PREDICTION OF SAND PRODUCTION FROM A SAUDI SANDSTONE RESERVOIR

M. N. AL-AWAD and S. E. M. DESOUKY

Petroleum Engineering Department,
College of Engineering[1]

PRÉVISION DE LA PRODUCTION DE SABLE POUR UN RÉSERVOIR GRÉSEUX D'ARABIE SAOUDITE

La production de sable est un phénomène rencontré dans certains gisements pétroliers saoudiens. L'étude a porté sur six échantillons de sable provenant de différents puits d'exploitation d'un réservoir gréseux. Des échantillons de grès issus de ce même réservoir ont été soumis à des essais de compression uniaxiale et triaxiale. Les débris des échantillons de grès et les échantillons de sable ont fait l'objet d'un examen minéralogique par diffractométrie aux rayons X et granulométrie sur tamis standards. Les méthodes d'analyse statistique ont été employées pour vérifier si la différence statistique entre les échantillons de sable provenant des puits et les débris des échantillons de grès est significative ou non. On a également calculé les taux critiques de production du gisement saoudien pour différents angles d'inclinaison des puits. Les résultats font apparaître qu'il n'existe pas de différence significative entre les échantillons de sable et les débris de grès, avec un niveau de confiance de 95 %. Deux mécanismes de rupture évidents, traction et cisaillement, sont responsables de la production de sable par les puits de ce réservoir. On estime que la production maximale sans sable pour le réservoir étudié varie entre 960 et 4080 barils par jour.

PREDICTION OF SAND PRODUCTION FROM A SAUDI SANDSTONE RESERVOIR

Sand production is encountered in some Saudi oil fields. Six sand samples produced from different wells in a Saudi oil reservoir were obtained. Sandstone samples obtained from the same reservoir were subjected to uniaxial and triaxial failure tests. The debris produced from the sandstone samples and the six sand samples were characterized for their mineralogy using X-ray diffractometer and grain size distribution using standard sieves. Statistical analyses were employed to check whether a statistical difference between the sand samples produced from oil wells and debris collected from the failed sandstone specimens is significant or not. The critical oil rates of the Saudi oil reservoir were also calculated for different well inclination angles. Results show that, no significant statistical difference between the sand samples and debris exists at a confidence level of 95%. Two obvious failure mechanisms, splitting and shear failure, are responsible for sand production from the studied Saudi oil reservoir. The maximum sand-free production for the studied oil reservoir range from 960 to 4080 barrels per day.

PREVISIÓN DE LA PRODUCCIÓN DE ARENA PARA UN DEPÓSITO QUE CONTIENE ARENISCA EN ARABIA SAUDÍ.

La producción de arena constituye un fenómeno con que se tropieza en ciertos yacimientos petrolíferos de Arabia Saudí. El

(1) King Saud University,
PO Box 800, Riyadh 11421 - Saudi Arabia

estudio se refiere a seis muestras de arena procedentes de diversos pozos de explotación de un depósito con cierto contenido de arenisca. Se han sometido a pruebas de compresión uniaxial y triaxial diversas muestras de arenisca procedentes de este mismo depósito. Los residuos de las muestras de arenisca y las muestras de arena han sido objeto de un examen mineralógico por difractometría de rayos X y granulometría en tamices estándar. Se han aplicado los métodos de análisis estadístico para verificar si la diferencia estadística entre las muestras de arena procedentes de los pozos y los residuos de arenisca es o no significativa. También se han calculado los valores críticos de producción del yacimiento saudí para distintos ángulos de inclinación de los pozos. Los resultados obtenidos indican de no existe ninguna diferencia significativa entre las muestras de arena y los residuos de arenisca, y ello con un nivel de confianza de un 95 %. Dos mecanismos de ruptura evidentes, escisión y cizallamiento, son responsables de la producción de arena por los pozos de este depósito. Se piensa que la producción máxima sin arena para el depósito estudiado oscila entre 960 y 4080 barriles diarios.

INTRODUCTION

Sand production is the production of small or large amounts of sand together with the reservoir fluids. These amounts of produced sand vary from a few grams or less per ton of produced reservoir fluid to huge amounts possibly leading to complete filling of the borehole [1]. When oil or gas wells are drilled into unconsolidated or poorly cemented sandstone reservoirs, the decision about the need for sand control is almost clear. However, the decision is difficult in moderately hard and competent sandstone reservoirs. If sanding occurs and no sand control is implemented, potential problems associated with sand production arise. Typical problems associated with the sand production are [2]:

- wear of downhole and surface equipment,
- borehole instability,
- casing collapse,
- workover and sand separation costs,
- environmental problems of disposing dirty sand.

Sand production is considered as reservoir permeability self enhancement. There are three possible ways for reservoir permeability enhancement caused by sand production [3]:

- the establishment of new flow channels,
- the initiation of localized shear surfaces,
- the dilatancy over a large volume of the reservoir.

Production of reservoir fluids at high rates causes an increase in the induced effective tangential stresses concentrated on the face of an open hole or on the walls of perforations in a cased borehole. If these induced stresses exceed formation *in situ* strength, then the formation will fail and sand will be produced form the initiated failure surfaces.

In the present work, failure mechanisms causing sand production from a Saudi oil reservoir are experimentally determined. The critical production rates, above which sand is produced from the studied reservoir are also calculated.

1 FAILURE MECHANISMS CAUSING SAND PRODUCTION

Sand produced from unconsolidated or poorly cemented sandstone reservoirs due to perforation or cavity failure is known as a sand arch failure mechanism [4] as shown in Figure 1.

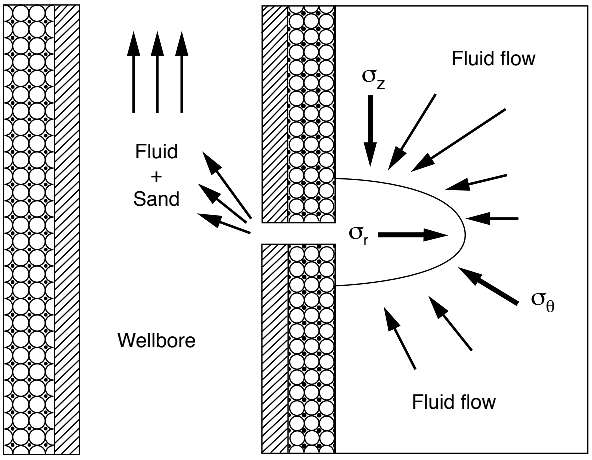

Figure 1

Sand arch failure mechanism [4].

In a sand arch failure mechanism, the induced tangential stress is concentrated on the walls of the perforation due to the lack of supporting pressure that is initially provided by the reservoir fluid pressure. The near borehole fluid pressure declines too fast when high production rate is continued. This leads to perforation collapse or detachment of thin shells of sand inside the perforation cavity. In addition, the collapsed sand is moved along with the reservoir fluids towards the wellbore. The problem of sand arch failure mechanism has been studied analytically and experimentally by several researchers [5-11]. Whilst sand production is mostly associated with unconsolidated or poorly cemented sandstone reservoirs, the production of solids has also been observed from formations which could be considered strong and competent [12-14]. Failure mechanism in competent sandstones is different from the sand arch failure mode which prevails in unconsolidated and poorly cemented sandstones, therefore it needs to be investigated.

2 MATERIALS AND TESTING SET-UP

Six produced sand and sandstone core samples were obtained from a Saudi oil reservoir. A compression machine equipped with Hoek cell and constant confining pressure system was used to measure the sandstone mechanical properties and to establish its failure criteria. The tests were conducted according to the standard procedures outlined by the *International Society for Rock Mechanics* for samples preparation and testing procedures [15]. The obtained sand samples and debris produced from compressive tests were characterized for their mineralogy using X-ray diffractometer and grain size distribution using standard sieves. To investigate the source of sand production in this Saudi sandstone reservoir, thirteen sandstone specimens were employed in this study. These specimens were characterized for their mechanical properties as follows:

– Three specimens (0.8 × 1.5 inch) for the indirect tensile strength measurements.
– Three specimens (1.5 × 3.5 inch) for the uniaxial compressive strenght measurements.
– Seven specimens (1.5 × 3.5 inch) for the triaxial compressive strength measurements.

3 STATISTICAL ANALYSIS

As in the case of most experimental work, the t-test is used to check whether a statistical difference between two sets of data is significant or not. This can be carried out by calculating the value of (t_c) from the following equation [16]:

$$t_c = \left\{ \left|\overline{X}_1 - \overline{X}_2\right| / \left[S(x) \sqrt{\frac{1}{n_1} - \frac{1}{n_2}} \right] \right\} \quad (1)$$

where S(x) is given by

$$S(x) = \left\{ \left[S_1^2 \left(n_1 - 1\right) + S_2^2 \left(n_1 - 1\right) \right] / \left(n_1 + n_2 - 2\right) \right\}^{0.5} \quad (2)$$

The value of (t_c) calculated from Eq. (1) is then compared with the value of (t_t) obained from statistical tables at a specified confidence level. If (t_c) is equal to or less than (t_t), no significant statistical difference between the two sets of data exists and they can be grouped into one set of data.

4 RESULTS AND DISCUSSION

The granulometric analysis of the sand samples and debris obtained from compressive tests are plotted in Figure 2. These histograms ensure that the grain sizes are uniformly distributed and lie between 600 and 40 μm. Eq. (1) was used to calculate (t_c) for these data.

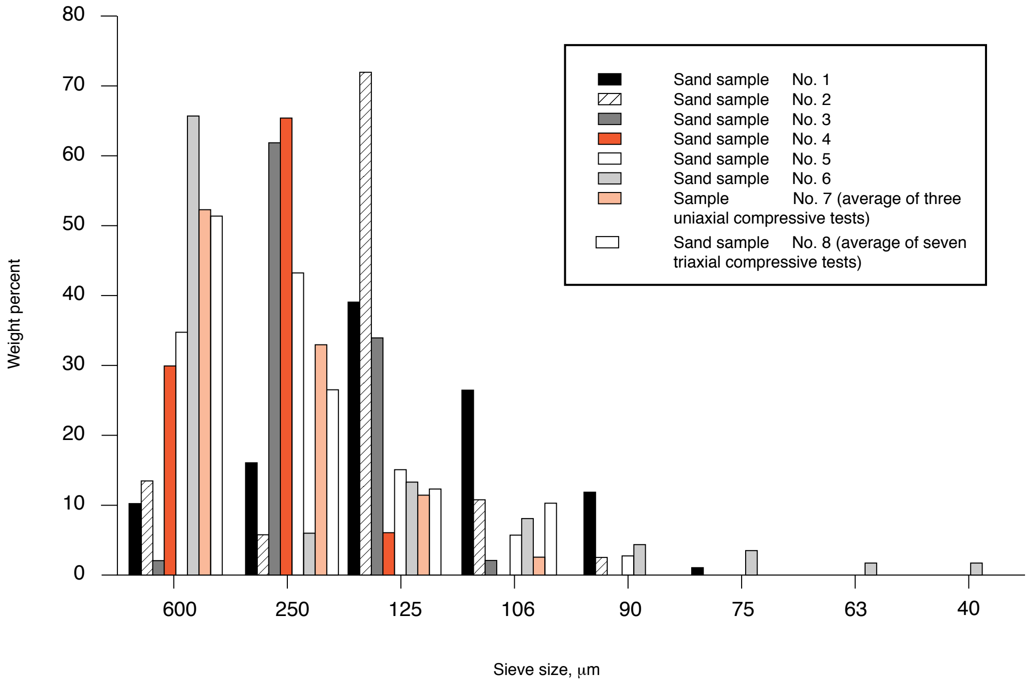

Figure 2

Histograms of sand samples and debris.

The values of t-test are given in Table 1. This table shows that, the values of (t_c) are less than that of (t_t) obtained from statistical tables at a confidence level of 95%. Thus, no statistical difference exists between the sand samples and debris. Hence, the sandstone samples can be used to determine the proper failure mechanism which causes the sand production. The X-ray diffraction analysis shows that, the sand and sandstone samples are mostly composed of quartz, feldspar and traces of apatite and pyroxene. The mechanical properties of the sandstones are given in Table 2. The tested sandstone specimens were observed to fail in a splitting mode during the uniaxial compressive tests and in shear mode during the triaxial compressive tests as shown in Figure 3. The failure criteria of the tested sandstone is shown in Figure 4. Since no statistical difference exists between the sand samples and laboratory collected debris, the shear and splitting failure modes are the potential sources for sand production in the studied Saudi sandstone reservoir. From the experimental data, the average sand production capability factor (ψ) for this reservoir is equal to 2.45 for shear failure mode and 4.9 for splitting failure mode. Sand production capability factor can be calculated by dividing the weight of the generated debris by the specimen's initial

TABLE 1

Statistical analysis results using t-test

Combination	t_c*	t_t**	Combination	t_c*	t_t**
Sample 1 vs. 2	0.25	1.833	Sample 3 vs. 5	0.31	1.895
Sample 1 vs. 3	0.61	1.86	Sample 3 vs. 6	0.83	1.812
Sample 1 vs. 4	1.17	1.895	Sample 3 vs. 7	0.00	1.943
Sample 1 vs. 5	0.36	1.833	Sample 3 vs. 8	0.00	1.943
Sample 1 vs. 6	0.41	1.782	Sample 4 vs. 5	0.79	1.943
Sample 1 vs. 7	0.75	1.86	Sample 4 vs. 6	1.26	1.833
Sample 1 vs. 8	0.82	1.86	Sample 4 vs. 7	0.42	2.015
Sample 2 vs. 3	0.25	1.895	Sample 4 vs. 8	0.44	2.015
Sample 2 vs. 4	0.60	1.943	Sample 5 vs. 6	0.64	1.796
Sample 2 vs. 5	0.00	1.86	Sample 5 vs. 7	0.37	1.895
Sample 2 vs. 6	0.52	1.796	Sample 5 vs. 8	0.41	1.895
Sample 2 vs. 7	0.28	1.895	Sample 6 vs. 7	0.92	1.812
Sample 2 vs. 8	0.29	1.895	Sample 6 vs. 8	0.96	1.812
Sample 3 vs. 4	0.36	2.015	Sample 7 vs. 8	0.80	1.943

* Calculated value of t-test using Eq. (1).

** Tabulated value of t-test obtained from statistical tables [16] at a confidence level of 95%.

TABLE 2

Mechanical properties of the studied Saudi sandstone

Uniaxial compressive strength	=	714 psi
Uniaxial tensile strength	=	135 psi
Angle of internal friction	=	21 degree
Apparent cohesion	=	590 psi
Triaxial stress factor	=	2.1
Average sand production capability factor (uniaxial)	=	3.6%
Average sand production capability factor (triaxial)	=	2.5%

Triaxial failure data

Axial stress at failure (σ_1), psi	2240	2983	3021	3967	4864	6212	8015
Confining pressure at failure (σ_3), psi	250	500	750	1000	1500	2000	3000

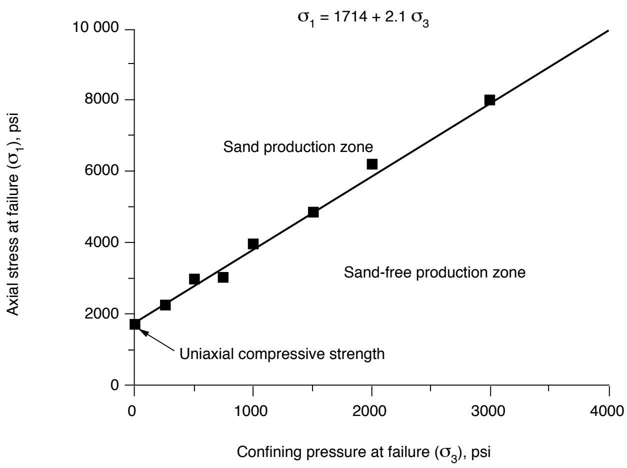

Figure 4

Mohr-Coulomb failure criteria for the tested Saudi sandstone.

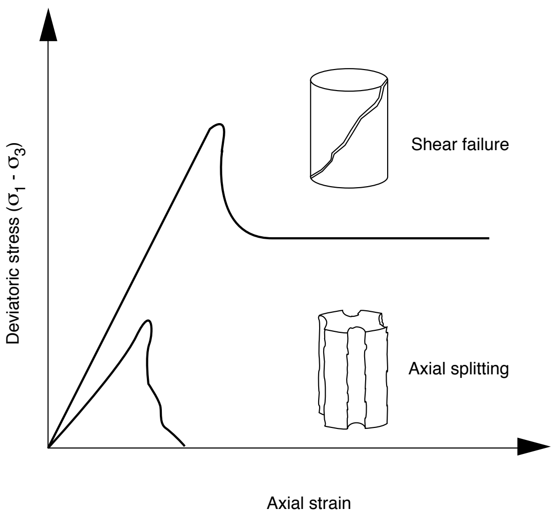

Figure 3

Types of failure modes causing sand production.

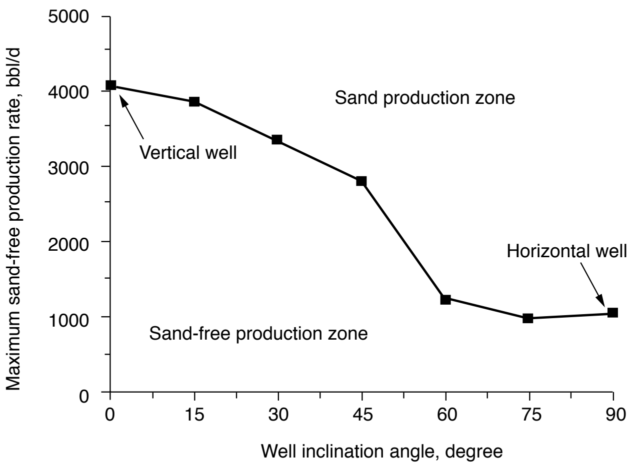

Figure 5

Prediction of the critical production rates for various well inclinations for the studied Saudi reservoir.

weight. Based on the reservoir physical and mechanical data as well as the *in situ* stress state of the reservoir presented in Table 3, the effect of production rate on shear or splitting failure mechanisms is initiated when the reservoir fluids are produced at high rates. A suitable reservoir stability (i.e. sand production initiation) was predicted using the elaborated mathematical model presented in Appendix (A) [17]. It might be advisable to note that the calculated induced stresses are approximate ones. Figure 5 shows the effect of production rate on reservoir stability and sand production initiation for various well inclinations. Production rate must be chosen according to the reservoir rock mechanical properties (failure criteria), *in situ* stress state and well inclination and orientation to avoid sand production problem in the studied Saudi sandstone reservoir.

TABLE 3

Reservoir stress state, mechanical and physical properties

In-situ stress state	Reservoir data	Formation strength data
σ_v = 72 psi/100 ft	Permeability = 17 mD	Apparent cohesion = 589 psi
σ_h = 19.2 psi/100 ft	r_w = 0.375 ft	Angle of internal fraction = 21°
σ_H = 19.2 psi/100 ft	r_e = 912 ft	Poisson's ratio (ν) = 0.21
α = zero to 90°	P_e = 0.51 psi/100 ft	
β = 90°	μ_o = 1.18 cp.	
θ = 0°	h = 136 ft	
	TVD = 7200 ft	

Predicted sand-free production rates

Well inclination (degree)		Max. production (BPD)
0	Vertical well	4080
15	Inclined well	3850
30	Inclined well	3360
45	Inclined well	2800
60	Inclined well	1220
75	Inclined well	970
90	Horizontal well	960

Other data:

Reservoir temperature 190°F
Gas-oil ratio (GOR) = 425 SCF/STB
Oil formation volume factor ($ß_o$) = 1.27 bbl/STB
Connate water saturation = 0.15
Average productivity index = 3 bbl/day/psi
Saturation pressure = 1761 psi
API = 32°
Average porosity = 21%

CONCLUSIONS

Based on the analysis performed in this study, the following conclusions are arrived with:

- Two obvious failure mechanisms namely, splitting and shear failure mechanisms are responsible for sand production from the studied saudi sandstone reservoir.
- Debris collected due to splitting failure mechanism is two folds greater than that collected from shear failure mode.
- No a statisical difference exists between the grain size distributions of the sand obtained from the studied Saudi oil reservoir and the laboratory collected debris.
- Production of reservoir fluids with high rate is a major reason beyonds the initiation of failure surfaces and responsible for sand production.
- The critical production rates for the studied Saudi reservoir range from 960 to 4080 bbl/day for well inclination angles ranging from 0° (vertical well) to 90° (horizontal well).

NOMENCLATURE

h	reservoir thickness, ft
k	reservoir rock permeability, mD
n_1, n_2	number of data point
P_e	reservoir pressure, psi/100 ft
P_{wc}	critical wellbore pressure, psi/100 ft
q_c	critical production rate, bbl/day
r_w, r_e	wellbore and reservoir radii respectively, ft
S_1, S_2	standard deviations of data set-1 and data set-2 respectively
TVD	total vertical depth, ft
t_c, t_t	calculated and tabulated values of the t-test
x_1, x_2	average values of data set-1 and data set-2, respectively
α, β	well inclination and orientation angles respectively, degree
θ	angular position around the borehole, degree.
ψ	sand production capability factor, fraction.
μ_o	reservoir fluid viscosity, cp

σ	normal stress at failure, psi/100 ft
σ_o	uniaxial compressive strength, psi
σ_H, σ_h, σ_v	*in situ* principal stresses, psi/100 ft
σ_x, σ_y, σ_{zz}	transformed *in situ* stress in cartesian form, psi/100 ft
σ_r, σ_θ, σ_z	induced stresses in polar form, psi/100 ft
σ_1, σ_2, σ_3	principal stresses acting on the wall of a borehole, psi/100 ft
ν	Poisson's ratio, fraction
ϕ	rock angle of internal friction, degree
τ_f	shear stress at failure, psi/100 ft
τ_{Max}	calculated shear stress from the in situ stresses, psi/100 ft
τ_o	apparent cohesion of the reservoir rock, psi
τ_{xy}, τ_{xz}, τ_{yz}	induced shear stresses acting on the wall of a borehole, psi/100 ft
$\tau_{r\theta}$, τ_{rz}, $\tau_{\theta z}$	induced stresses acting on the wall of a borehole, psi/100 ft.

REFERENCES

1 Sanfilipo F., M. Ripa and F.J. Santarelli (1995), Economical management of sand production by a methodology validated on an extensive database of field data. *SPE Paper* No. 30472 presented at the 70th *SPE Annual Technical Conference & Exhibition,* Dallas, Texas.

2 Fjaer E., R.M. Holt, P. Horsud, A.R. Raaen and R. Risnes (1992), *Petroleum Related Rock Mechanics*. 1st edition, Elsevier Science Publishers BV, p. 338, Amsterdam, The Netherlands.

3 Wong R.C.K., A.M. Samieh and R.L. Kuhlemeyer (1994), Oil sand strenght parameters at low effective stress. Its Effects on Sand Production. *The Journal of Canadian Petroleum Technology*, **33**, 5, pp. 44-49.

4 *Log Interpretation Principles/Applications* (1991), Handbook Published by Schlumberger Educational Services, Section 13, Houston, Texas.

5 Tronvoll J. and E. Fjaer (1994), Experimental study of sand poduction from perforation cavities. *International Journal of Rock Mechanics and Mining Sciences & Geomechanical Abstracts*, **31**, 5, pp. 393-410.

6 Tronvoll J. (1992), Experimental Investigation of Perforation Cavity Stability. *Rock Mechanics*, Balkema, Rotterdam, pp. 365-373.

7 Tronvoll J., M. Nobuo and F.J. Santarelli (1992), Perforation cavity stability: comprehensive laboratory experiments and numerical analysis. *SPE paper* No. 24799 presented at the *67th. Annual Technical Conference,* Washington DC.

8 Morita N. (1994), Field and laboratory verification of sand-production prediction models. *SPE Drilling & Completion*, pp. 227-235.

9 Cook J.M., I.D.R. Bradford and R.A. Plumb (1994), A study of the physical mechanisms of sanding and application to sand prodution prediction. *SPE Paper* No. 18852 presented at the *European Petroleum Conference,* London, UK.

10 Sarda J-P, N. Kessler., E. Wicquart, K. Hannaford and J.P. Deflandre (1993), Use of porosity as a strength indicator for sand production evaluation. *SPE paper* No. 36454 presented at the *68th Annual Technical Conference,* Houston, Texas.

11 Stein N., A.S. Odeh and L.G. Jones (1974), Estimating maximum sand-free production rates from friable sands for different well completion geometries. *Journal of Petroleum Technology*, pp. 1156-1158.

12 Risnes R., R.K. Bratli and P. Horsud (1982), Sand stresses around a wellbore. *SPE Journal,* pp. 883-898.

13 Peden J.M. and A.A.M. Yassin A.A.M. (1986), The determination of optimum completion and production conditions for sand-free oil production. *SPE Paper* No. 15406, *SPE 61st. Annual Technical Conference and Exhibition*, New Orleans, LA.

14 Desai S (1988), Sand production model for sanfanyia field in saudi arabia. *M.Sc. Thesis*, Louisiana Technical University, Ruston, LA, USA.

15 Kovari K., A. Tisa, H.H. Einstein and J.A. Franklin (1983), Suggested methods for determining the strength of rock materials in triaxial compression. Revised Version, *The International Society for Rock Mechanics Abstrac*t, **20**, 6.

16 Volk W (1969), *Applied Statistics for Engineers.* McGraw-Hill Book Company, London.

17 Al-Awad M.N. and O.A. AlMisned (1996), Rock failure criteria a key for predicting sand-free production rates. Considered for publication in the *Journal of the Society of Egyptian Engineers*.

18 Santarelli F.J., H. Ouadfel and J.P. Zundel (1991), Optimizing the completion procedure to minimize sand production risk. *SPE Paper* No. 22797, 14 p.

19 Kessler N., Y. Wang and F.J. Santarelli (1993), A simplified pseudo-3D model to evaluate sand production risk in deviated cased holes. *SPE Paper N*o. 26542 presented in the *68th Annual Technical Conference of the SPE*, Houston, Texas.

Final manuscript received in June 1997

APPENDIX (A)

One of the most famous and applied rock failure criterion is the Mohr-Coulomb failure criteria [1-4, 8, 15, 17-19]. This criteria is defined as follows [2]:

$$\tau_f = \tau_o + \sigma \tan \phi \tag{A-1a}$$

or

$$\sigma_1 = \sigma_o + k\,\sigma_3 \tag{A-1b}$$

The *in situ* principal stresses can be transformed parallel to the wellbore axis (for inclined or horizontal wells) using the following matrices:

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_{zz}\end{bmatrix} = \begin{bmatrix}\mathrm{Cos}^2\beta\ \mathrm{Cos}^2\alpha & \mathrm{Sin}^2\beta\ \mathrm{Cos}^2\alpha & \mathrm{Sin}^2\alpha\\ \mathrm{Sin}^2\beta & \mathrm{Cos}^2\beta & 0\\ \mathrm{Cos}^2\beta\ \mathrm{Sin}^2\alpha & \mathrm{Sin}^2\beta\ \mathrm{Sin}^2\alpha & \mathrm{Cos}^2\alpha\end{bmatrix}\begin{bmatrix}\sigma_H\\ \sigma_h\\ \sigma_v\end{bmatrix} \tag{A-2}$$

$$\begin{bmatrix}\tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{bmatrix} = \frac{1}{2}\begin{bmatrix}\mathrm{Sin}2\beta\ \mathrm{Sin}\alpha & -\mathrm{Sin}2\beta\ \mathrm{Sin}\alpha & 0\\ \mathrm{Sin}2\alpha\ \mathrm{Cos}\beta & \mathrm{Sin}^2\beta\ \mathrm{Sin}2\alpha & -\mathrm{Sin}2\alpha\\ \mathrm{Cos}^2\beta\ \mathrm{Sin}^2\alpha & -\mathrm{Sin}2\beta\ \mathrm{Cos}\alpha & 0\end{bmatrix}\begin{bmatrix}\sigma_H\\ \sigma_h\\ \sigma_v\end{bmatrix} \tag{A-3}$$

The *in situ* principal stresses acting on the wall of a borehole or a perforation cavity then can be computed as follows:

$$\begin{aligned}
\sigma_r &= P_{wc}\\
\sigma_\theta &= (\sigma_x + \sigma_y - P_{wc})\\
&\quad - 2(\sigma_x - \sigma_y)\cos 2\theta - 4\tau_{xy}\,\mathrm{Sin}2\theta\\
\sigma_z &= \sigma_{zz} - 2\nu(\sigma_x - \sigma_y)\cos 2\theta - 4\nu\tau_{xy}\,\mathrm{Sin}2\theta\\
\tau_{r\theta} &= \tau_{rz} = 0\\
\tau_{\theta z} &= 2[-\tau_{zx}\,\mathrm{Sin}\theta + \tau_{yz}\,\mathrm{Cos}\theta]
\end{aligned} \tag{A-4}$$

The critical bottom hole pressure is calculated using the following Darcy equation:

$$q_c = \frac{7.082\ kh\ (P_e - P_{wc})}{\mu_o\ \ln\left(\frac{r_e}{r_w}\right)} \tag{A-5}$$

Using the calculated bottom hole pressure from Eq. (A-5), then the induced principal stresses acting on the wall of a borehole or a perforation cavity can be computed:

$$\begin{aligned}
\sigma_1 &= \sigma_r + P_{wc}\\
\sigma_2 &= \frac{1}{2}(\sigma_\theta + \sigma_z) - \frac{1}{2}\sqrt{(\sigma_\theta - \sigma_z)^2 + 4\tau_{\theta z}}\\
\sigma_3 &= \frac{1}{2}(\sigma_\theta + \sigma_z) + \frac{1}{2}\sqrt{(\sigma_\theta - \sigma_z)^2 + 4\tau_{\theta z}}
\end{aligned} \tag{A-6}$$

The maximum and minimum induced stresses acting on the wall of a borehole or perforation will be as follows:

$$\overline{\sigma}_1 = \text{Maximum of } [\sigma_1\ ,\ \sigma_2\ ,\ \sigma_3] \tag{A-7}$$

$$\overline{\sigma}_3 = \text{Maximum of } [\sigma_1\ ,\ \sigma_2\ ,\ \sigma_3] \tag{A-8}$$

Finally the borehole or perforation stability can be predicted by comparing the computed and the experimentally measured shear stresses as follows:

$$\tau_f = \tau_o + \left[\frac{\overline{\sigma}_1 + \overline{\sigma}_3}{2}\right] \tan\phi \tag{A-9}$$

$$\tau_{Max} = \left[\frac{\overline{\sigma}_1 - \overline{\sigma}_3}{2}\right] \tag{A-10}$$

If $\tau_{Max} \geq \tau_f$ then unstable conditions will take place and sand will be produced.