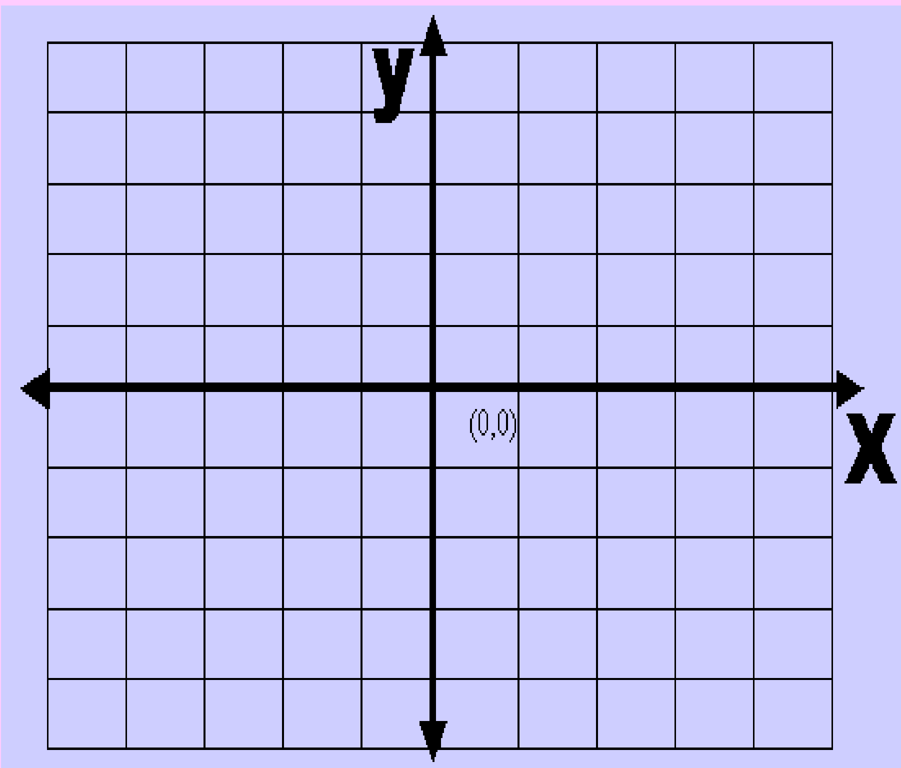
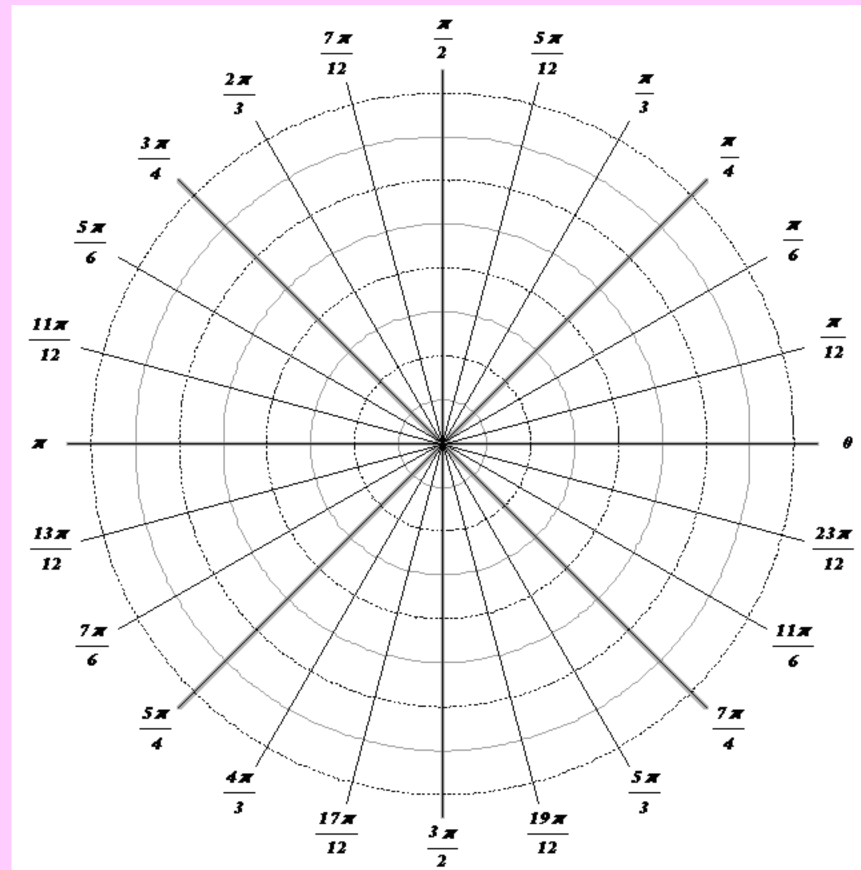
REVIEW 9.1-9.4

Polar Coordinates and Equations

You are familiar with plotting with a rectangular coordinate system.

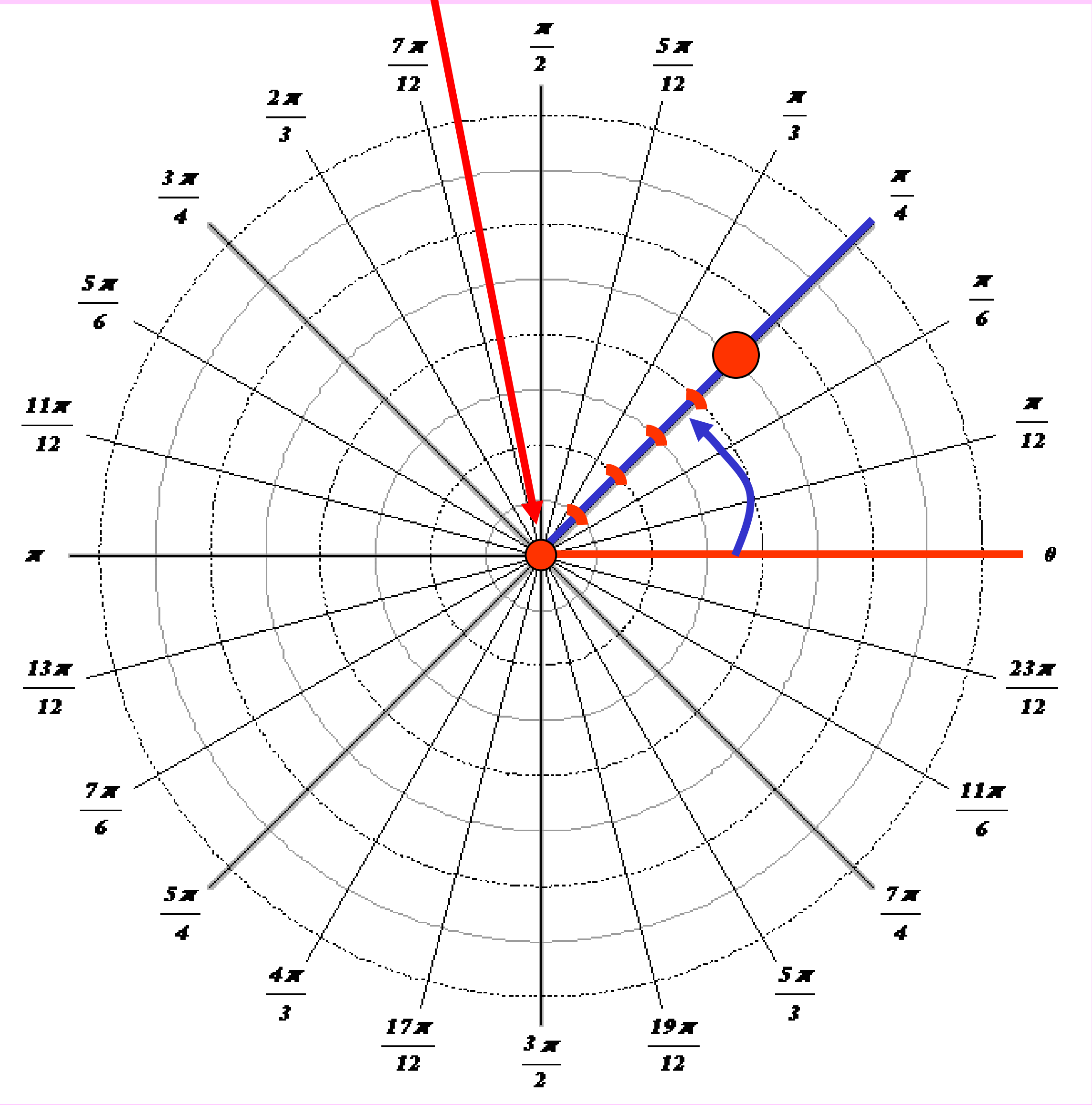
We are going to look at a new coordinate system called the polar coordinate system.





The center of the graph is called the pole.

Angles are measured from the positive *x* axis.



Points are represented by a radius and an angle

$$(r, \theta)$$

To plot the point

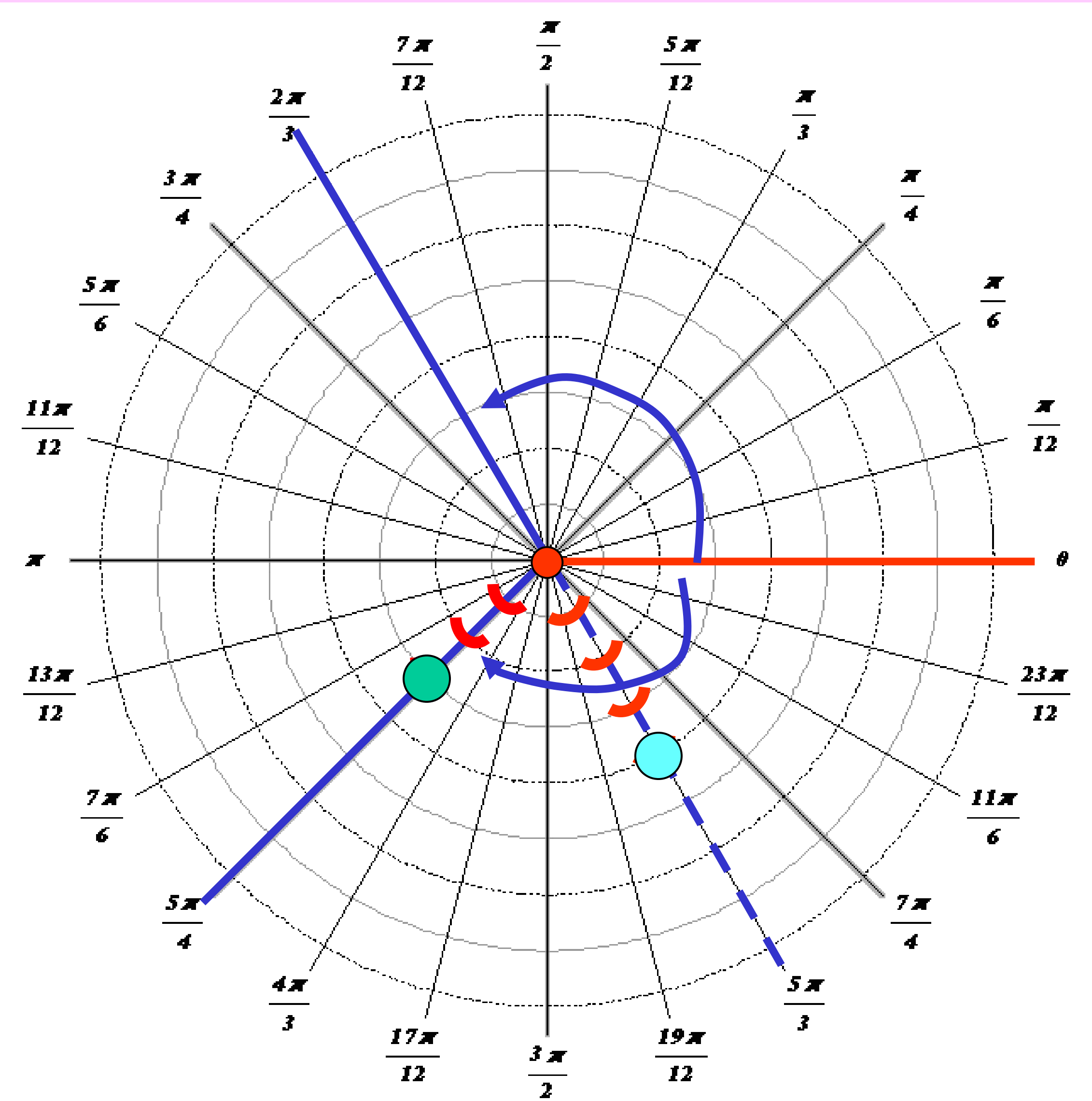
$$\left(5, \frac{\pi}{4}\right)$$

First find the angle

Then move out along the terminal side 5

A negative angle would be measured clockwise like usual.

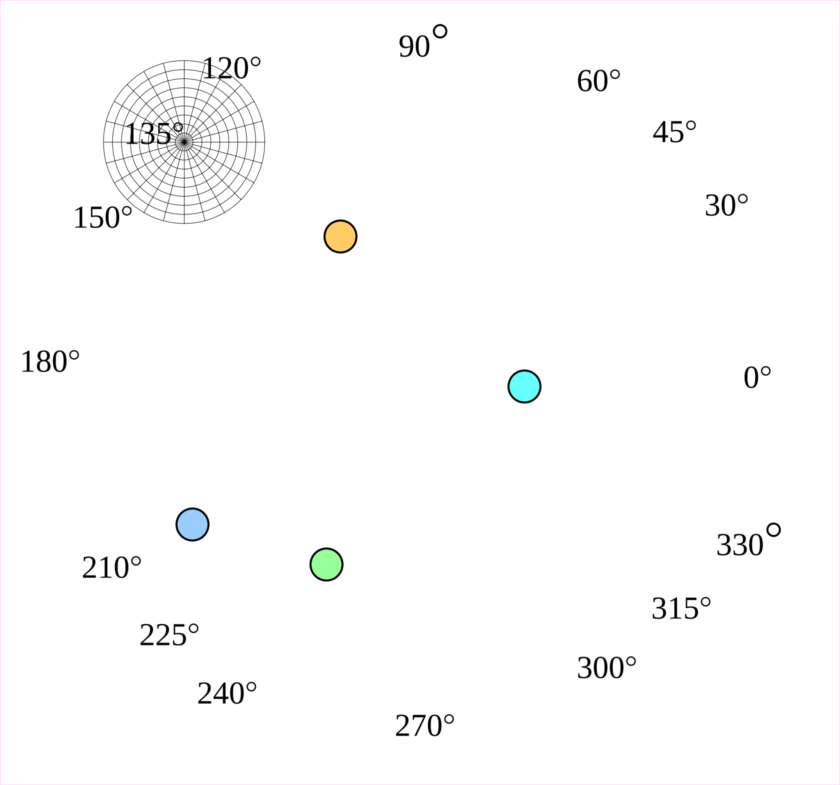
$$\left(3, -\frac{3\pi}{4}\right)$$



To plot a point with a negative radius, find the terminal side of the angle but then measure from the pole in the negative direction of the terminal side.

$$\left(-4, \frac{2\pi}{3}\right)$$

Polar coordinates can also be given with the angle in degrees.



(8, 210°)

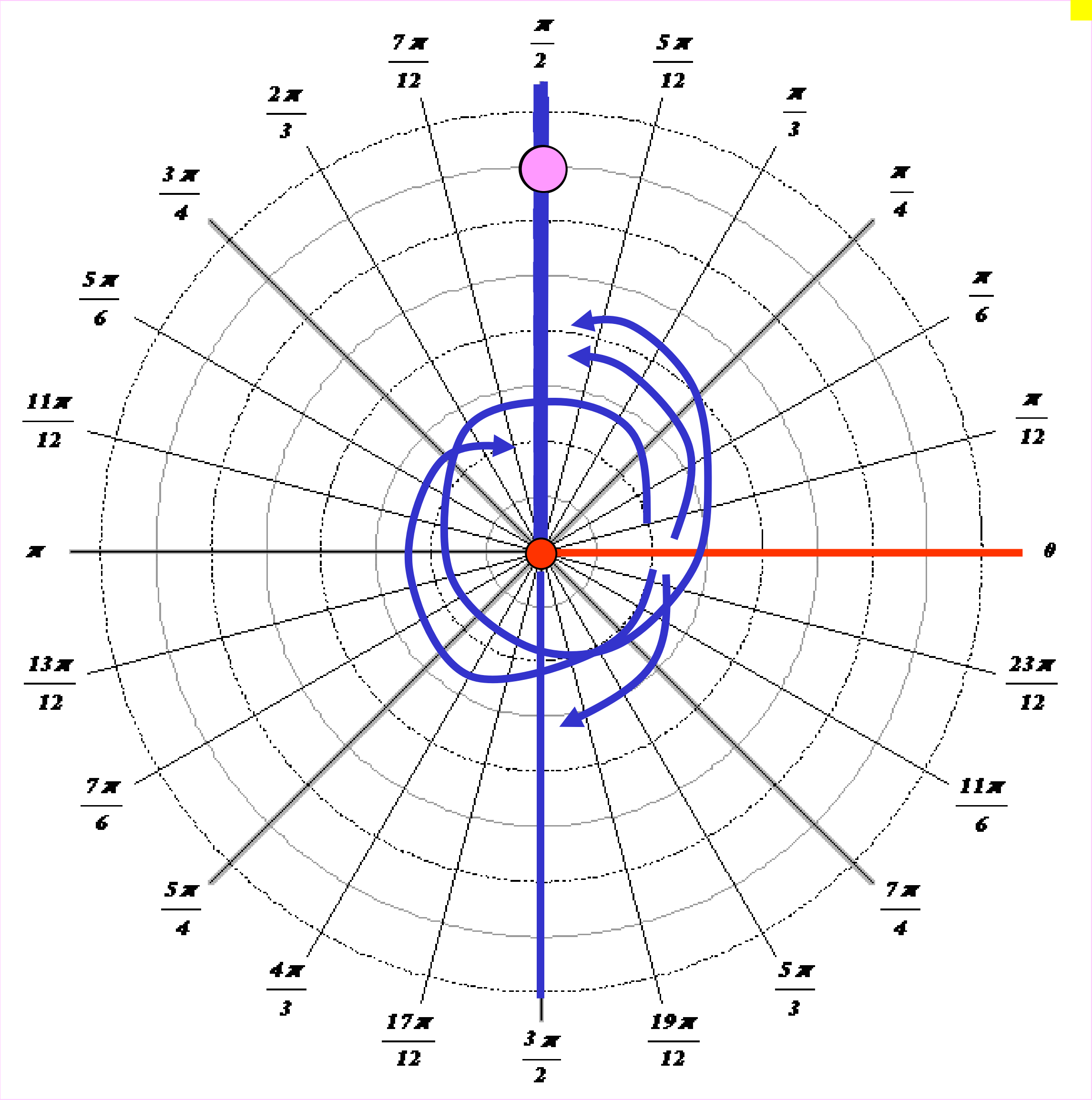
(6, -120°)

(-5, 300°)

(-3, 540°)

Let's plot the following points:

$$\left(7, \frac{\pi}{2}\right) \quad \left(-7, -\frac{\pi}{2}\right) \quad \left(7, \frac{5\pi}{2}\right) \quad \left(7, -\frac{3\pi}{2}\right)$$



Notice unlike in the rectangular coordinate system, there are many ways to list the same point.

Name three other polar coordinates that represent that same point given:

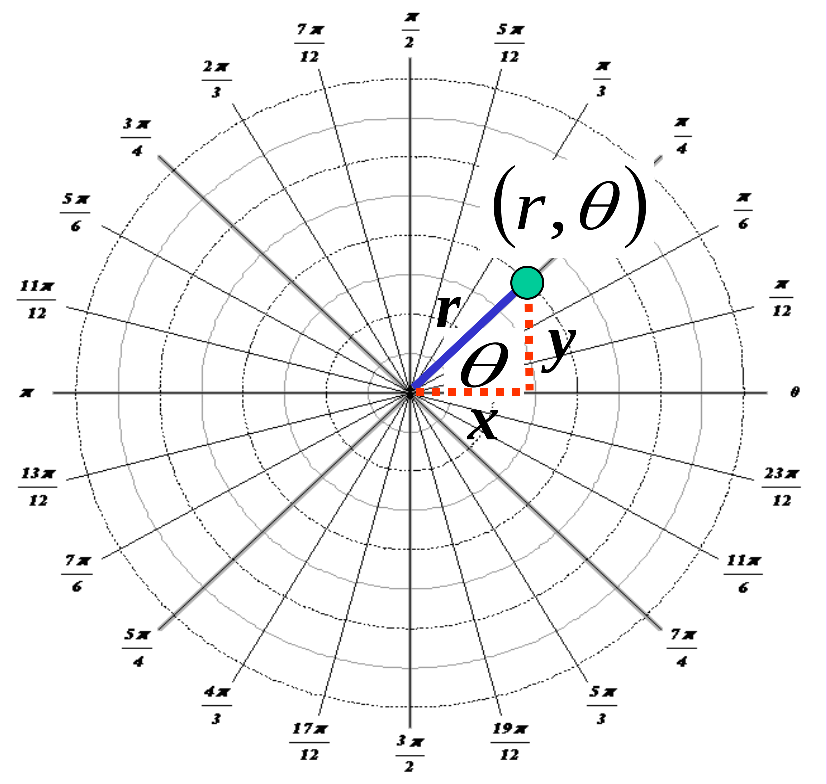
1.) $(2.5, 135°)$

$(2.5, -225°)$
$(-2.5, -45°)$
$(-2.5, -315°)$

2.) $(-3, 60°)$

$(-3, -310°)$
$(3, 240°)$
$(3, -120°)$

Let's generalize the conversion from polar to rectangular coordinates.



$$\cos\theta = \frac{x}{r}$$

$$x = r\cos\theta$$

$$\sin\theta = \frac{y}{r}$$

$$y = r\sin\theta$$

Steps for Converting Equations from Rectangular to Polar form and vice versa

Four critical equivalents to keep in mind are:

$$x = r\cos\theta \qquad y = r\sin\theta$$

$$r^2 = x^2 + y^2$$

$$\theta = Arc\tan\left(\frac{y}{x}\right) \qquad \theta = Arc\tan\left(\frac{y}{x}\right) + \pi$$

If x > 0 If x < 0

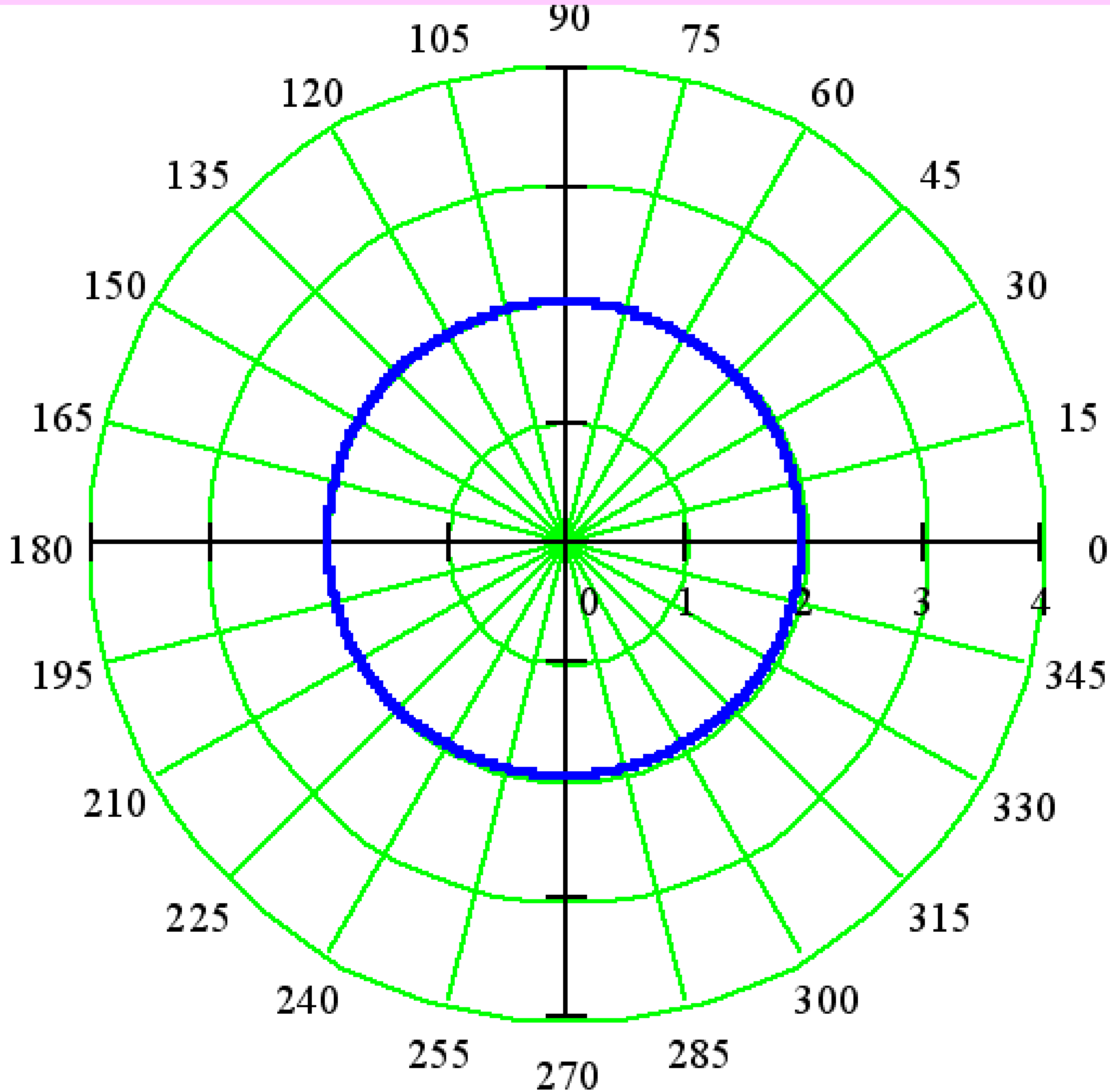
Identify, then convert to a rectangular equation and then graph the equation:

$$r = 2$$

$$r^2 = 4$$

$$\boxed{x^2 + y^2 = 4}$$

Circle with center at the pole and radius 2.

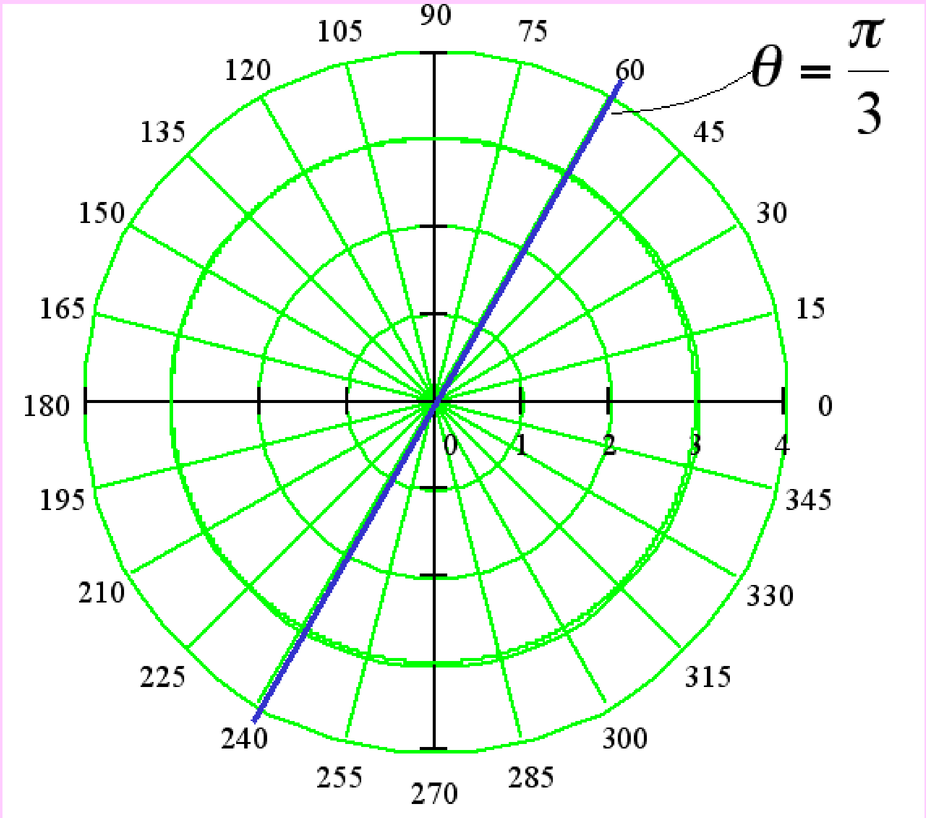

90
105
75
120
60
135
45
150
30
165
15
180
0
0
1
2
3
4
195
345
210
330
225
315
240
300
255
285
270

IDENTIFY and GRAPH:

$$\theta = \frac{\pi}{3}$$

The graph is a straight line at extending through the pole.

π



Now convert that equation into a rectangular equation:

$$\theta = \frac{\pi}{3}$$

$$Arc\tan\left(\frac{y}{x}\right) = \frac{\pi}{3}$$

Take the tan of both sides:

$$\frac{y}{x} = \tan\left(\frac{\pi}{3}\right)$$

$$\frac{y}{x} = \tan\left(\frac{\pi}{3}\right)$$

$$\frac{y}{x} = \sqrt{3}$$

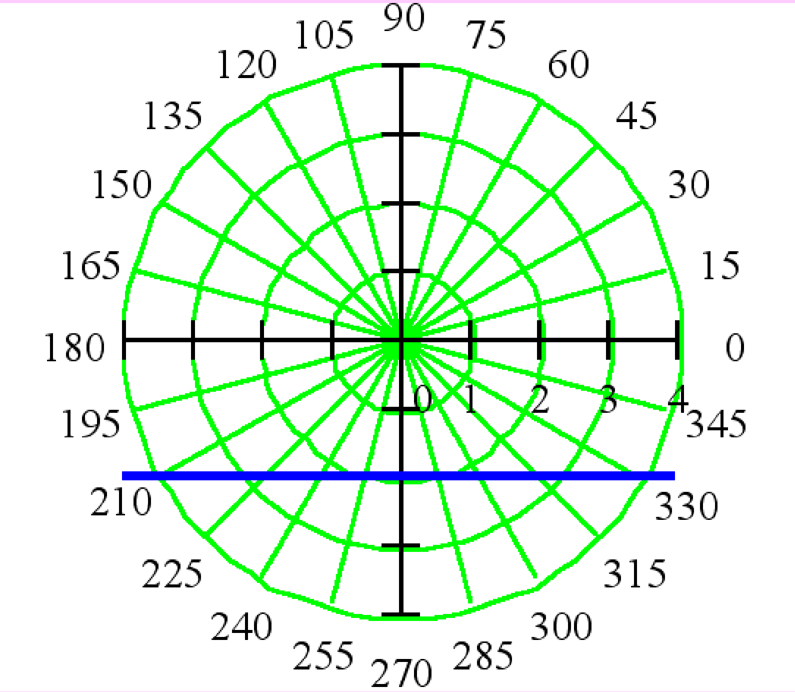
Cross multiply:

$$y = \sqrt{3}x$$

$$\sqrt{3}x - y = 0$$

IDENTIFY, GRAPH, AND THEN CONVERT TO A RECTANGULAR EQUATION:

$$r\sin\theta = -2$$



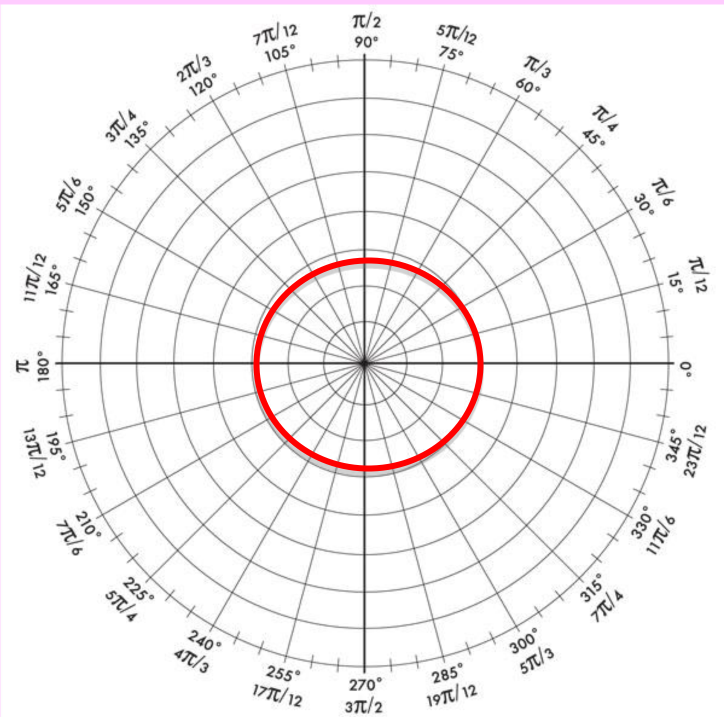
$$\boxed{y = -2}$$

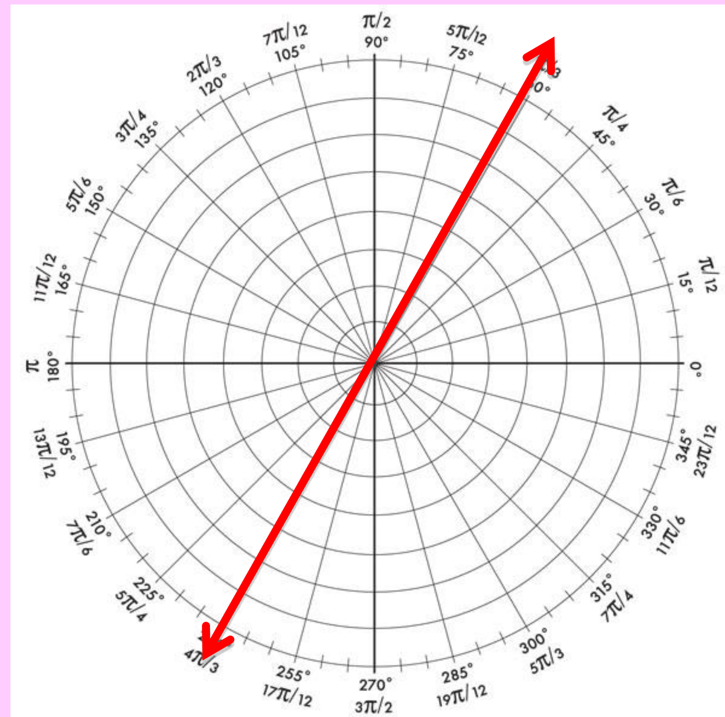
The graph is a horizontal line at y = -2

GRAPH EACH POLAR EQUATION.

1.) $r = 3$

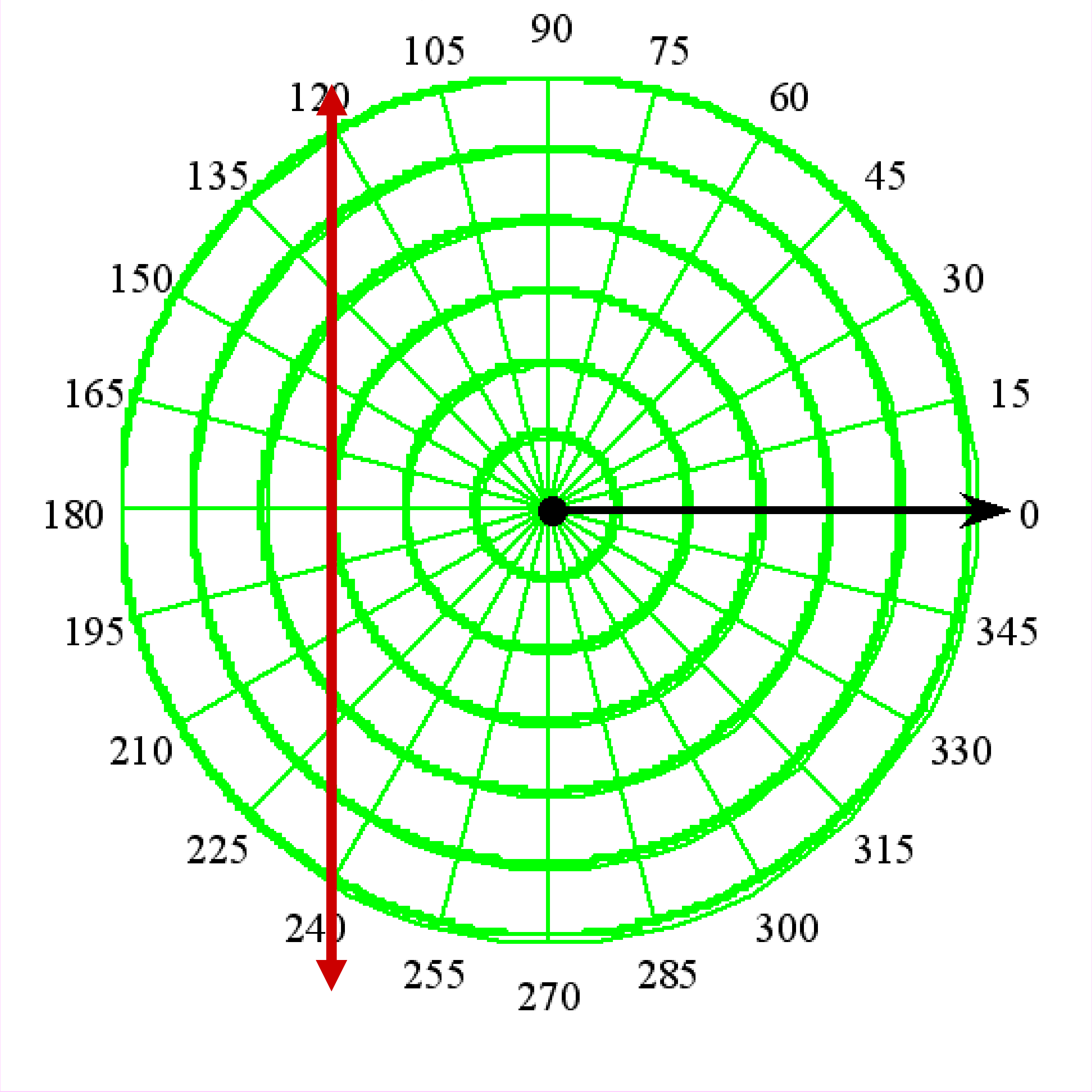
2.) $\theta = 60^\circ$





Graph: $r\cos\theta = -3$ $\quad r = \dfrac{-3}{\cos\theta}$

θ	r
0°	-3
30°	-3.5
60°	-6
90°	UD
120°	6
150°	3.5
180°	3
210°	3.5
240°	6
270°	UD
300°	-6
330°	-3.5
360°	-3

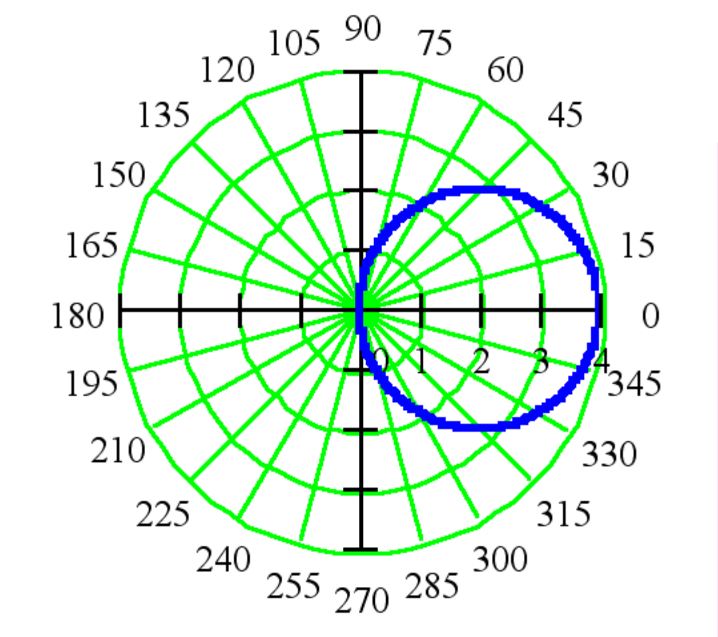


IDENTIFY, GRAPH, AND THEN CONVERT TO A RECTANGULAR EQUATION: $r^2 = 4r\cos\theta$

θ	r
0°	4
30°	3.5
60°	2
90°	0
120°	-2
150°	-3.5
180°	-4
210°	-3.5
240°	-2
270°	0
300°	2
330°	3.5
360°	4

$$r = 4\cos\theta$$

$$r^2 = 4r\cos\theta$$



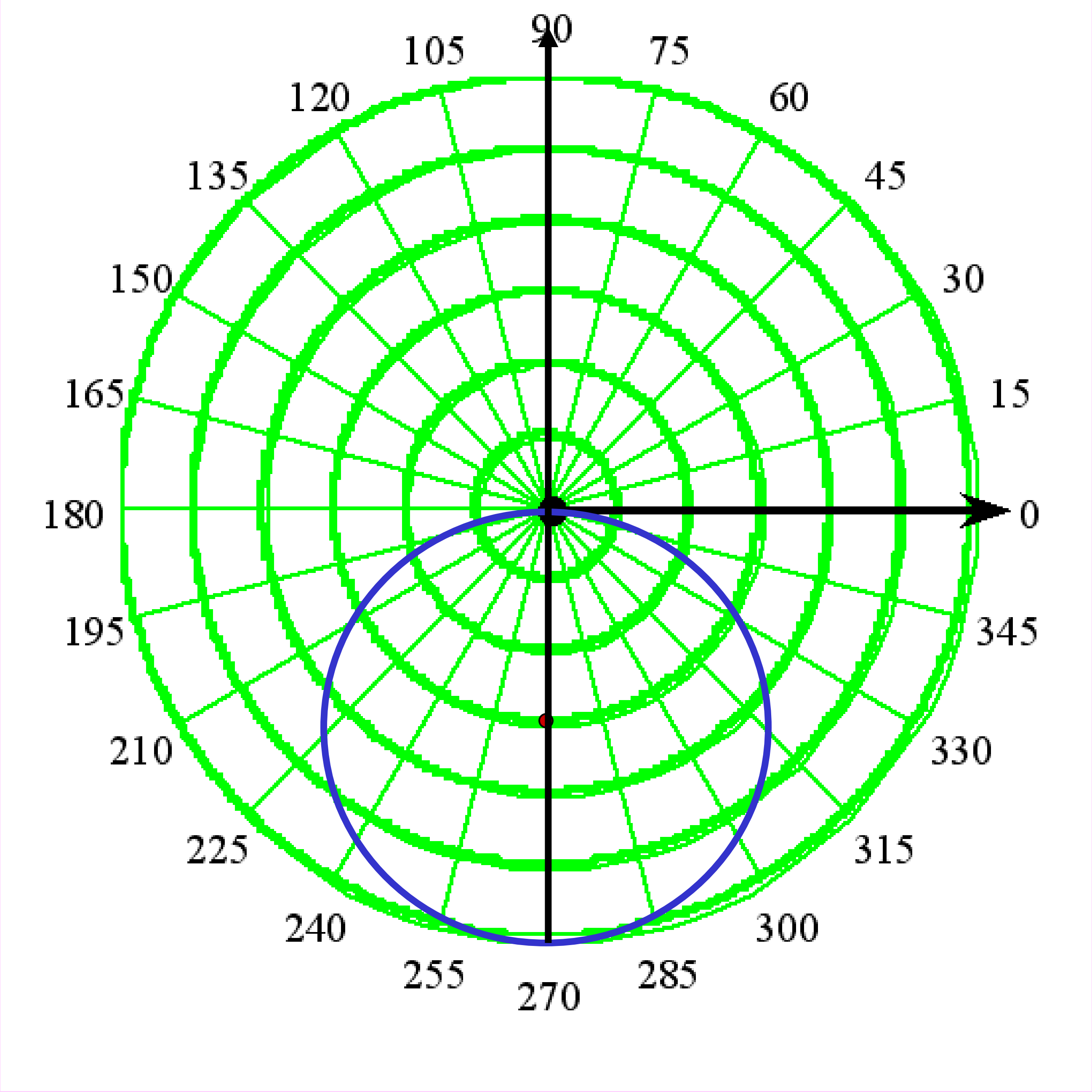
$$x^2 + y^2 = 4x$$

$$x^2 - 4x + y^2 = 0$$

$$x^2 - 4x + 4 + y^2 = 4$$

$$\boxed{(x-2)^2 + y^2 = 4}$$

$$r = -6\sin\theta$$

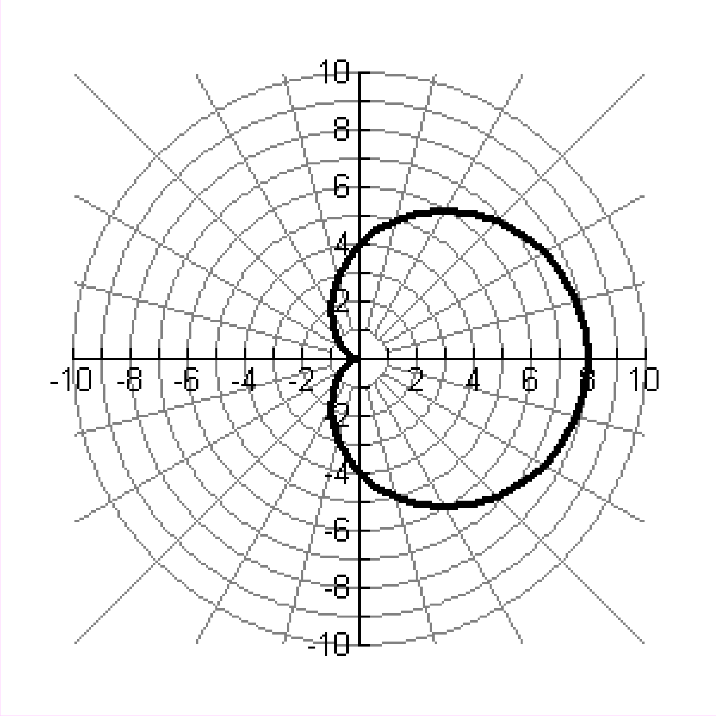


90
105
75
120
60
135
45
150
30
165
15
180
0
195
345
210
330
225
315
240
300
255
285
270

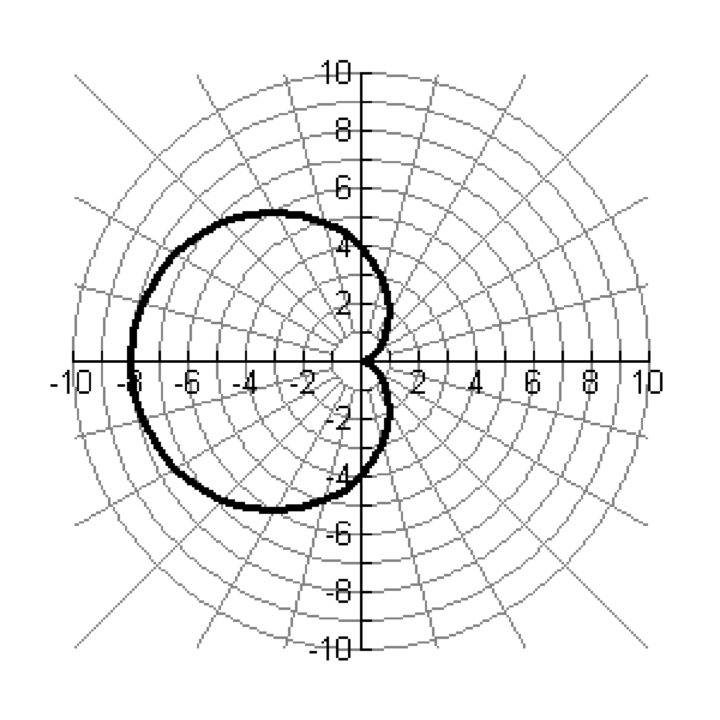
Cardioids (heart-shaped curves) where a > 0 and passes through the origin

$$r = a + a\cos\theta \qquad\qquad r = a - a\cos\theta$$

$$r = 4 + 4\cos\theta$$

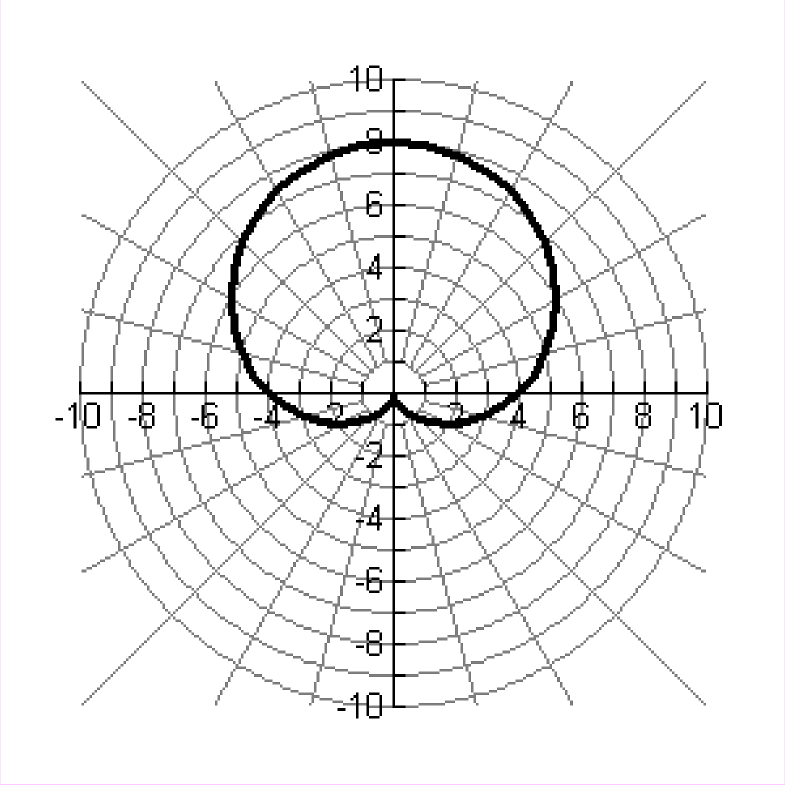


$$r = 4 - 4\cos\theta$$

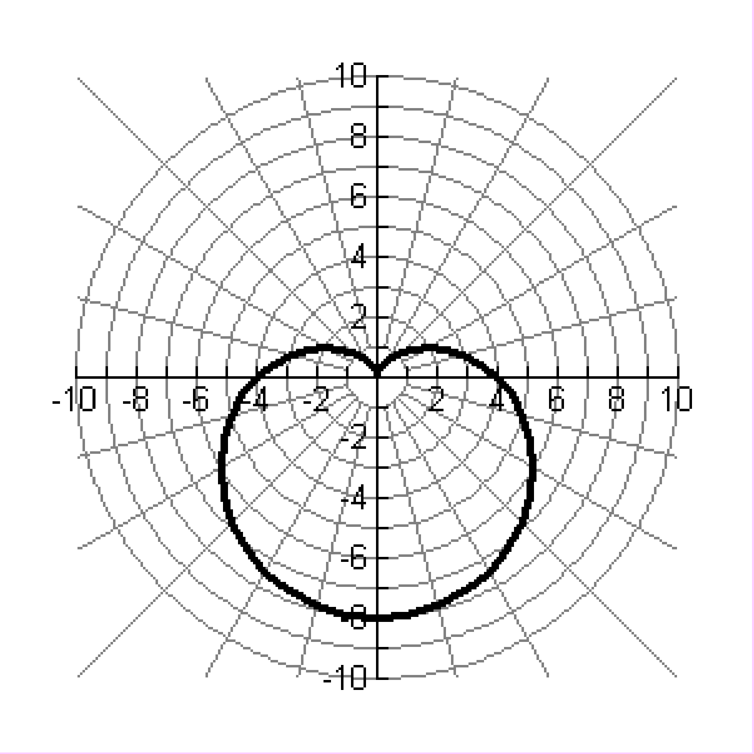


$$a + a\sin\theta \qquad\qquad a - a\sin\theta$$

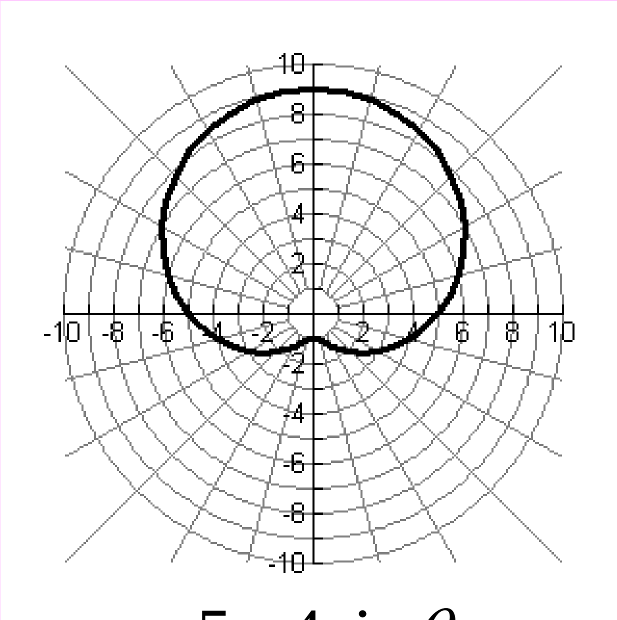
$$4 + 4\sin\theta$$



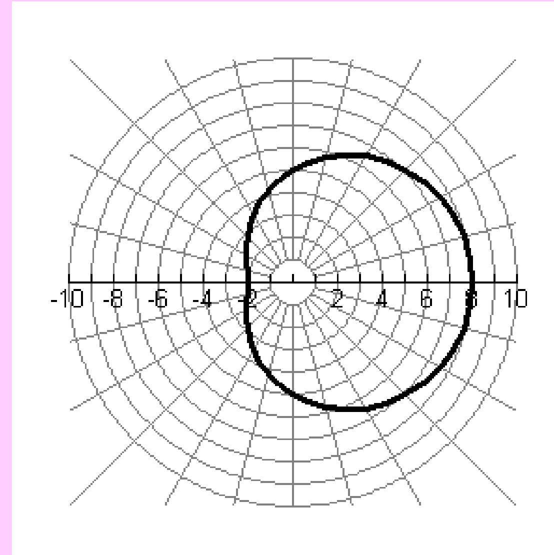
$$4 - 4\sin\theta$$



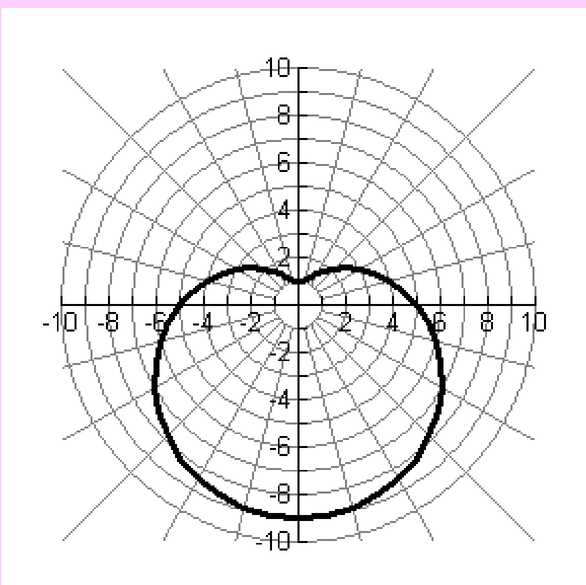
$r = 5 + 4\sin\theta$



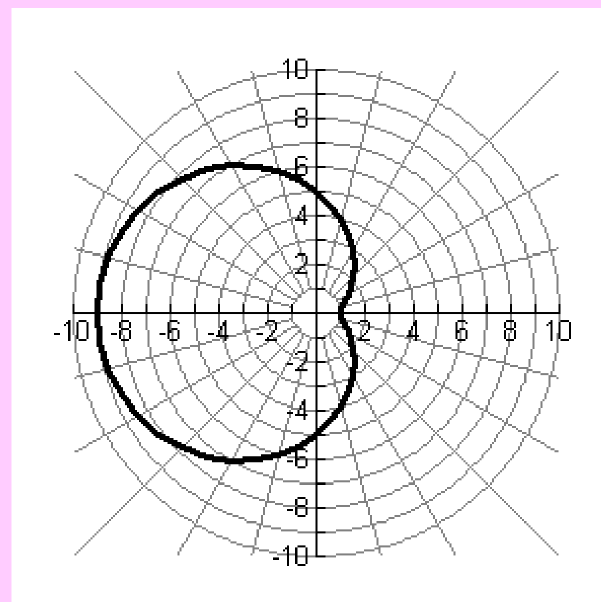
$r = 5 + 3\cos\theta$



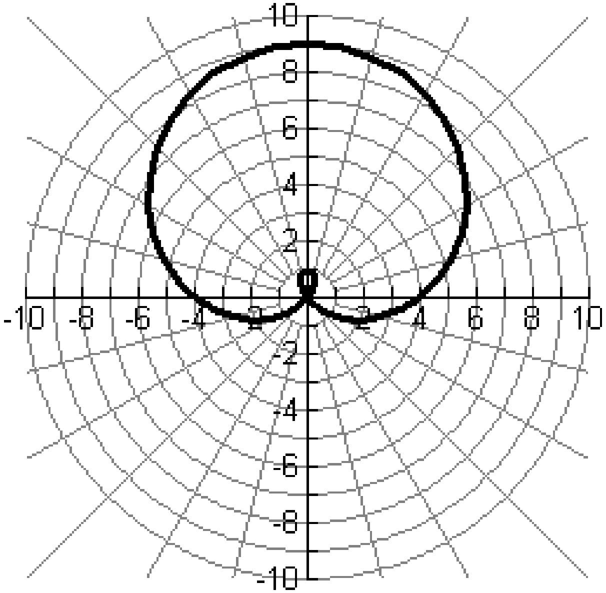
$r = 5 - 4\sin\theta$



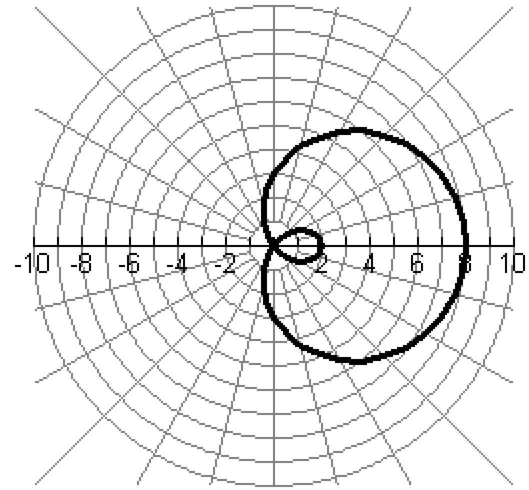
$r = 5 - 4\cos\theta$



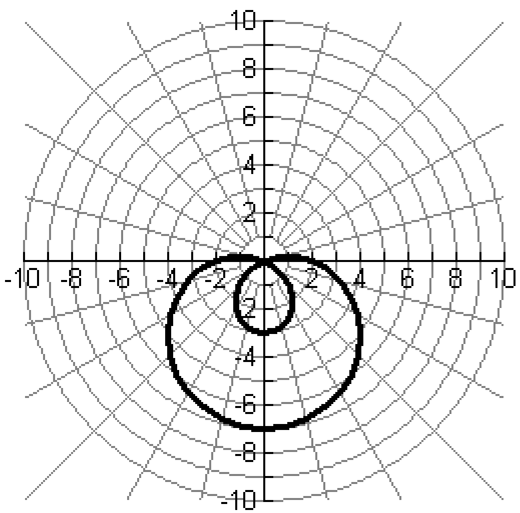
$$r = 4 + 5\sin\theta$$



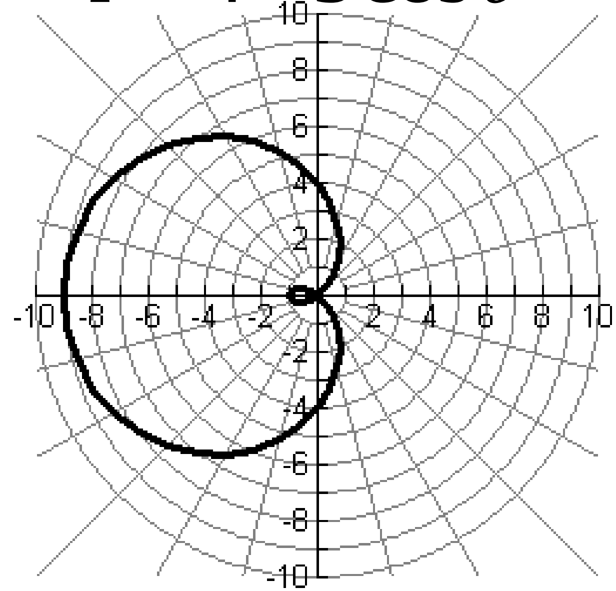
$$r = 3 + 5\cos\theta$$



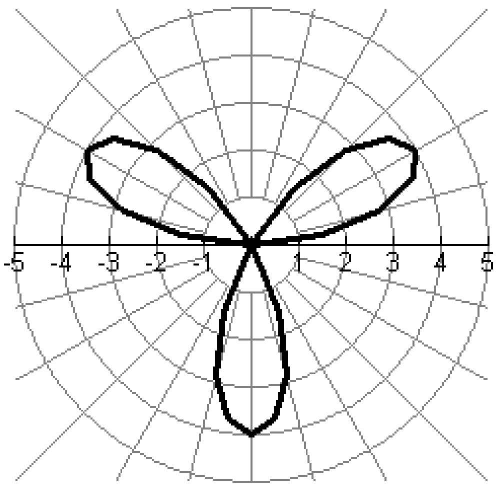
$$r = 2 - 5\sin\theta$$



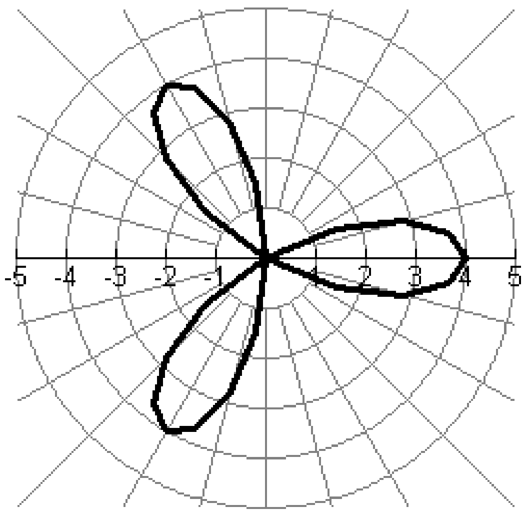
$$r = 4 - 5\cos\theta$$



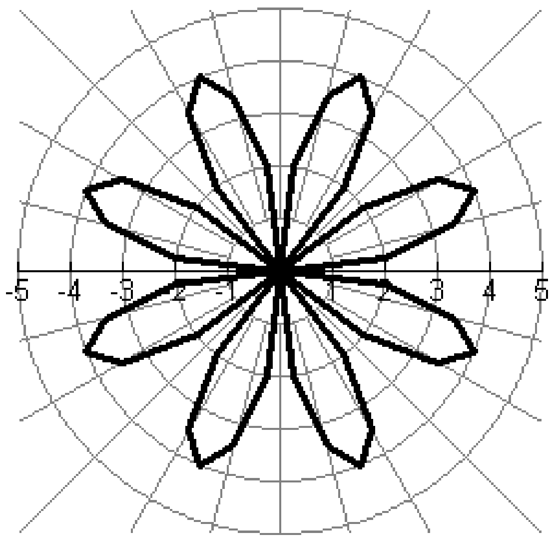
$$r = 4\sin 3\theta$$



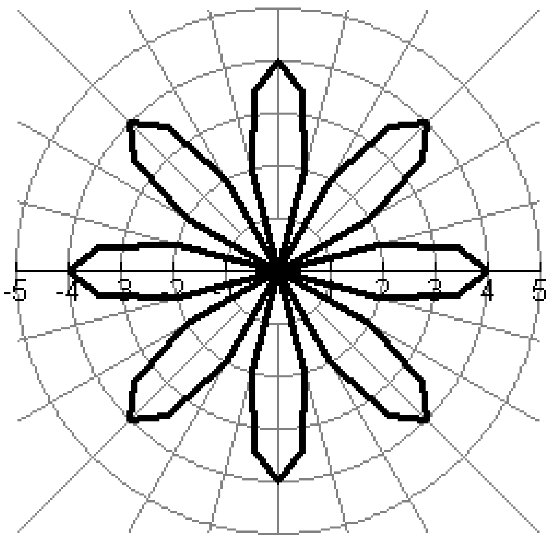
$$r = 4\cos 3\theta$$



$$r = 4\sin 4\theta$$

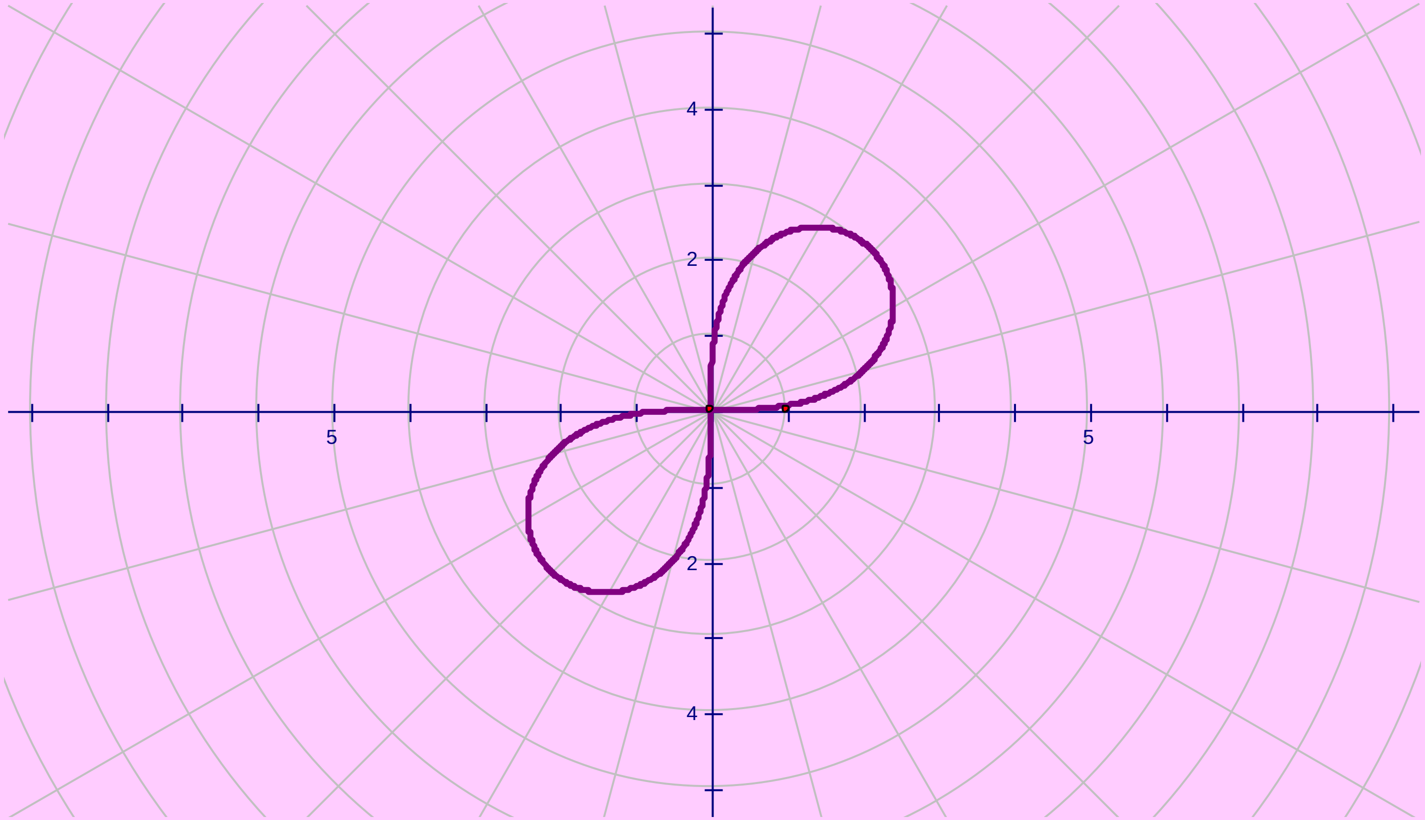


$$r = 4\cos 4\theta$$



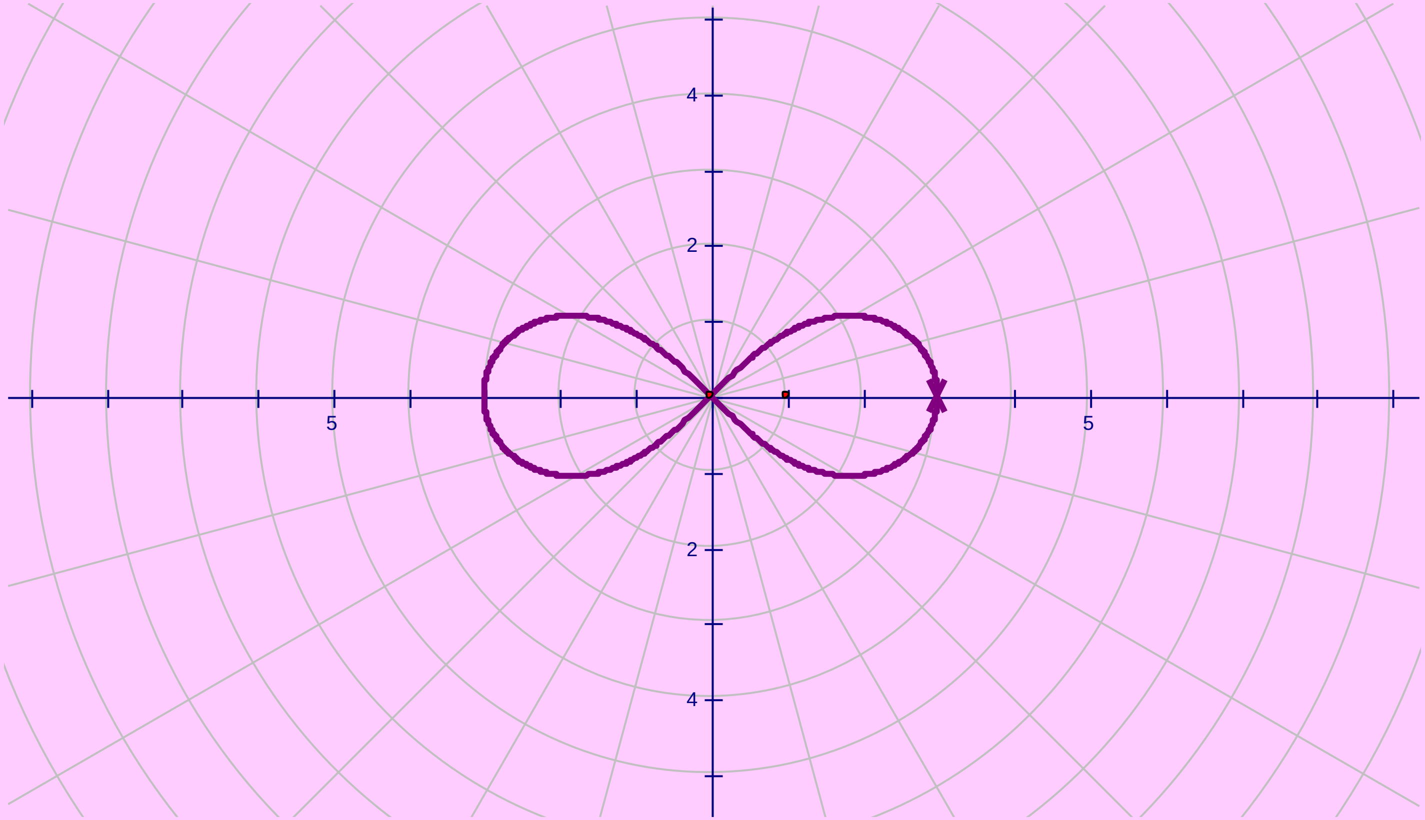
$$r^2 = 9\sin 2\theta$$

Not on our quiz!

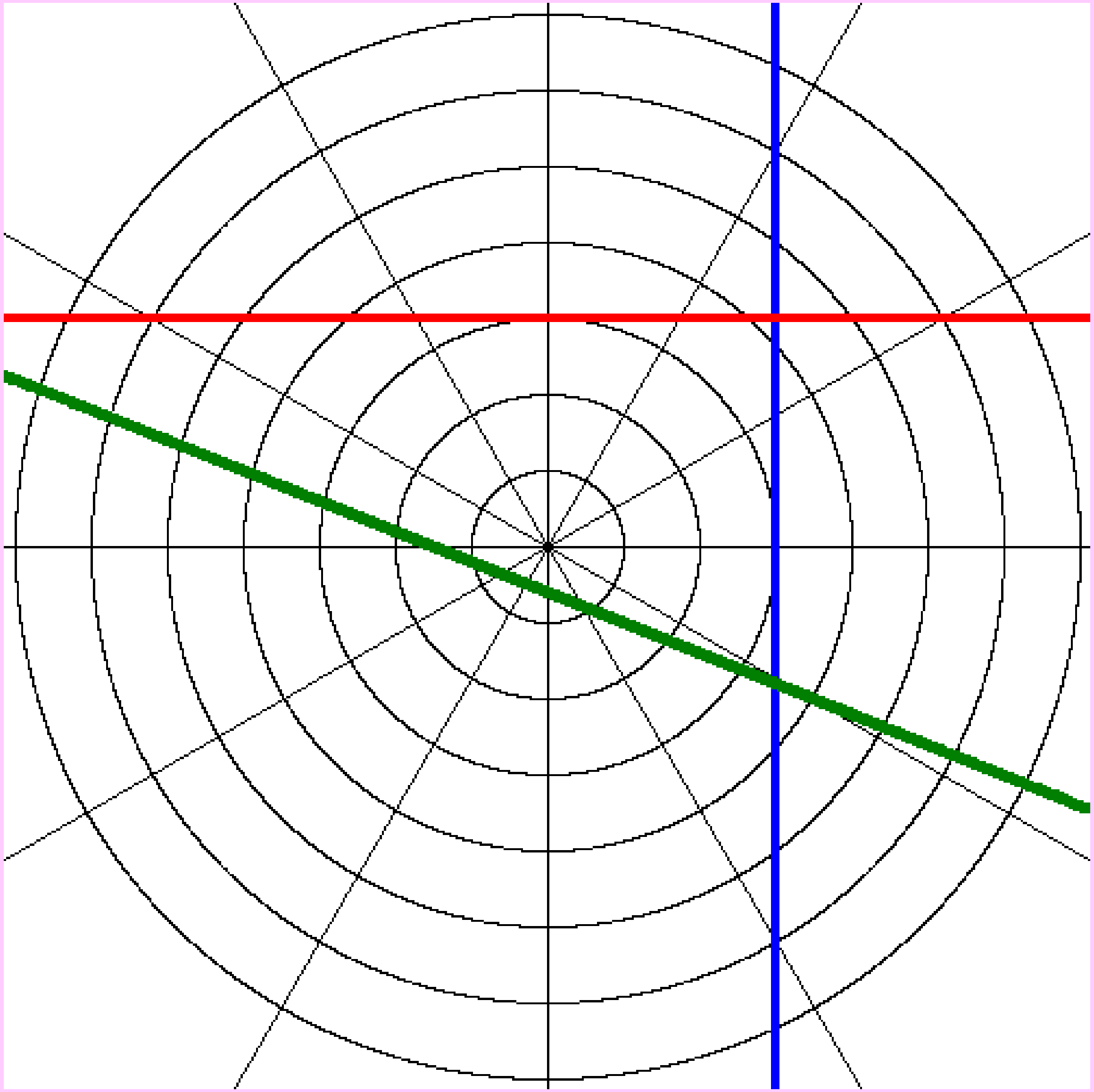


Not on our quiz!

$$r^2 = 9\cos 2\theta$$



Can you graph each equation on the same graph?



$$r\cos\theta = 3$$

$$r = \frac{3}{\cos\theta}$$

Vertical Line

$$r\sin\theta = 3$$

$$r = \frac{3}{\sin\theta}$$

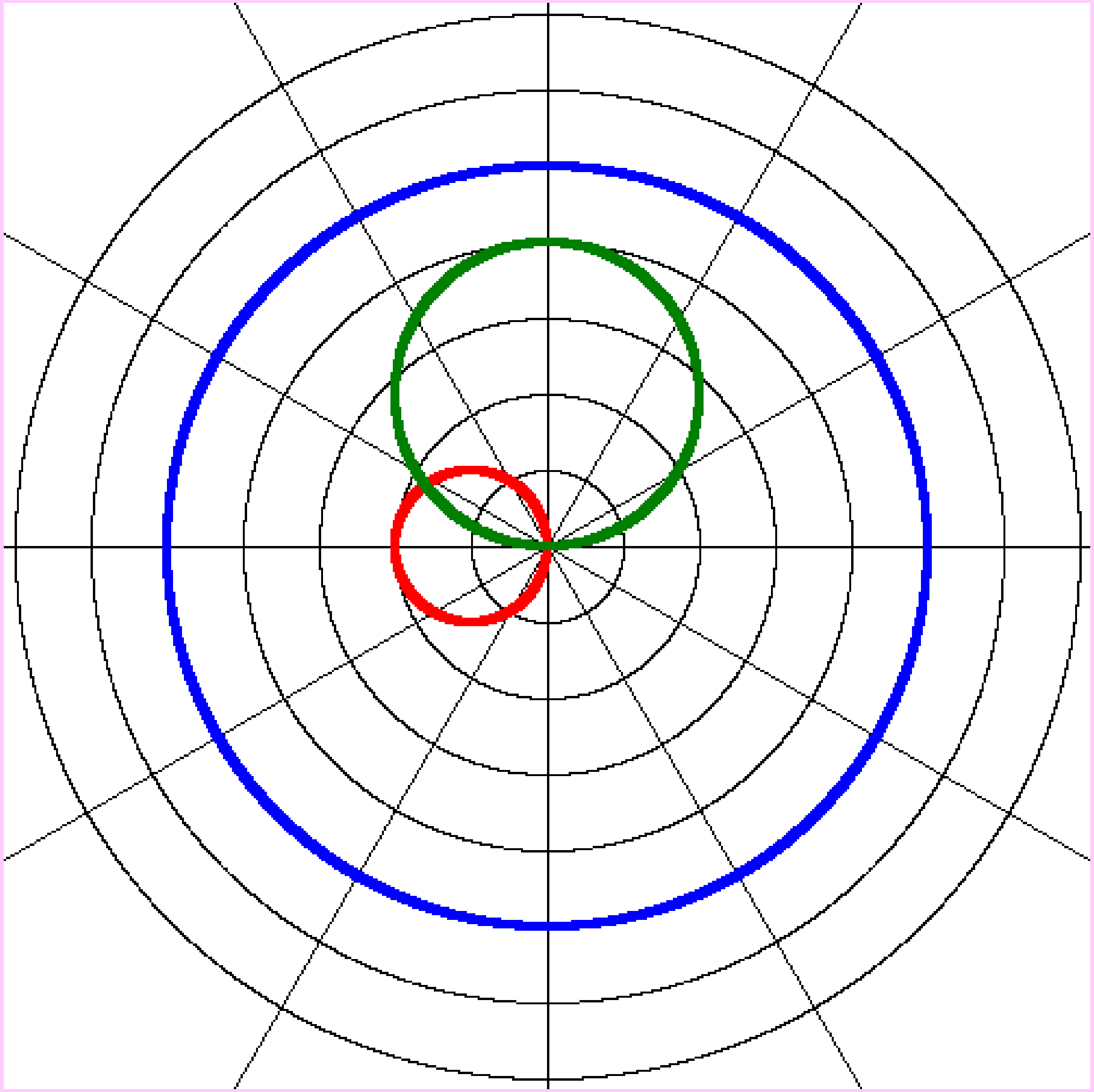
Horizontal Line

$$r\left(2\cos\theta + 5\sin\theta\right) + 3 = 0$$

$$r = \frac{3}{2\cos\theta + 5\sin\theta}$$

General Line

Can you graph each circle on the same graph??

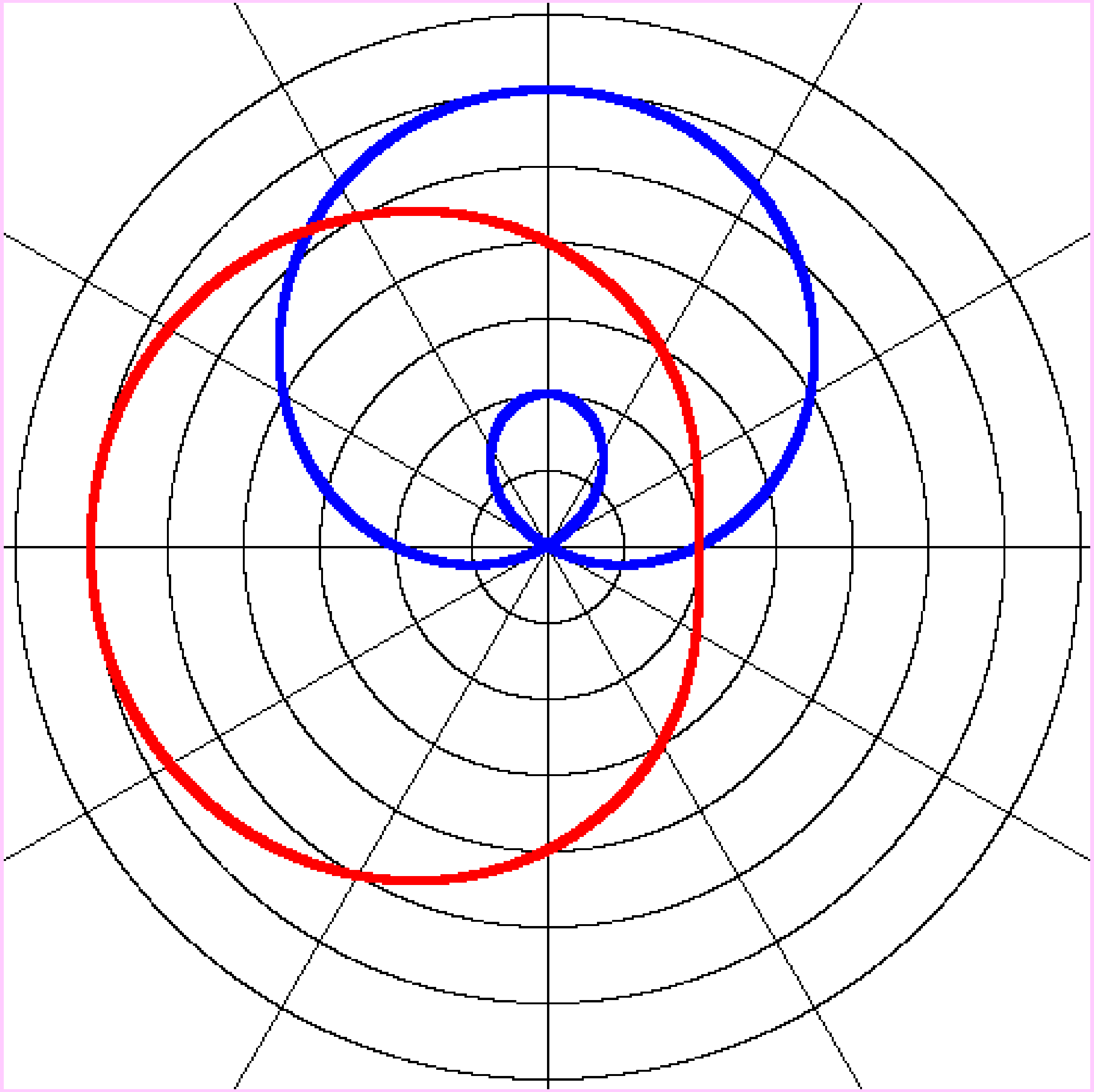


$r = 5$

$r = -2\cos\theta$

$r = 4\sin\theta$

Can you graph both equations on the same graph??



$r = 2 + 4\sin\theta$

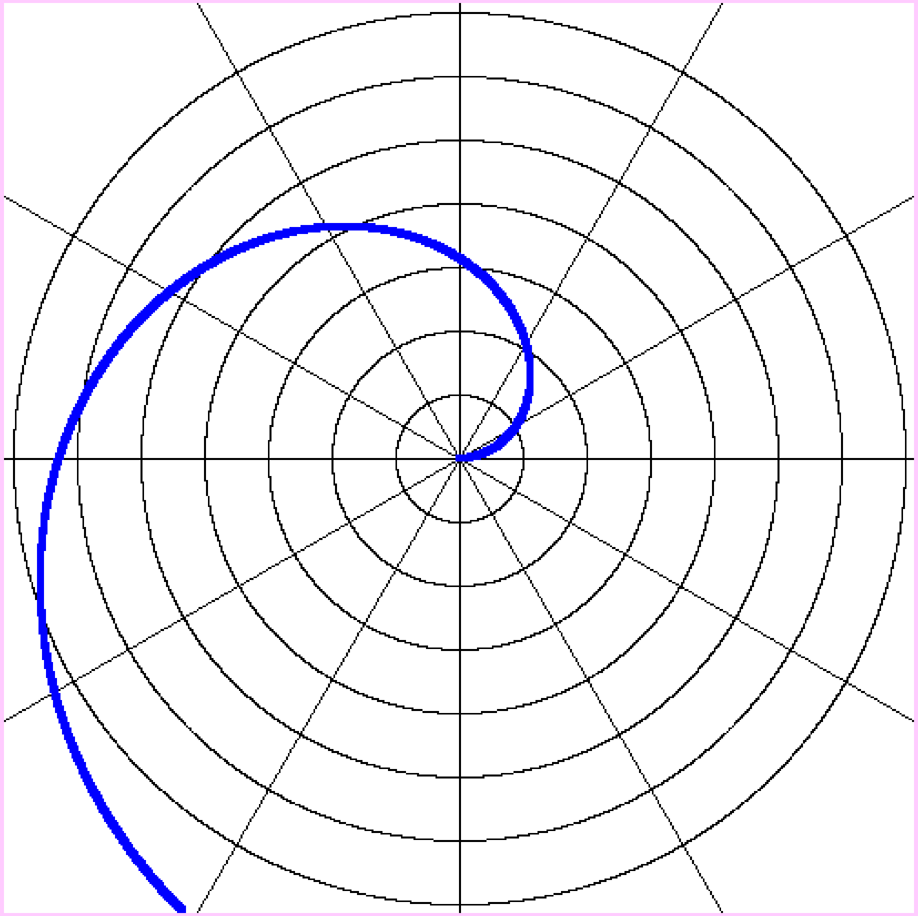
$r = 4 - 2\cos\theta$

$r = a \pm b\sin\theta$

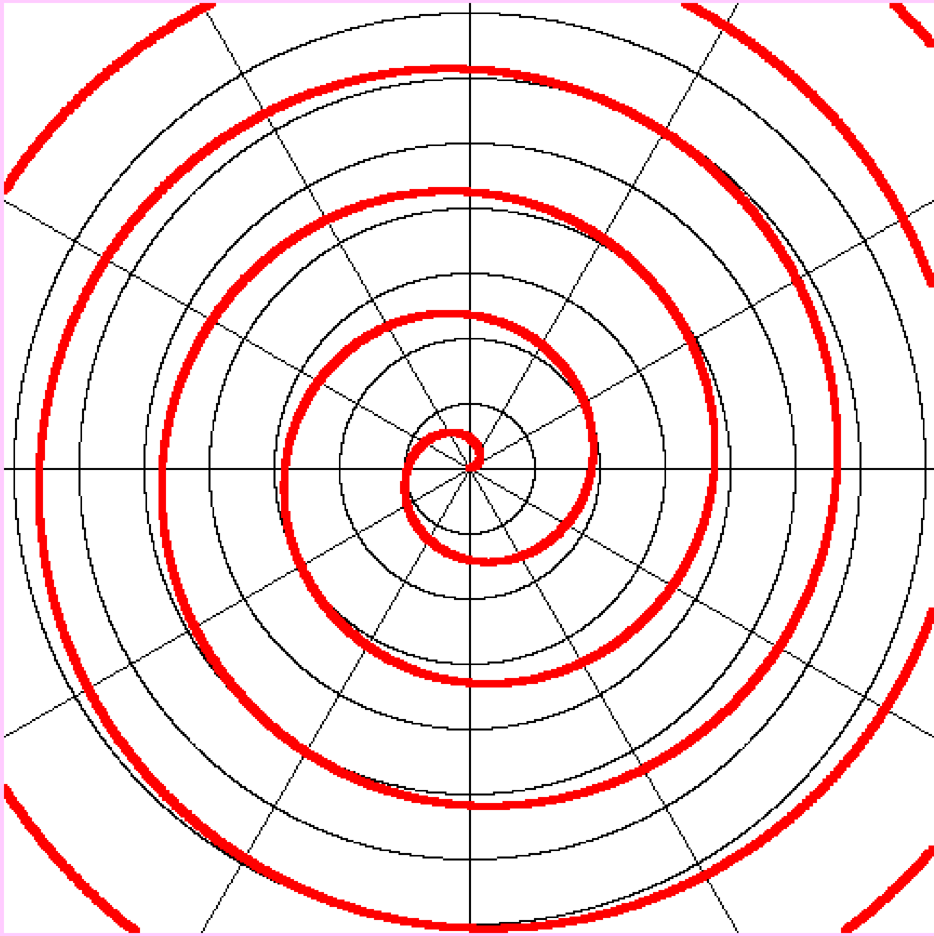
$r = a \pm b\cos\theta$

Note: INNER loop
Only if $a < b$

Can you graph each spriral below?

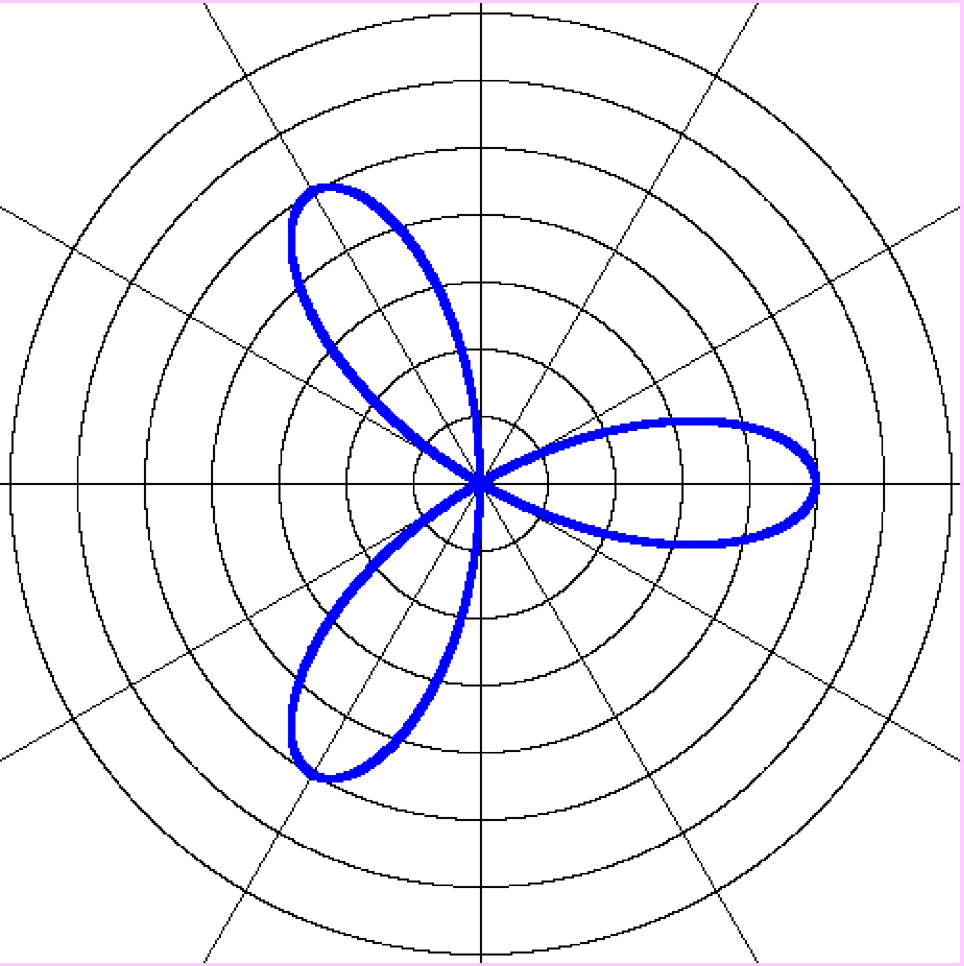


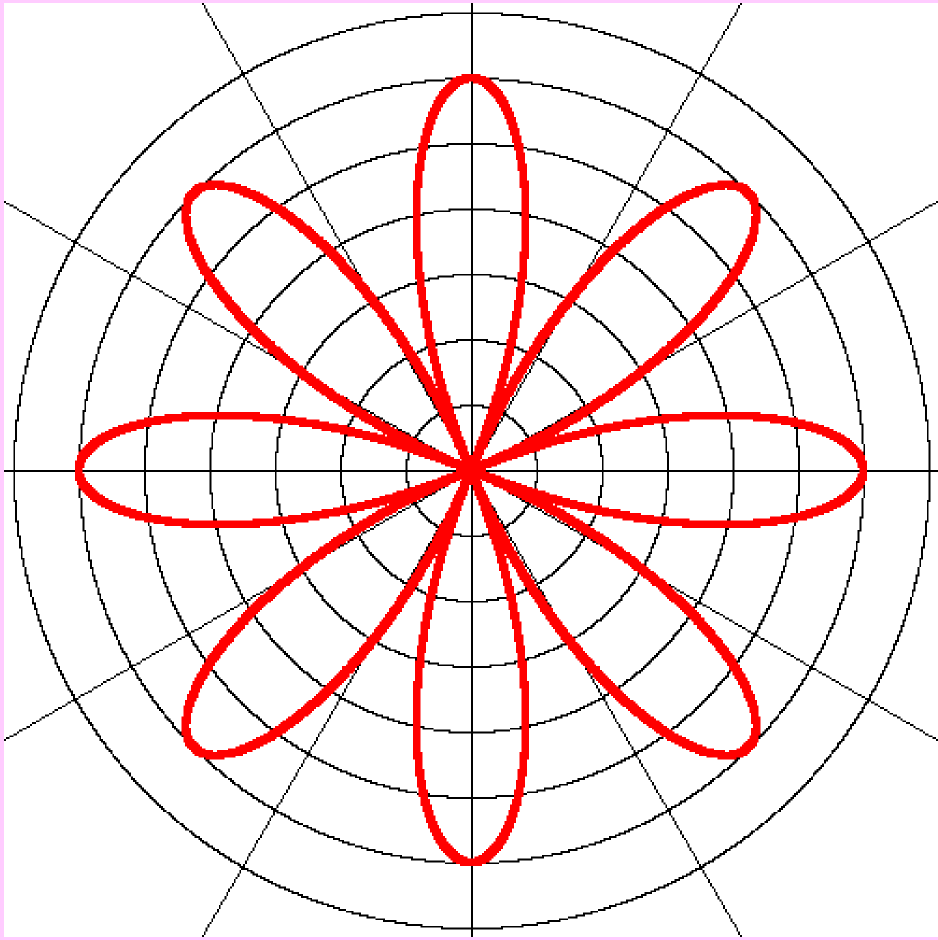
$$r = 2\theta$$



$$r = \frac{\theta}{3}$$

Graph each equation





$r = 5\cos 3\theta$

$r = 6\sin 4\theta$

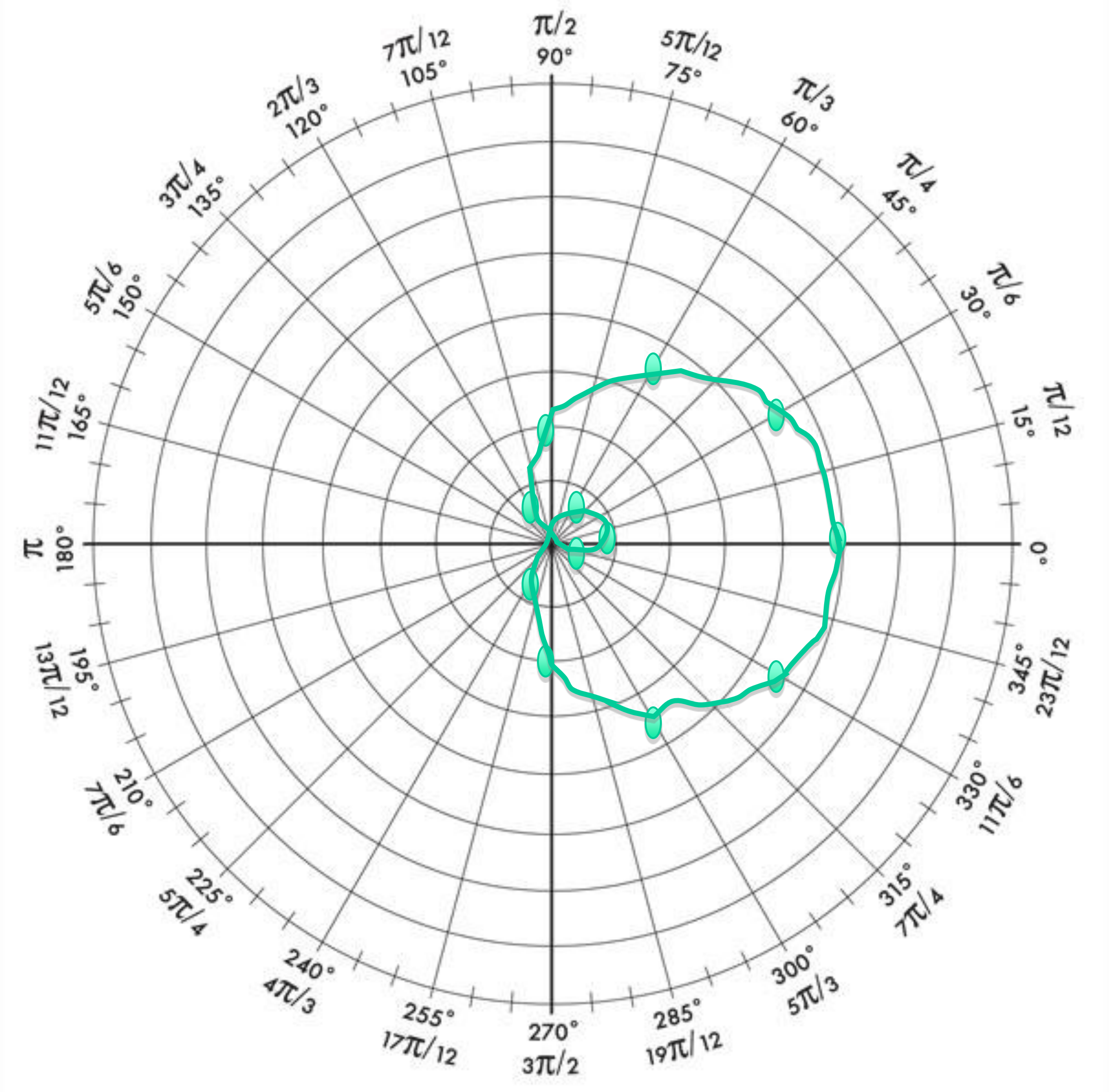
$r = a\sin(n\theta)$ n even – 2n pedals

$r = b\sin(n\theta)$ n odd – n pedals

Graph: $r = 2 + 3\cos\theta$

LIMACON WITH INNER LOOP

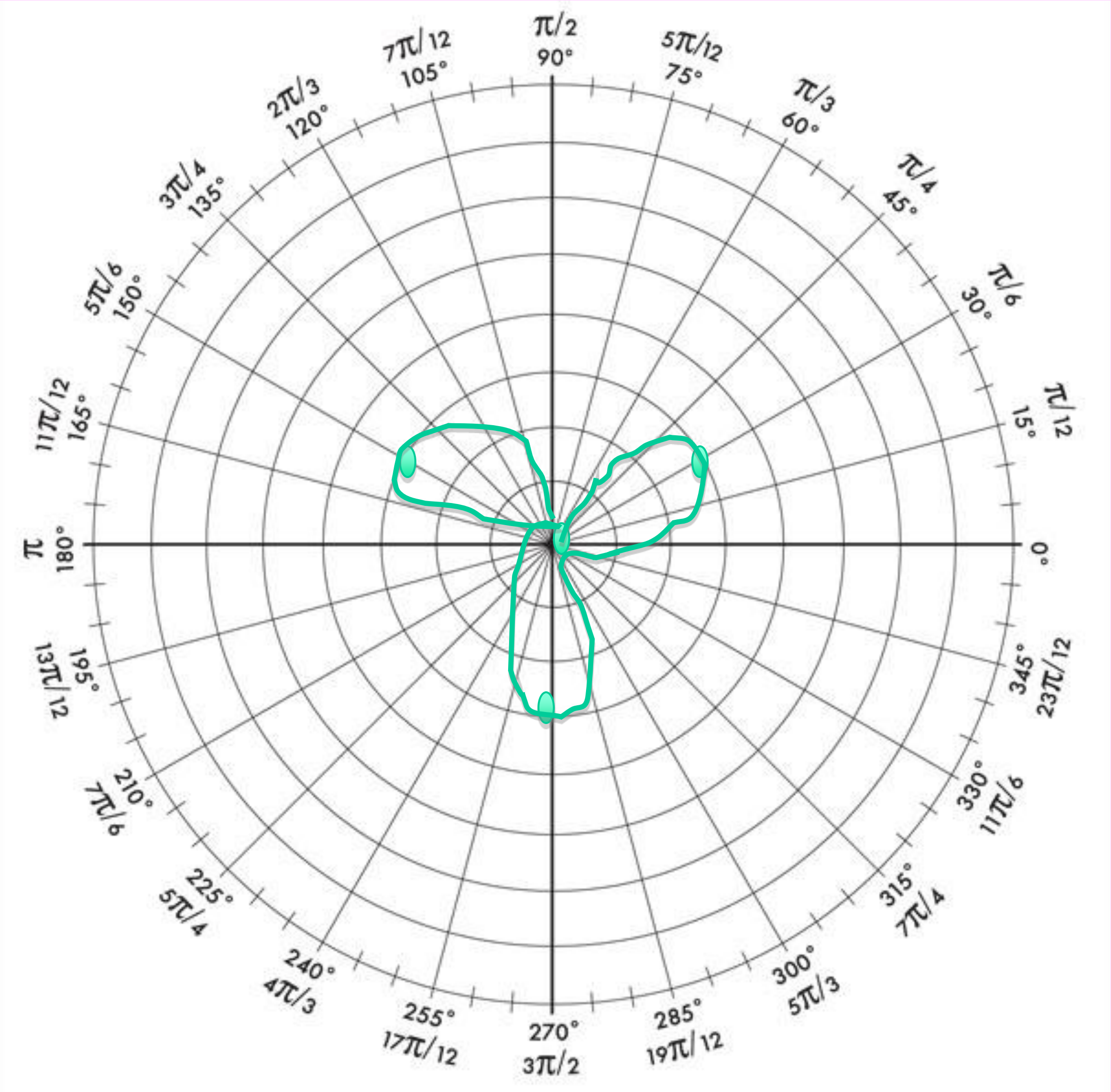
θ	r
0˚	5
30˚	4.6
60˚	3.5
90˚	2
120˚	0.5
150˚	-0.6
180˚	-1
210˚	-0.6
240˚	0.5
270˚	2
300˚	3.5
330˚	4.6
360˚	5



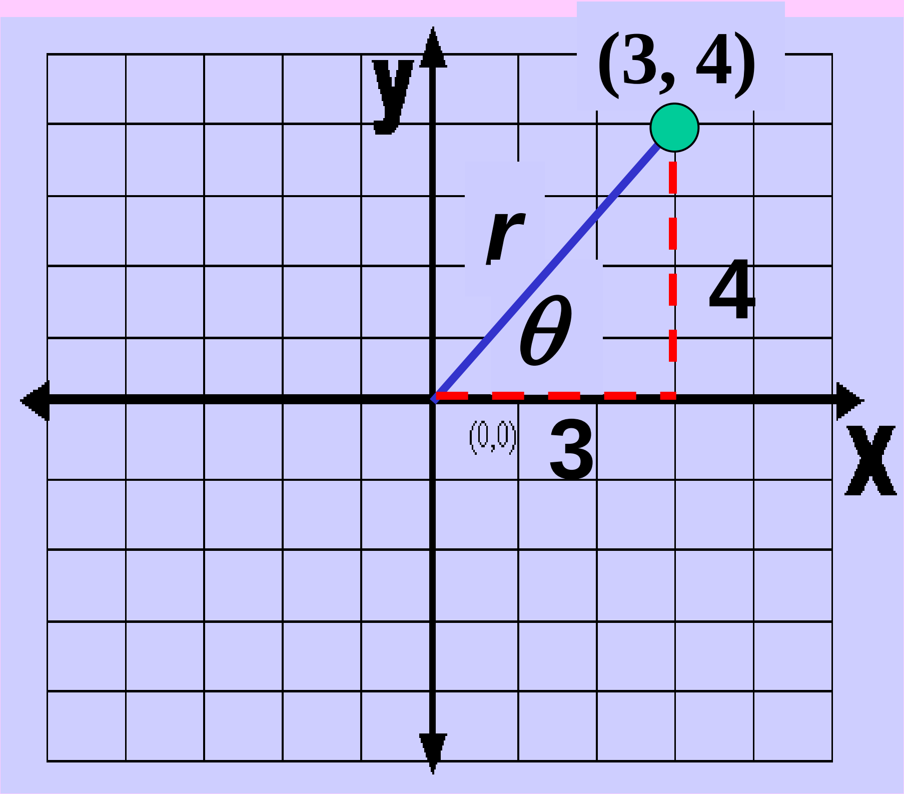
Graph: $r = 3\sin 3\theta$

Rose with 3 petals

θ	r
0°	0
30°	3
60°	0
90°	-3
120°	0
150°	3
180°	0
210°	-3
240°	0
270°	3
300°	0
330°	-3
360°	0



Let's take a point in the rectangular coordinate system and convert it to the polar coordinate system.



Based on the trig you know can you see how to find *r* and *θ*?

$$3^2 + 4^2 = r^2$$

$$\boldsymbol{r = 5}$$

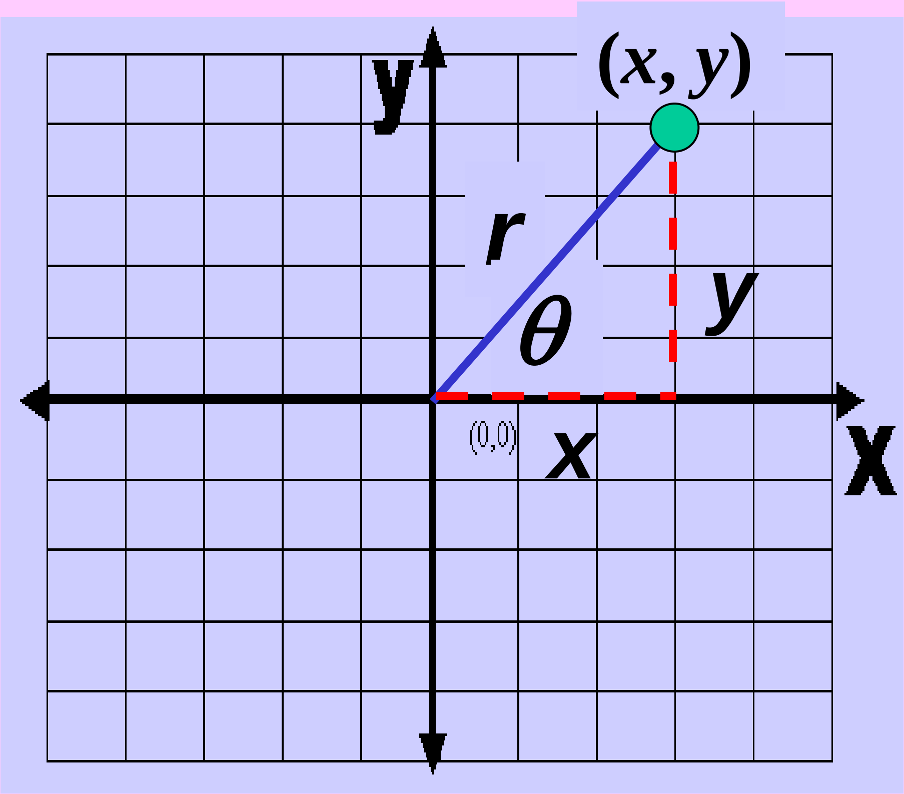
$$\tan\theta = \frac{4}{3}$$

We'll find θ **in radians**

$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 0.93$$

polar coordinates are: $(5, 0.93)$

Let's generalize this to find formulas for converting from rectangular to polar coordinates.



$$x^2 + y^2 = r^2$$

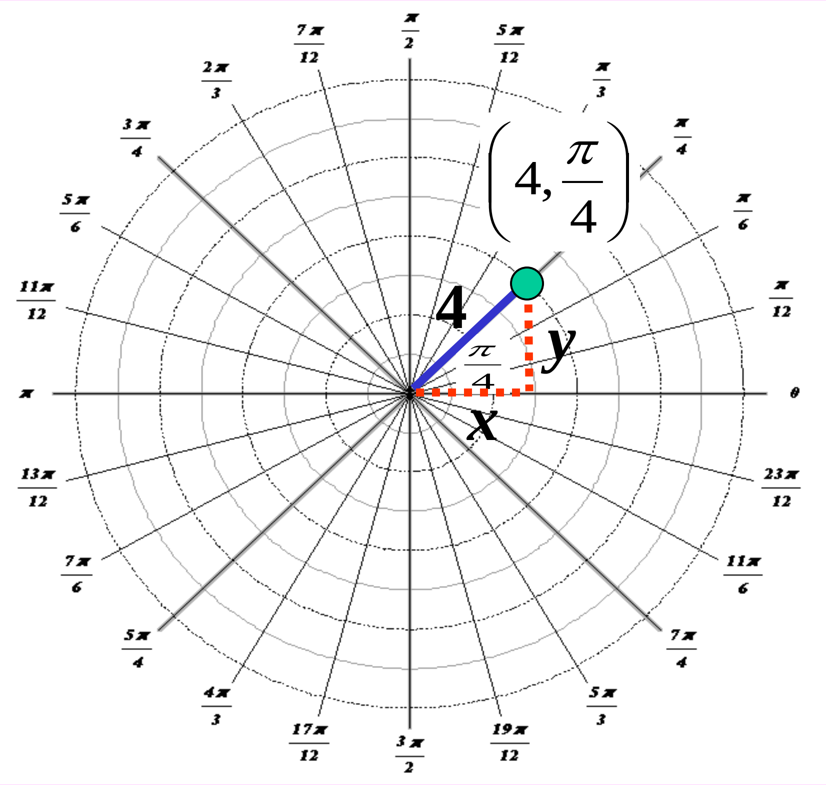
$$r = \sqrt{x^2 + y^2}$$

$$\tan\theta = \frac{y}{x}$$

If x > 0 $\theta = \tan^{-1}\left(\frac{y}{x}\right)$ or

$\theta = \tan^{-1}\left(\frac{y}{x}\right) + \pi$ If x < 0

Now let's go the other way, from polar to rectangular coordinates.



Based on the trig you know can you see how to find *x* and *y*?

$$\cos\frac{\pi}{4} = \frac{x}{4}$$

$$x = 4\left(\frac{\sqrt{2}}{2}\right) = 2\sqrt{2}$$

$$\sin\frac{\pi}{4} = \frac{y}{4}$$

$$y = 4\left(\frac{\sqrt{2}}{2}\right) = 2\sqrt{2}$$

rectangular coordinates are:

$$\boxed{\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)}$$

Convert each of these rectangular coordinates to polar coordinates:

1.) $(0, 4)$

2.) $\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$

$\left(4, \pi/2\right)$

$\left(1, -\pi/3\right)$

3.) $\left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right)$

4.) $(4, 0)$

$\left(1, 5\pi/6\right)$

$(4, 0)$

5.) $\left(-1, -\sqrt{3}\right)$

6.) $(2, 2)$

$\left(2, 4\pi/3\right)$

$\left(2\sqrt{2}, \pi/4\right)$

Convert each of these polar coordinates to rectangular coordinates:

1.) $(6, 120°)$

$(-3, 3\sqrt{3})$

2.) $(-4, 45°)$

$(-2\sqrt{2}, -2\sqrt{2})$

3.) $(3, 300°)$

$\left(\frac{3}{2}, \frac{-3\sqrt{3}}{2}\right)$

4.) $(4, \pi/6)$

$(2\sqrt{3}, 2)$

5.) $(0, 13\pi/3)$

$(0, 0)$

6.) $(3, -3\pi/4)$

$\left(\frac{-3\sqrt{2}}{2}, \frac{-3\sqrt{2}}{2}\right)$

Steps for Converting Equations from Rectangular to Polar form and vice versa

Four critical equivalents to keep in mind are:

$$x = r\cos\theta \qquad\qquad y = r\sin\theta$$

$$r^2 = x^2 + y^2$$

$$\theta = Arc\tan\left(\frac{y}{x}\right) \qquad\qquad \theta = Arc\tan\left(\frac{y}{x}\right) + \pi$$

If x > 0 | If x < 0

Convert the equation: $r = 2$ to rectangular form

Since we know that $r^2 = x^2 + y^2$, square both sides of the equation.

$$r^2 = 4$$

$$\boxed{x^2 + y^2 = 4}$$

Transform the equation $r = \cos\theta - \sin\theta$ from polar coordinates to rectangular coordinates.

$$r = \cos\theta - \sin\theta$$

We still need r^2, but is there a better choice than squaring both sides?

$$r^2 = r\cos\theta - r\sin\theta$$

$$x^2 + y^2 = x - y$$

$$x^2 - x + y^2 + y = 0$$

Convert the following equation from rectangular to polar form.

$$x^2 + y^2 = x$$

Since

$$r^2 = x^2 + y^2$$

and

$$x = r\cos\theta$$

$$r^2 = r\cos\theta$$

$$r = \cos\theta$$

Convert the following equation from rectangular to polar form.

$$2x^2 + 2y^2 = 3$$

$$2(x^2 + y^2) = 3$$

$$x^2 + y^2 = \frac{3}{2}$$

$$r^2 = \frac{3}{2}$$

$$r = \pm\sqrt{\frac{3}{2}}$$

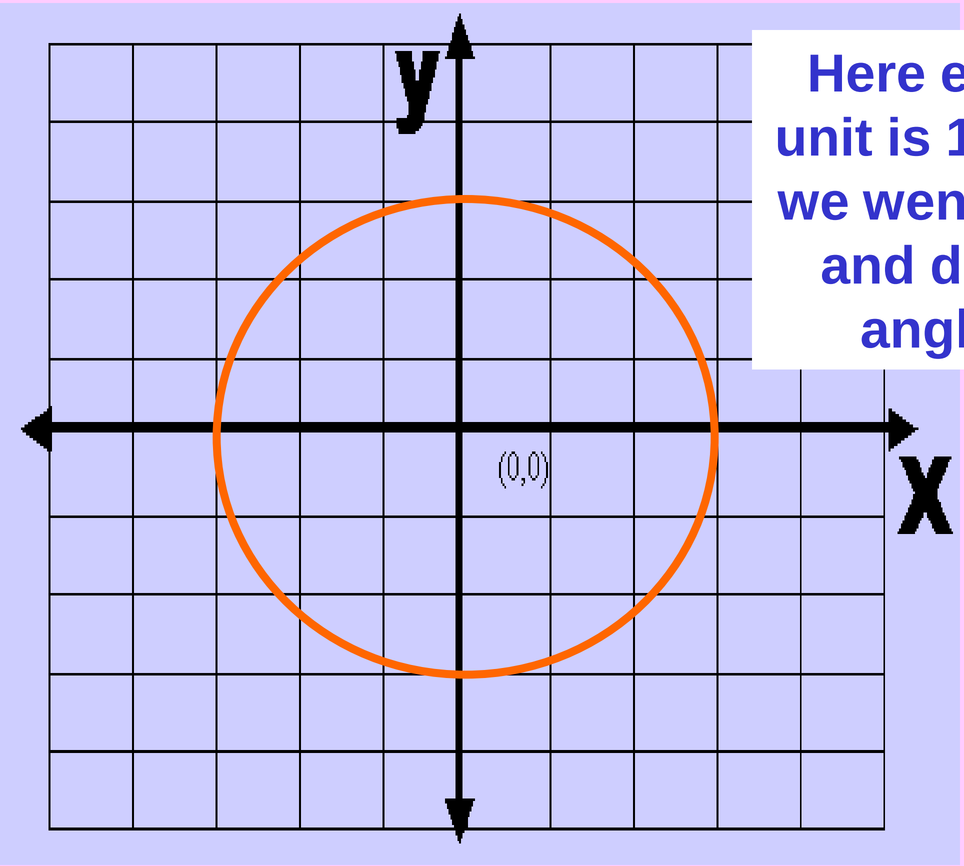
Convert the rectangular coordinate system equation to a polar coordinate system equation.

$$\sqrt{x^2+y^2}=\pm\sqrt{9}$$

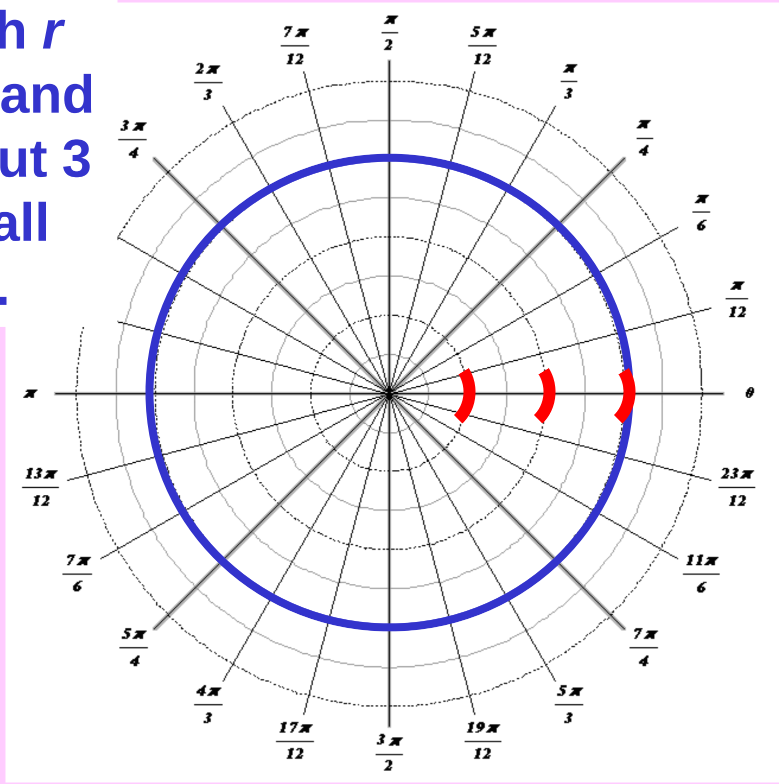
From conversions, how was r related to x^2 and y^2?

$$r=\sqrt{x^2+y^2}$$

$$r=\pm 3$$



Here each *r* unit is 1/2 and we went out 3 and did all angles.



Before we do the conversion let's look at the graph.

***r* must be $\pm$ 3 but there is no restriction on θ so consider all values.**

Convert the rectangular coordinate system equation to a polar coordinate system equation.

$$x^2 = 4y$$

What are the polar conversions we found for *x* and *y*?

$$x = \boxed{r\cos\theta}$$

$$y = \boxed{r\sin\theta}$$

substitute in for *x* and *y*

$$(r\cos\theta)^2 = 4(r\sin\theta)$$

$$r^2\cos^2\theta = 4r\sin\theta$$

$$r\cos^2\theta = 4\sin\theta$$

$$r = \frac{4\sin\theta}{\cos^2\theta}$$

$$r = 4\tan\theta\sec\theta$$

WRITE EACH EQUATION IN POLAR FORM:

1.) $2 = y - 2x$

2.) $x = 11$

$$\frac{2\sqrt{5}}{5} = r\cos(\theta - 153°)$$

$$11 = r\cos\theta$$