Thermodynamics of Quantum-Corrected Black Holes

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Introduction

The connection between thermodynamics and geometry has became a key aspect in quantum gravity. In fact, it was shown by Jacobson [1] that Einstein equations of general relativity (GR) can be retrieved from the second law of thermodynamics, using Raychaudhuri equation (RE). Along with other conceptual problems, like the problem of time in GR. Indicating the need to study the space-time as an ensemble of some geometric subsystems with their own microstates. Such that GR is recovered in the statistical limit.

Inspired by Jacobson's formalism, Das [2] and followed by Alsaleh *et al* [3] thought of RE as a fundamental equation of gravity instead of Einstein equations. Later, RE was canonically quantised. Leading to a discovery of a new dynamical system for gravity called geometric flows [3]. Studying quantum geometric flows lead to proving rigorously that singularities only exist as a classical limit of the quantum space-time [4]. Moreover, an analogous equation to Wheeler DeWitt equation was derived for quantum geometric flows (known as the Schrödinger-Raychaudhuri-Das equation), that contains a real Hamiltonian that generates time-evolution, solving the problem of time.

Objectives

- Studying space-time as an ensemble of geometric flows, making statistical mechanics as the bridge between geometric flows and Einstein field equations.
- Studying the thermodynamic properties of quantum Schwarzschild black hole .
- Compare the temperature and entropy obtained from the quantum geometric flows with the results obtained from first order quantum corrections to Schwarzschild geometry from quantum RE.

Geometric flows

We start by Studying the congruence of test particles moving on an $n+1$ dimensional space-time $\mathcal{M}$. We can use the proper time for the test particles λ as a dynamical foliation parameter, such that we foliate the space-time as in figure 1.

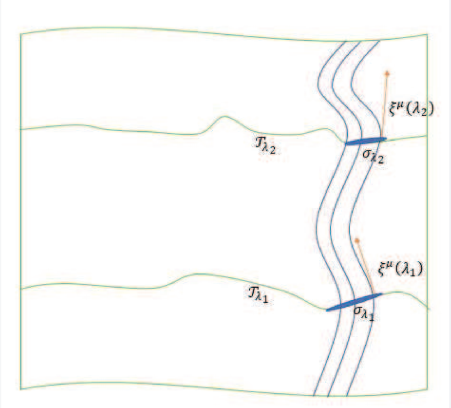

Figure 1: *The dynamic foliation of the space-time $\mathcal{M}$ by the flow of geodesic congruences. The cross-sectional hypersurface σ_λ represent the geometric flow.*

The dynamical system resembling the geometric flow is the cross-section of the congruence σ_λ, and the dynamical degree of freedom is the volume of this cross-section, which is given by:

$$\rho = 2\int d^n x\sqrt{\det h}. \qquad (1)$$

With $h_{\alpha\beta}$ being the metric of the hypersurface σ_λ. We may also define the variable $\chi = 2\sqrt{\rho}$.

The canonically-conjugate momentum (CCM) is called ϖ, and it is related to the expansion parameter θ by the relation $\theta = \frac{\varpi}{\chi}$. Using the CCM, we can write the Hamiltonian:

$$H = \frac{1}{2n}\varpi^2 + \left(\frac{1}{2}\mathcal{R}\chi^2 - \Sigma\right). \qquad (2)$$

with $\mathcal{R}$ being the Raychaudhuri scalar and Σ is the shear potential. We have chosen the dynamic foliation such that the tangent vector field to the congruence ξ^μ is always orthogonal to σ_λ. Such that the rotation Ω vanishes. Raychaudhuri equation can be recovered from Hamilton's equation

$$\{\theta, H\} = -\dot{\theta} = -\frac{1}{n}\theta^2 - \mathcal{R} + 2\sigma^2. \qquad (3)$$

Which is the expected result, as the motion of test particles on geodesics is generated by the dynamics of geometric flows.

Canonical quantisation

We can now quantise the system by introducing the operators $\hat{\chi}$ and $\hat{\varpi}$, that obey the CCR:

$$[\hat{\chi}, \hat{\varpi}] = i\hbar^{-1/2}\hat{I}. \qquad (4)$$

In the χ representation, we define the geometric flow wavefunctionals $\Psi[\chi;\lambda]$ that obey the Schrödinger-Raychudhri-Das (SRD) equation:

$$\left(\frac{-\hbar^2}{2n}\frac{\delta^2}{\delta\chi^2} + \frac{1}{2}\mathcal{R}\chi^2 + \Sigma\right)\Psi[\chi;\lambda] = i\hbar\frac{\partial}{\partial\lambda}\Psi[\chi;\lambda] \qquad (5)$$

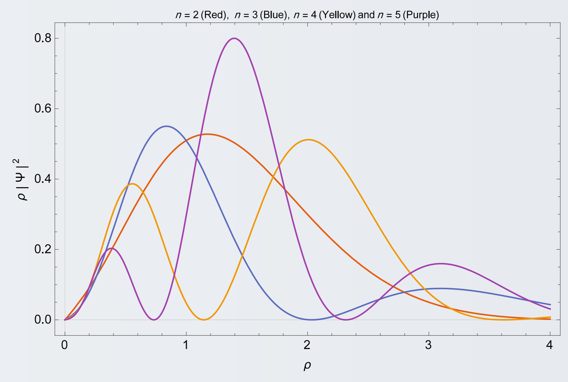

Figure 2: *A plot of the probability density function $|\Psi|^2$ vs $\rho = \chi^2$ obtained by solving (5) with $\mathcal{R} = \Sigma = 0$. The plot indicates that the probability density function rapidly decreases as $\rho \to 0$ and vanishes identically at the singularity.*

Statistical mechanics

We can consider geometric flows playing the role of subsystems in Jacobson's formalism [1]. Therefore, the statistical mechanical study of geometric flows is supposed to reproduce the physics of space-time, in this study we considered Schwarzschild black holes.

We can consider the event horizon of a black hole as an ensemble of N geometric flows, each of them correspond to a single generator, from holographic principle, there is a bound on N, given by Bekenstein limit [6].

$$N \sim \frac{A}{4\hbar} \qquad (6)$$

Since the classical Schwarzschild geometry is Ricci flat, we may assume a time-independent quantum fluctuations of geometry. Manifesting themselves as a constant $\mathcal{R}$. With this assumption, the SRD equation becomes similar to a SHO with half potential, with angular frequency $\omega = (\mathcal{R}/n)^{1/2}$. The spectrum of 'analogous' energy is given by:

$$\epsilon_m = \hbar\omega\left(m + \frac{1}{2}\right), \qquad (7)$$

with the modes taking only odd numbers $m = 1, 3, 5, \dots$.

Figure 3: *A plot of the probability density function $|\Psi|^2$ vs $\rho = \chi^2$ obtained by solving (5) with $\mathcal{R} = \mathrm{const}$. In 4 dimensional space-time.*

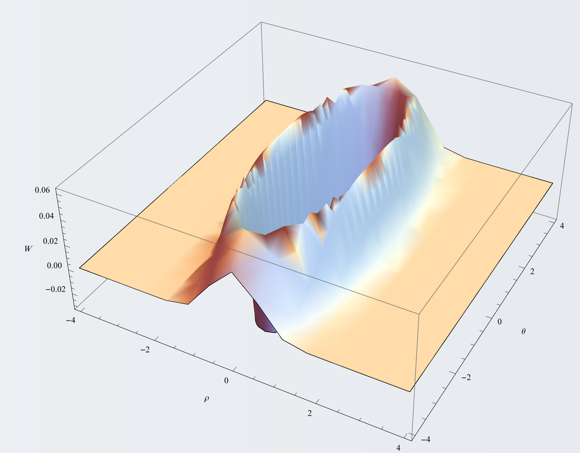

Figure 4: *A plot of the Wagner quasi-probability distribution $W(\rho;\theta)$, over the quantum phase space. For the ground state solution $m = 1$ and $n = 3$.*

The explicit solution in terms of the variable ρ is given by,

$$\Psi_m[\rho] = A\exp\left(-\sqrt{\frac{\mathcal{R}}{n\hbar^2}}\rho\right)L_m\left(2\sqrt{\frac{\mathcal{R}}{n\hbar^2}}\rho\right) \qquad (8)$$

Thus, we may calculate the partition function of the quantum geometric flows ensemble

$$Z = \frac{e^{-\frac{3}{2}\beta N\hbar\omega}}{(1 - e^{-\beta\hbar\omega})^N}, \qquad (9)$$

from which we can fully characterise the statistical mechanics of geometric flows.

Quantum-corrected black holes

We can calculate the entropy of the horizon using (9)

$$S = N\left(1 + \ln\left(\frac{T\hbar}{\omega}\right) + \frac{\omega^2\hbar^2}{24T^2} + \dots\right), \qquad (10)$$

from which we can recover the Area-Entropy law $S = A/4\hbar$ from taking the leading term of (10). Moreover, we define the BH mass as the average energy $M = \langle E\rangle$, that is given by

$$\langle E\rangle = M = N\left(2T + \frac{\omega^2\hbar^2}{6T} + \dots\right). \qquad (11)$$

By taking the leading term, we also recover the relation between the mass of BH temperature $T = \hbar/8\pi M$. Moreover by taking sub-leading terms of (11), we get to calculating quantum correction to temperature:

$$T = \frac{\sqrt{M^2 - 4\pi^2\sqrt{\mathcal{R}\hbar}}}{2\pi\left(\sqrt{M^2 - 4\pi^2\sqrt{\mathcal{R}\hbar}} + M\right)^2}, \qquad (12)$$

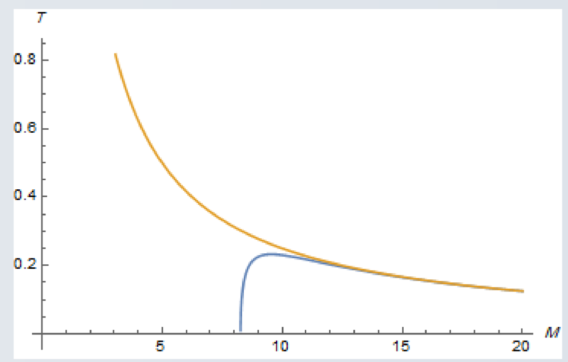

Figure 5: *A graph of M vs T for classical (blue) and quantum (red) black holes. We assumed area fluctuation of Plankian order $\omega \sim$. The graph indicates the existence of remnant for the quantum black hole.*

and to the entropy, as well

$$S = 2\pi\left(M\sqrt{M^2 - 4\pi^2\sqrt{\mathcal{R}\hbar}} + M^2\right). \qquad (13)$$

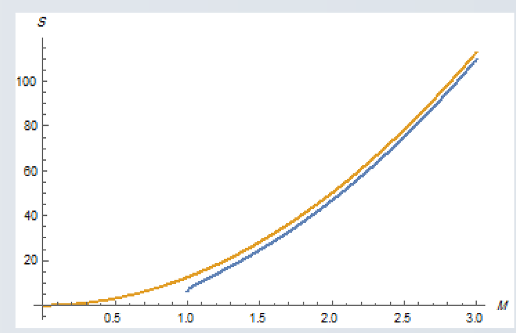

Figure 6: *Graphs of M vs T (up) and M vs S (down). For classical (blue) and quantum (red) black holes. We assumed area fluctuation of Plankian order $\omega \sim 1$. The graph indicates the existence of remnant for the quantum black hole.*

Conclusion

We can see from figures 5 and 6. That the quantum correction to Schwarzschild geometry using the statistical mechanics of geometric flows reproduces the same quantum corrections obtained in [5], by considering the quantum Raychaudhuri equation (QRE) [2]. Indicating that the initial conjecture about the space-time being an ensemble of geometric flows is true, and showing a correspondence between geometric flows and QRE. Recovering the standard formulae for BH temperature and entropy is a great test for creditability of quantum geometric flows as a potential approach for quantum gravity.

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