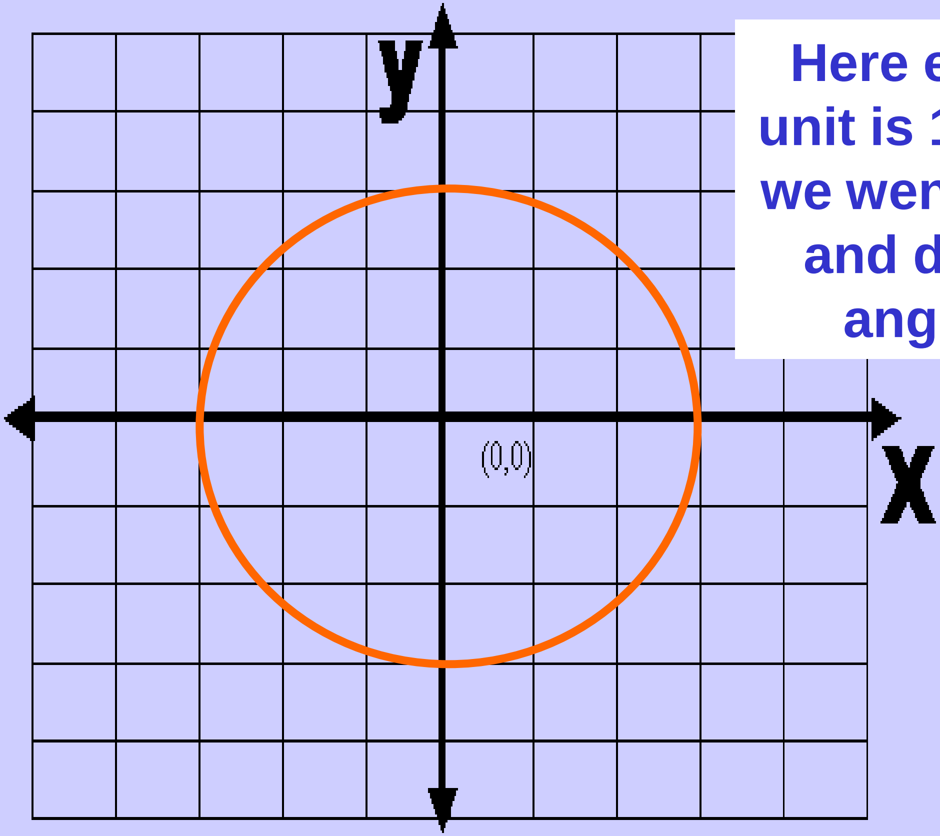
Convert the rectangular coordinate system equation to a polar coordinate system equation.

$$\sqrt{x^2 + y^2} = \pm\sqrt{9}$$

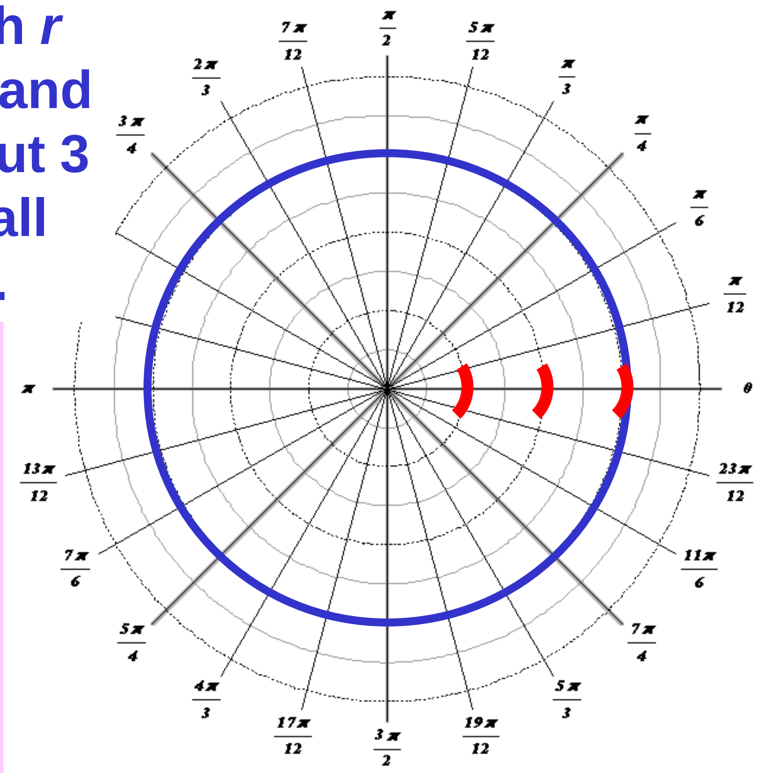
$$r = \sqrt{x^2 + y^2}$$

From conversions, how was r related to x^2 and y^2?

$$r = \pm 3$$



Here each *r* unit is 1/2 and we went out 3 and did all angles.



Before we do the conversion let's look at the graph.

***r* must be $\pm$ 3 but there is no restriction on θ so consider all values.**

Convert the rectangular coordinate system equation to a polar coordinate system equation.

$$x^2 = 4y$$

What are the polar conversions we found for *x* and *y*?

substitute in for *x* and *y*

$$x = \boxed{r\cos\theta}$$

$$y = \boxed{r\sin\theta}$$

$$\left(r\cos\theta\right)^2 = 4\left(r\sin\theta\right)$$

$$r^2\cos^2\theta = 4r\sin\theta$$

We wouldn't recognize what this equation looked like in polar coordinates but looking at the rectangular equation we'd know it was a parabola.

Acknowledgement

I wish to thank Shawna Haider from Salt Lake Community College, Utah USA for her hard work in creating this PowerPoint.

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Shawna has kindly given permission for this resource to be downloaded from www.mathxtc.com and for it to be modified to suit the Western Australian Mathematics Curriculum.

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