Chapter SEVEN
The Net Premiums and the Mathematical Reserve of Life Insurance Policies

What did you study?

Basics of risk and insurance have been studied in the first part. Moreover, we had studied classifications of risks and risk management. Furthermore, the legal principles of insurance contract had been studied. Finally we had studied fundamentals of life insurance, in the last chapter

What are you going to study in this chapter?

In this chapter, the reader will study the following points:

- The net single premium (P) and the net annual premium (AP) and the difference between P and AP
- Finding the net single premium formulas of the different life insurance policies (whether survival policies or death policies)
- Finding the net annual premium formulas of the different life insurance policies (whether survival policies or death policies)
- Finding the mathematical reserve of life insurance policies by retrospective method and prospective method.

Objectives of studying this chapter:

After study this chapter, the reader has to be able to answer the following questions:

- What is the net single premium and what is the net annual premium? Explain the difference between of them.
- How can you find the net single premium for a life insurance policy?
- How can you calculate the net annual premium for a life insurance policy?
- How can you compute the reserve for a life insurance policy ,Whether by retrospective method or by prospective method.

Chapter SEVEN
The Net Premiums and the Mathematical Reserve of Life Insurance Policies

7.1-Introduction

As pointed out earlier, the cost of life insurance policy is *net premium*. And, the latter is made either by paying a single amount called *single premium* or by paying equal annual payments, called the *net annual premiums*. So, the following questions may be raised:

What is the net single premium (P) of life insurance policy?

What is the net annual premium (AP) of life insurance policy?

What is the difference between P and AP?

The net single premium (P) is the present value of future death benefits. In other words, a sum is paid at the beginning of the policy which under certain basic assumptions, would be sufficient to provide the scheduled benefit payments.

The basic assumptions may be summarized as follows:

1- The <u>mortality table</u> employed will represent precisely the mortality to be experienced by group of lives connected.
2- All <u>net premiums</u> are paid at the beginning policy year and immediately invested in income-producing assets.
3- Death claims (<u>benefits</u>) are paid at the end of the policy year
4- The <u>death rate</u> is uniform throughout the year.
5- All <u>net single premiums</u> collected from insureds plus all interest earned, will be paid in the form of benefits.
6- The <u>all life insurance policies</u> charge the premiums on the basis of purchaser's birthday (i.e. birthday nearest to the date of issue of the policy)

By using the <u>foregoing assumptions</u>, the relationship between the net single premiums and future benefits *<u>for example</u>* "<u>term life insurance policy</u>" may be expressed at the beginning of policy as indicated in both the following equation and figure (7.1)

برواز

PV of *<u>all net single premiums</u>* to be paid = PV of *<u>all benefits</u>* to be received (at the beginning of the policy) (1)
Where
PV is present value
PV = P = Net single premium

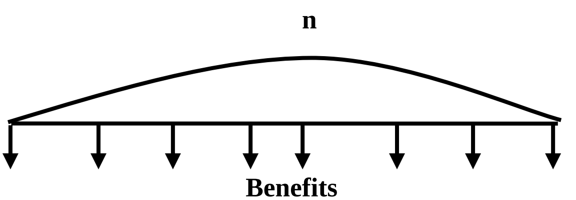


Figure (7.1) relationship between single premium and future benefits of term life insurance policy

<u>The net annual premium (AP):</u>

In effect, most life insurance policies are not purchased with a single premium because of the large amount of cash, that is, required. So, the consumers prefer to purchase the policy in <u>installment payments</u>. The premiums that are paid annually are called <u>*the net annual premiums (AP).*</u> The present value of the net annual premiums must be mathematically equivalent to the net single premium (P).

Hence, if the net annual premiums are equal, the purchasers (i.e. insureds) will pay life annuity due (where annual premium = life annuity due) either over the period of the policy or death whichever is earlier or for a period less than the period of the policy or death whichever is earlier. Consequently, net annual premium (AP) can be determined by the following equation and figure (7.2) as below:

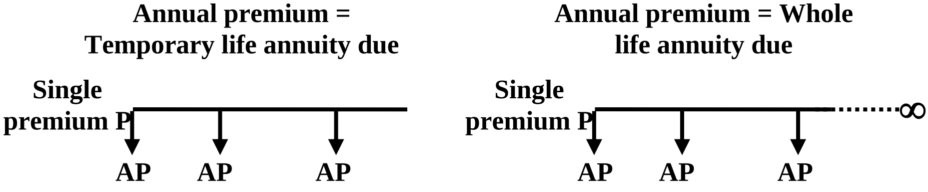


Figure (7.2) Relationship between Net single Premium (p) and Net Annual Premium (AP)

The net single premium (P) = Present value of life annuity due (ä) *(present value of net annual premiums (AP) on purchase date)* That is,

برواز

$$P = AP \times ä \qquad (2)$$

Where:

ä is the present value of life annuity due either whole life annuity due ($ä_x$) or temporary life annuity due ($\mathbf{ä_{x:n]}}$)

From the preceding formula, we may conclude

برواز

$$AP = \frac{\mathbf{P}}{\mathbf{ä}} \qquad (3)$$

That is:

The net annual premium of any policy =

$$\frac{\textbf{Net single premium of the policy}}{\textbf{Present value of life annuity due}} \qquad (ä_x \text{ or } ä_{x}\text{: } n)$$

From equation (3) we conclude that:

$$AP = \frac{\textbf{numerator of single premium of the policy}}{\mathbf{N_x \text{ or } N_{x+n}}} \qquad (3)$$

Where:

N_x if the premium is paid for life time, but $N_x = N_{x+n}$ if the premium us paid for a period n

بروار

Notices

1- The net annual premium payments (as indicated in figure (7.2) can be viewed as being similar to a life annuity except the payment flow from the insured to the insurance company.
2- Both life annuity payments and annual premium payments are similar to that; both are paid during lifetime (i.e. forever) of a specified individual, or for a stated period of the time (i.e. temporary period.
3- Both life annuity payments and annual premium payments cease on death. Moreover they are discounted for compound interest.

After study of the relationship between *net single premium (P)* and *net annual premium (AP)* our concern in the next sections is calculation of both (P) and (AP) for the following policies:

i) The policies that cover the survival risk
ii) The policies that cover the death risk
iii) The policies that cover a combination of death risk and survival risk.

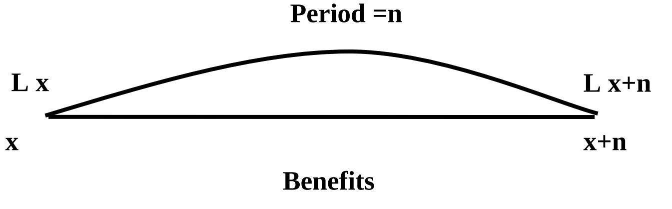
7.2-The net single premium and the net annual premium of the policies that cover the survival risk:

As we have already studied in the first chapter (fundamental life insurance) the policies that cover the survival risk are:

1- Pure endowment policy
2- Life annuities policies

7.2.1-Pure Endowment policy

We have already defined this policy in section 5.6.2.1, whereby insurance company pays sum insured to the insured if he is a live at the end of policy period. So, both the net single premium and the net annual premium for that policy can be calculated according to some assumptions indicated in the following diagram.



Given that:

Net single premium = ${}^{n}E_{x}$

Sum insured = face amount = 1 L.E

Net annual premium = ${}^{n}P_{x}$

A- Finding the net single premium

The assumptions

i) L_x represent the persons who buy this policy at age x
ii) The period of the policy n years
iii) The face amount = 1 L.E

iv) The international symbols of the net single premium and the net annual premium are ${}^{n}E_{x}$ and ${}^{n}P_{x}$ respectively

Required: Finding

1- Net single premium = ${}^{n}E_{x}$

2- Net annual premium = ${}^{n}P_{x}$

Prove

In order to calculate the net single premium, we may apply the equation (1).i.e.

PV of all net single premiums to be paid = PV of all benefits to be received (at the beginning of the policy)

By looking at the previous diagram, we may notice that:

- *Insureds obligations* = PV of all net single premiums = $L_{x} \cdot {}^{n}E_{x}$ (L_{x} is number of persons who buy pure endowment policy × the value of net single premium)
- *Insurance company's obligations* = PV of all benefits to be received (after n years) $= L_{x+n} \cdot V^{n}$ (where $v^{n}_{i\%}$ is the present value of 1 at interest rate i% and for period n years, and L_{x+n} is the number of survivors at age x + n.)

Hence, $L_{x} \cdot {}^{n}E_{x} = L_{x+n} V^{n} \times 1$ (on purchase date or at the beginning of the policy)

Where L_{x+n} are the survivors insureds who will be living at the end of n-year period, 1 is sum insured and $L_{x+n} V^{n}$ is the present value of the benefits that are paid by insurance company to survivors at age x+n

By dividing both sides by L_x, we get:

$$ {}^{n}E_x = \frac{L_{x+n} V^n}{L_x} \times \frac{V^x}{V^x} = \frac{L_{x+n} V^{x+n}}{L_x V^x} \qquad (4)$$

Now the following question may be raised:

What is meaning of $L_x V^x$ *and* $L_{x+n} V^{x+n}$?

The answer L_x is number of living in morality table, V^x is present value of sum insured which equal 1 for a period x in compound interest table.

To reduce the numerical work required to solve any formula has these symbols ($L_x V^x$ or $L_{x+n} V^{x+n}$ or others … etc) ***the commutation functions*** had been constructed in a table is called ***commutation columns*** - interest at 3% based on 1958 CSO Mortality Table (see appendix).

The commutations functions that are used for calculating net single premium or net annual premium, of any life policy might be summarized as follows:

i) Symbols are used for survival policies:

$$D_x = L_x V^x \qquad (5)$$

For example:

$D_{40} = L_{40} V^{40}$ and $D_{40+5} = L_{40+5} V^{40+5}$

Where:

L_{40} is number of living at age 40

V^{40} is present value of 1 L.E, that is due at the end of 40 years at interest rate 3%

$$N_x = \sum_{x=x}^{w-1} D_x = D_x + D_{x+1} + D_{x+2} + \dots\dots\dots + D_{w-1} \qquad (6)$$

For example

$N_{50} = D_{50} + D_{51} + D_{52} + \ldots\ldots + D_{99}$

<u>Also</u>: $N_{50} - N_{60} = D_{50} + D_{51} + \ldots\ldots\ldots\ldots\ldots + D_{59}$

$$\mathbf{S_x = \sum_{x}^{w-1} N_x = N_x + N_{x+1} + N_{x+2} + \ldots\ldots\ldots + D_{W-1}} \qquad (7)$$

For example

$S_{50} = N50 + N51 + N52 + \ldots\ldots\ldots.. + N99$

<u>Also</u>: $S_{50} - S_{60} = N_{50} + N_{51} + \ldots\ldots + N_{59}$

ii) **<u>Symbols are used for death policies</u>**

$$C_X = \mathbf{d_x V^{x+1}} \qquad (8)$$

For example

$C_{40} = d_{40} V^{41}$

Where:

d_{40} is the number of dying at age 40

V^{41} is the present value of 1 that is due at the end of 41 years at interest rate 3%

$$\mathbf{M_x = \sum_{x=x}^{w-1} C_x = C_x + C_{x+1} + C_{x+2} + \ldots\ldots\ldots + C_{w-1}} \qquad (9)$$

For example

$M_{30} = C_{30} + C_{31} + C_{32} + \ldots\ldots + C_{99}$

<u>Also</u>

$M_{30} - M_{40} = C_{30} + C_{31} + \ldots\ldots + C_{39}$

$$\mathbf{R_x = \sum_{x=x}^{w-1} M_x = M_x + M_{x+1} + M_{x+2} + \ldots\ldots\ldots + M_{w-1}} \qquad (10)$$

For example

$R_{35} = M_{35} + M_{36} + \ldots.. + M_{99}$

Also
$R_{40} - R_{50} = M_{40} + M_{41} + M_{42} + \ldots\ldots\ldots + M_{49}$

Now, by substituting commutation functions, in equation (4) we find *the net single premium* ($^{n}E_{x}$) as follows:

برواز

$$\text{Net single premium} = {}^{n}E_{x} = \frac{D_{x+n}}{D_{x}} \times 1 \qquad (11)$$

Where 1 = sum insured or face value of the policy

B-finding the net annual premium
Using equation (3) the net annual premium of pure endowment policy will be equal

$$^{n}P_{x} = \frac{\text{Net Single premium of pure endowment policy}}{\text{Present value of temporary life annuity due}}$$

So,
a) If the annual premium is paid for period of the policy (n) or death whichever is earlier:

$$^{n}P_{x} = \frac{D_{x+n}}{D_{x}} \div \frac{N_{x} - N_{x+n}}{D_{x}} = \frac{D_{x+n}}{D_{x}} \times \frac{D_{x}}{N_{x} - N_{x+n}}$$

(Where
So

$$^{n}p_{x} = \frac{D_{x+n}}{N_{x} - N_{x+n}} \times \text{sum insured}$$

b) If the annual premium is paid for a period m where m < n

$$^{m}P_{x} = \frac{D_{x+n}}{N_{x} - N_{x+m}} \times \text{sum insured}$$

Solved problems

Example 1

Find both the net single premium and the net annual premium of pure an endowment policy of 20,000 L.E payable in 10 years to a person now aged 30

Solution

a) ***Net single premium***

$$ {}^{n}E_x = \frac{D_{x+n}}{D_x} \text{ x } 20000 $$

$$ {}^{10}E_{30} = \frac{D_{30+10}}{D_{30}} \text{ x } 20000 = \frac{D_{40}}{D_{30}} \text{ x } 20000 = \frac{2833001.8}{3905782} \text{ x } 20000 $$

$$ {}^{10}E_{30} = 14506.70 \text{ LE} $$

b) ***Net annual premium***

$$ \frac{D_{40}}{N_{30} - N_{40}} \text{ x } 20000 = \frac{2833001.8}{91698461.9 - 57719347.6} \text{ x } 20000 = 1667.49 $$

(Where $N_{30} - N_{40}$ is numerator of $\ddot{a}_{30:\overline{10|}}$)

Example 2:

A father paid 492.100 L.E for a pure endowment policy for his daughter now aged 35. The endowment policy will be payable to his daughter if and when she reaches aged 55. How much will the daughter receive if she is then alive?

Solution

Since $\quad {}^{n}E_x = \frac{D_{x+n}}{D_x} \text{ x sum insured}$

So $\quad 492.100 = \frac{D_{55}}{D_{35}} \text{ x sum insured}$

$$ 492.100 = \frac{1639329.7}{3331295.4} \text{ x sum insured} = 0.4920997 \text{ x sum insured} $$

Sum insured that will be received by the daughter

$$= \frac{492.100}{0.4920997} = 1000\,\text{L.E}$$

Exercises (3.1)

1- What is the single premium and the net annual premium of a pure endowment contract of 10000 payable in 7 years to a person now aged 23?

2- A woman now aged 30 is promised a gift of 10000 L.E when she reaches age 40. Find the present value of the promise?

3- A person aged 30 pays 400 L.E to a Misr insurance company for a 15- year pure endowment policy. How much will that person receive at age 60 if then alive?

4- Show that: ${}^{n}E_x \cdot {}^{n}E_{x+m} = {}^{m+n}E_x$

5- Compute the values ${}^{5}E_{30}$ and $1/\,{}^{10}E_{40}$

7.2.2-Life Annuities policies

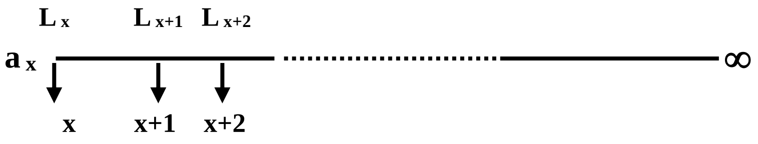
As we have already mentioned, life annuities policies are generally classified into two major types (a) whole life annuities and (b) temporary life annuities (see section 5.6.2.2).

I) Whole life annuities

These annuities are *four* types: they are

***1-Ordinary whole life annuity policy** (i.e. immediate whole life annuity)*

By virtue of this policy, insurance company will pay annuity 1 L.E to annuitant at the end of each year as long as he is alive as indicated in the following figure



A-Finding the single premium:

The single premium (**a** $_x$) of this annuity can be calculated according to the following assumptions:

i) The net single premium is **a** $_x$

ii) The value of the annuity is 1 L.E payable at the end of each year

iii) The first payment will be made one year later (i.e. after a year)

Required: *Finding the net single premium (a $_x$)*

Prove:

To compute the single risk premium (a $_x$) by equation (1) , and the preceding diagram, we can determine both insureds' obligations and insurance company obligations as follows:

- Insureds' obligations who buy ordinary while life annuity policy at age x = $L_x \cdot a_x$ = PV of all net single premiums to be paid.
- Also, insurance company's obligations may be determined as follows:

At the end of the first year, there will L_{x+1} survivors according to mortality table, each one gets 1 L.E.

Hence insurance company's obligation at the end of first year will be $L_{x+1} \times 1 = L_{x+1}$ L.E

Likewise, *at the end of the second year*, there will be L_{x+2} survivors, each one gets 1 L.E. Hence, insurance company's obligation at the end of second year will be $L_{x+2} \times 1 = L_{x+2}$ L.E, and so on ….. up to the end of mortality table (L_{w-1}).

Consequently insurance company's obligations (on purchase date) =

$L_{x+1} \times v^1 + L_{x+2} \times v^2 + L_{x+3} \times v^3 +$ ….. up to the end of mortality table = PV of all benefits to be received.

Using equation (1) we find:

PV of all net single premiums to be paid = PV of all benefits to be received, that is

$L_x \,.\, a_x = L_{x+1} V^1 + L_{x+2} v^2 + \ldots\ldots\ldots + L_{w-1} V^{w-x-1}$

By dividing both sides by L_x

$$a_x = \frac{L_{x+1}V^1 + L_{x+2}V^2 + \ldots\ldots + L_{w-1}V^{w-x-1}}{L_x}$$

Multiply both numerator and denominator by V^x then

$$a_x = \frac{L_{x+1}V^{x+1} + L_{x+2}V^{x+2} + \ldots\ldots + L_{w-1}V^{w-1}}{L_x V^x} \quad \text{(Where w=100)}$$

Using commutation functions the preceding equation will becomes

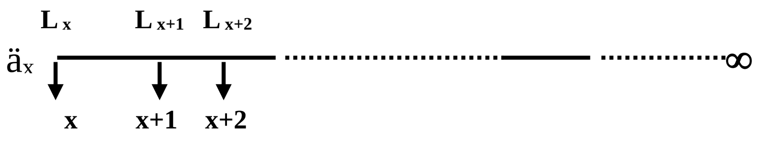
$$a_x = \frac{D_{x+1} + D_{x+2} + \ldots\ldots + L_{99}}{D_x}$$

$$a_x = \frac{N_{x+1}}{D_x} \times 1 \qquad (12)$$

(Where 1 = annual payment to the annuitant)

2-Whole life annuity due policy

If the first payment is made at once (i.e. at the time of purchase) as indicated in the following diagram, the annuity is called annuity due



Finding the single premium

If the net single premium is $\ddot{a}_x$, it can calculated by looking at the preceding diagram. Since the only difference between this annuity (i.e. whole life annuity due) and the ordinary whole life annuity described in the last section is the payments made to L_x persons at purchase date. So, we may say that

$L_x \ddot{a}_x = L_x V^0 + L_{x+1} V^1 + L_{x+2} V^2 + + \ldots\ldots.. L_{99} V^{99-x}$

Solving this equation and multiplying numerator and denominator by V^x

$$\ddot{a}_x = \frac{L_x V^x + L_{x+1} V^{x+1} + \ldots\ldots. + L_{99} V^{99}}{L_x V^x}$$

$$\ddot{a}_x = \frac{D_x + D_{x+1} + D_{x+2} + \ldots\ldots. + D_{99}}{D_x}$$

$$\ddot{a}_x = \frac{D_x}{D_x} + a_x = 1 + a_x$$

$$\ddot{a}_x = \frac{N_x}{D_x} \times 1 \qquad (13)$$

And consequently

$$a_x = \ddot{a}_x - 1$$

(i.e. ordinary whole life annuity = whole life annuity due -1)

برواز

A notice: The cost of both ordinary whole life annuity policy and whole life annuity due policy are charged by *single premium not annual premium*, because it is not logic the insured pays for the premium yearly and receives at the same time the annuity yearly. So, these policies have just single premium is paid on purchase date.

Solved problems

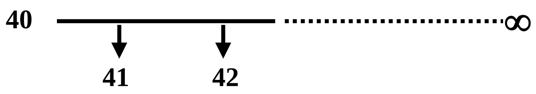
Example 1:

Find the net single premium for a life annuity of 200 L.E per year for a 40- year old man if:

a) the first payment is to be made in 1 year
b) the first payment is to be made now

Solution

a) If the first payment is to be made in 1 year, the annuity is ordinary whole life annuity:



So, the net single premium = $\ddot{a}_x = \frac{N_{41}}{D_{40}} x200$

$$= \frac{54886345.6}{2833001.8} x200 = 3874.78 \text{LE}$$

b) If the first payment is to be made now

The annuity is whole life annuity due

So, the net single premium

$$\ddot{a}_x = \frac{N_{40}}{D_{40}} x200 = \frac{57714347.6}{2833001.8} x200 = 4074.78$$

Example 2:

A worker was blinded by an explosion in his factory. He received 330000 L.E as compensation. If at age 45, he used $\frac{1}{3}$ of this amount to buy a whole life annuity due, what would be his annual income.

Solution

The third of amount = 330000 x $\frac{1}{3}$ = 110000 =

$\ddot{a}_{45} = \text{The net single preium}$,   and since

$$\ddot{a}_{45} = \text{annual income x} \frac{N_{45}}{D_{45}}$$

So $$110000 = \text{annual income x} \frac{44455164.3}{2392904.7} \times 200$$

$$\text{annual income} = \text{annual payment} = 5921.01 \text{LE}$$

Example 3

A man mentioned that $\frac{3}{4}$ of his company, after his death, will be used to purchase an ordinary life annuity for his single son and the remaining $\frac{1}{4}$ an ordinary life annuity for his wife. The company amount to 640000 Egyptian pound. When the annuities are purchased, the son is 15 and the widow is 55. Find the annual income of each.

Solution

Given that annual income = R

$\frac{3}{4}$ of amount = 640000 x $\frac{3}{4}$ = 480000 pound

$\frac{1}{4}$ of amount = 640000 x $\frac{1}{4}$ = 160000 pound

So, 480000000 = $R\,x\,\frac{N_{16}}{D_{15}} = R\,x\,\frac{161500785.1}{6253773.3}$

Annual income of son (R) = 247826.35 L.E

$$160000 = R\,x\,\frac{N_{56}}{D_{55}} = R\,x\,\frac{22392847.7}{1639329.7}$$

So, Annual income of wife (R) = 11713.24 L.E

<u>Exercise (3.2)</u>

1- A person aged 40 paid 40000 L.E for purchasing an ordinary whole life annuity with first payment to be made in one year. Find the value of the annual payment. Then if the first payment to be made at the time of purchasing what is the value of this annual payment.

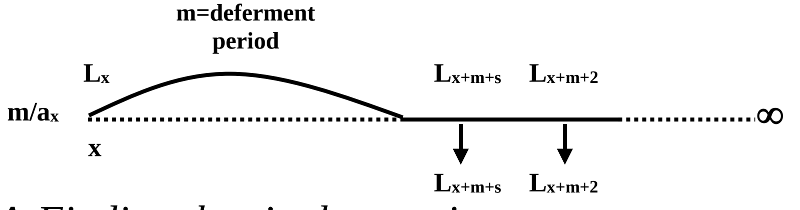
2- A person aged 40, purchase a life insurance policy. The net single premium of the policy is 1500 L.E. If the insurance company allows the premium to be paid by equal annual payments for a life, with first payment due new. Find the size of annual payment.

3- At age 65 an employee is to receive 1000 L.E per month, in a social insurance benefits. What is the approximate present value of his benefits?

4- A divorced woman who is the beneficiary of 30,000 insurance policy decides to take annual income for life. If she is 50 years old. Find the size of each payment if the first payment is to be made (a) after one year (b) Now.

3-Deferred ordinary whole life annuity policy

This policy is similar to ordinary whole life annuity policy, except the first payment starts after a period of more than one year has elapsed from the date of purchase. This period is called deferment period as indicated in the following diagram.



A-Finding the single premium

If the deferment period is m and the net single premium is $\mathbf{m/a_x}$. It can be calculated by the same procedure that had been followed in the preceding policies (ordinary whole life annuity – whole life annuity due).

Using equation (1) we may find (by looking at the previous diagram)

$L_x \,.\, m/a_x = L_{x+m+1} V^{m+1} + L_{x+m+2} V^{m+2} + \ldots. + L_{99} V^{99-x}$

Solving this equation and multiplying the numerator and denominator by $\boldsymbol{V^x}$, we obtain

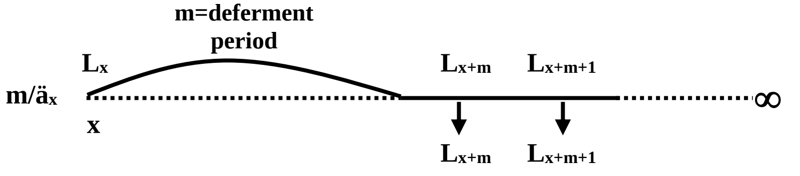
$$\mathbf{m/a_x = \frac{L_{x+m+1}V^{x+m+1} + L_{x+m+2}V^{x+m+2} + \ldots\ldots. + L_{99}V^{99}}{L_x V^x}}$$

$$\mathbf{m/a_x = \frac{D_{x+m+1} + D_{x+m+2} + \ldots\ldots. + D_{99}}{D_x}} \qquad (14)$$

$$\mathbf{m/a_x = \frac{N_{x+m+1}}{D_x} x1}$$

4-deferred whole life annuity due policy

The first annual payment in this policy starts directly after the deferment period (m) as indicated in the following diagram



A-Finding the single premium:

If the net single premium is **$m/\ddot{a}_x$**. Using equation (1) and the previous diagram, we may find

$L_x \,.\, m/\ddot{a}_x = L_{x+m} V^m + L_{x+m+1} V^{m+1} + \ldots.. + L_{99} V^{99-x}$

Solving this equation, we obtain *the net single premium ($m/\ddot{a}_x$)* as follows:

$$\mathbf{m/\ddot{a}_x = \frac{N_{x+m}}{D_x} x1} \qquad (15)$$

B-Finding the net annual premium:

Now, the following question may be raised. Do you think, the insured can pay annual premium for the deferred whole life annuities (ordinary – due).

The answer yes, he can, because it is possible to pay the annual premium over the deferment period or over a period less than it.

Consequently, *the net annual premium* can be calculated as follows:

i) The net annual premium of deferred ordinary whole life annuity policy

Using equations (3, 3 and 14)

$$\text{The net annual premium } \boldsymbol{AP\ (m/a_x)} = \mathbf{\frac{N_{x+m+1}}{D_x} x1} \qquad (16)$$

(Where m is period of payment = deferment period)

ii) ***The net annual premium of deferred whole life annuity due policy***

Using equations (3, 3 and 15)

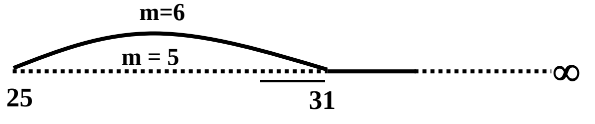
The net annual premium $\boldsymbol{AP\ (m/\ddot{a}_x)} = \dfrac{\mathbf{N_{x+m}}}{\mathbf{D_x}}\,\mathbf{x1}$ (17)

Solved problems

Example 4:

A 25 year-old son received an inheritance of 300,000 L.E and he paid $\frac{2}{3}$ this amount to buy a new car. If he would like to get annual payments for life starting on his 31st birthday, what is the size of the payment, if he paid $\frac{1}{3}$ of his inheritance?

Solution



This policy is either deferred whole life annuity due (where m = 6), or deferred ordinary whole life annuity (where m = 5).

Given that the annual payment = R and the policy is deferred whole life annuity due (m = 6)

$$\mathbf{100{,}000 = R\frac{N_{25+6}}{D_{25}} = R\frac{N_{31}}{D_{25}} = R\frac{87792679.9}{4573377.1} = 19.196L.E}$$

$$\mathbf{So, R(annual\ payment) = \frac{100{,}000}{19.196} = 209.29L.E}$$

برواز

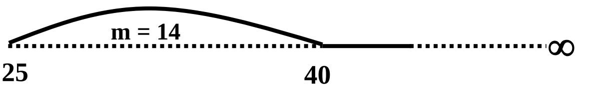
Training:

Please, *darling reader* solve the problem again using the deferred ordinary whole life annuity (m = 5) and compare the result by the preceding result, and what is your comment?

Example 5:

A young woman aged 25 has 20,000 L.E. If she desires to use the money to buy a whole life annuity with first payment to be made at age 40, how much can get yearly?

Solution



Given that policy is deferred ordinary whole life annuity (m = 14) and the yearly payment = R

Then:

$$20{,}000 = R\frac{N_{25+14+1}}{D_{25}} = R\frac{N_{40}}{D_{25}} = R\frac{57719347.6}{4573377.1} = 12.62R$$

$$\text{Yearly payment } = \frac{20{,}000}{12.62} = 1584.64\text{LE}$$

برواز

Training:

Solve the problem again using deferred whole life annuity due …. And what is your comment?

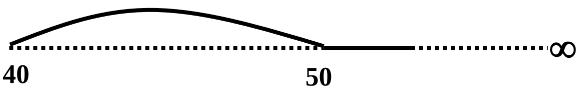
Example 6:

A person aged 40 wishes to buy a whole life annuity of 1000 per year, if the first pay is to be made 10 years

from now, find the net annual premium of this annuity if it is made:

a) over 5 years from purchase date
b) Over the deferred period

Solution



a) Given that, the policy is deferred whole life annuity due

Since the annual premium is made over 5 years and it is less than deferred period (10 years), so by using equation (17)

The net annual premium =

$$= \mathbf{1000}\frac{\mathbf{N_{40+10}}}{\mathbf{N_{40} - N_{40+5}}} = \mathbf{1000}\frac{\mathbf{N_{50}}}{\mathbf{N_{40} - N_{45}}}$$

$$= \mathbf{1000}\frac{\mathbf{33294951}}{\mathbf{57714347.6 - 44455164.3}}$$

$$= \mathbf{57719347.6 - 44455164.3}$$

$$= \mathbf{2510.139\ LE}$$

b) If the policy is deferred ordinary whole life annuity

Since the annual premium is made over the deferred period 10 years. So, by using equation (16)

, the net annual premium =

$$= \mathbf{1000}\frac{\mathbf{N_{40+9+1}}}{\mathbf{N_{40} - N_{50}}} = \mathbf{1000}\frac{\mathbf{N_{50}}}{\mathbf{N_{40} - N_{50}}}$$

$$= \mathbf{1000}\frac{\mathbf{33294954}}{\mathbf{57714347.6 - 33294951}}$$

$$= \mathbf{1363.184\ LE}$$

Exercise 3.3

1- A woman, who is the beneficiary of a 20,000 L.E insurance policy, decides to use this money to purchase a whole life annuity with the first payment due now. If the woman is 30 years old:
   a) What will be the size of annual payment?
   b) If the first payment is made at age 40, what is the net annual premium, if it is made over 6 years from purchase date?

2- A person aged 50 injured by car accident. A jury awarded 300,000 L.E in damages . If that person used $\frac{2}{3}$ of this amount to buy whole life annuity with the first payment to be made when he is 60, what is the size of annual payment.

3- A person has pure endowment policy. He decides when this policy mature at age 60, he will use the sum insured that amounts to 100,000 L.E to buy a life annuity policy beginning at age 65 , what is annual payment that will be provided by proceeds of the insurance policy?.

4- What is the net cost of a whole life annuity of 10,000 L.E a year for a 40 year old woman if the first payment is made 10 years hence? And what is the net annual premium if it is paid over 5 years?

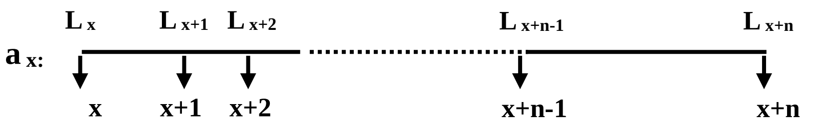
<u>II) Temporary life annuities</u>

When, the payments of a life annuity cease at the end of a certain number of years, even the annuitant is still living, the annuity is called <u>a temporary life annuity</u>.

Temporary life annuities classified into four types. These types are:

1-Ordinary temporary life annuity policy (i.e. immediate temporary life annuity)

By virtue of this policy, insurance company will pay annuity 1 L.E to annuitant at the end of each year for a limited period ***n*** as long as he is alive over this period. This annuity ceases at the end of a certain number of years or at the death of the annuitant, whichever occurs first as indicated in the following diagram



A-Finding the net single premium:

Let the net single premium of this annuity = $a_{x:\overline{n|}}$.

So, it can be calculated by the same assumptions as indicated under whole life annuity as follows:

Prove:

By application equation (1), we may notice from the preceding diagram the following:

- Insureds' obligation who buy ordinary temporary life annuity policy at age x = $L_x \cdot a_{x:\overline{n|}}$ = PV of all net single premium
- Insurance company's obligations may be determined as follows:

At the end of the first year, there will be L_{x+1} survivors according to mortality table, each one obtains 1 L.E. Hence insurance company's

obligation at the end of first year will be $L_{x+1} \times 1 =$ L_{x+1} L.E and its present value on purchase date $= L_{x+1} V^1$

Likewise, <u>at the end of the second year</u>, insurance company's obligations will be L_{x+2} L.E and its present value on purchase date $L_{x+2}V^2$ and so on ---- up to <u>the end of **n** year</u>, insurance company's obligations will be L_{x+n} L.E and its present value on purchase date $= L_{x+n}V^n$

Using equation (1) we find that:-

$$L_x \, . \, a_{x:\overline{n|}} = L_{x+1}V^1 + L_{x+2}V^2 + \ldots\ldots + L_{x+n}V^n$$

By solving this equation and multiplying numerator and denominator by V^x and substituting commutation symbols, we have

$$\mathbf{a_{x:\overline{n|}} = \frac{D_{x+1} + D_{x+2} + \ldots + D_{x+n}}{D_x}}$$

Since $N_{x+1} = D_{x+1} + D_{x+2} + \ldots\ldots + D_{x+n} + D_{x+n+1} + \ldots. + D_{99}$

And

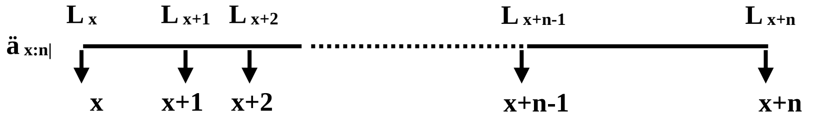
$$N_{x+n+1} = \qquad\qquad + D_{x+n+1} + \ldots. + D_{99}$$

So, the numerator can be written as $N_{x+1} - N_{x+n+1}$

Consequently $\mathbf{a_{x:\overline{n|}} = \dfrac{N_{x+1} - N_{x+n+1}}{D_x} \, x1}$ (18)

<u>2-Temporary life annuity due policy:</u>

In this policy the first payment is made on the date of purchase. So, we have a temporary life annuity due as shown in the following diagram



<u>Finding the net single premiums:</u>

Let $\ddot{a}_{x:\overline{n}|}$ = the net single premium or present value at age x of a temporary life annuity due 1 L.E payable each year for n annual payments by the preceding assumptions Given that the proceeding in the usual manner ,we may get the following equation for calculating <u>the net single premium of temporary life annuity due</u> as follows

$L_x \, . \, \ddot{a}_{x:\overline{n}|} = L_x V^0 + L_{x+1} V^1 + \ldots\ldots + L_{x+n-1} V^{n-1}$

By solving this equation as pointed out earlier, we get the net single premium ($\ddot{a}_{x:\overline{n}|}$) as follows:

$$\ddot{a}_{x:\overline{n}|} = \frac{N_x - N_{x+n}}{D_x} \times 1 \qquad (19)$$

برواز

A notice:

As pointed out earlier, the non-deferred temporary life annuities (ordinary – due) have not annual premiums, for the same reason indicated before.

<u>Example 7:</u>

How much would a woman aged 22 have to pay for a 10 year temporary life annuity of 500 L.E per year if the first payment is to be made when she is 23?

<u>Solution</u>

This annuity is ordinary temporary life annuity as indicated in the following diagram

<u>The net single premium</u>

$$= 500 \times \frac{N_{22+1} - N_{22+10+1}}{D_{22}} = 500 \times \frac{N_{23} - N_{32}}{D_{22}}$$

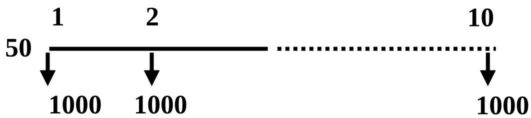
$$= 500 \times \frac{122779580.3 - 84008735.5}{5025845.1}$$

$$= 3857.147 \text{ LE}$$

<u>Example 8:</u>

Calculate the net single premium of a 10 year ordinary life annuity of 1000 L.E per year for a person aged 50, if the first payment is made on the date of purchase.

<u>Solution</u>



From the previous diagram, we can say, the annuity is temporary life annuity due.

So, <u>the single premium</u>

$$= 1000 \times \frac{N_{50} - N_{60}}{D_{50}} = 1000 \times \frac{33294951 - 16510078.8}{1948744}$$

$$= 8.613 \text{ L.E}$$

<u>Exercise (3.4)</u>

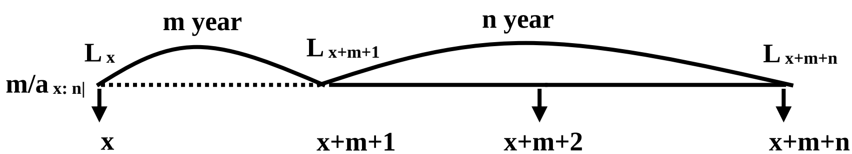
1- How much would a 10- year ordinary temporary life annuity of 1500 L.E yearly, cost a woman aged 35 if the first payment is to be made at age 36 ?
2- A person aged 20 has a pure endowment policy; if he reaches to age 30, he will get sum insured 10,000 L.E. Insurance company permits to him, to pay this sum insured if he is alive at age 30 for purchasing 20 equal annual payments, if the first

payment is due at age 31, what is the size of annual payment?

3- Refer to problem 2, if the first of the 20 annual payments is due at age 30; find the size of annual payment?

<u>3-Deferred ordinary temporary life annuity policy</u>

In this policy, the first annual payment is made after a period is called <u>deferment period (m)</u> as shown in the following diagram.



<u>Finding the net single premium</u>

The first payment will pay at age x+m+1 let $m/a_{x:n|}$ = <u>the net single premium</u> at age x.

Using equation (1) and the preceding diagram, the following equation can be concluded for calculating the net single premium ($m/\ddot{a}_{x:n|}$) as follows :-

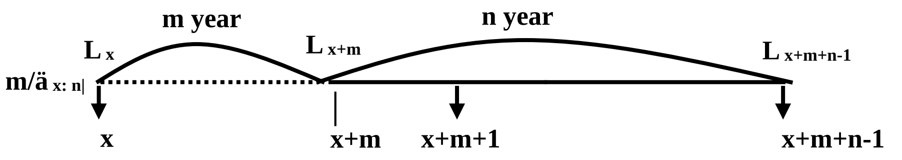
$m/\ddot{a}_{x:n|} = L_{x+m+1}V^{m+1} + L_{x+m+2}V^{m+2} + \ldots\ldots + L_{x+m+n}V^{m+n}$

Solving this equation we obtain

$$m/\ddot{a}_{x:n|} = \frac{N_{x+m+1} - N_{x+m+n+1}}{D_x} \times 1 \qquad (20)$$

<u>4-Deferred temporary life annuity due policy</u>

In this policy the first annual payment begins directly after the deferment period (m) as shown in following diagram



A-Finding the net single premium

Hence, if the net single premium is ($m/\ddot{a}_{x:n|}$). Using equation (1) and the preceding diagram we may find

$L_x \, m/\ddot{a}_{x:n|} = L_{x+m} V^m + L_{x+m+1} V^{m+1} + \ldots\ldots + L_{x+m+n-1} V^{m+n-1}$

Solving this equation, we get the following net single premium:

$$m/\ddot{a}_{x:n|} = \frac{N_{x+m} - N_{x+m+n}}{D_x} \times 1 \qquad (21)$$

B-Finding the net annual premium

A notice:

It is worthwhile to mentioned that the insured can pay the annual premium for the deferred temporary life annuities (ordinary – due). This premium may be made over the deferment period or a period less than it. Hence, *the annual premium* for:

I) *The deferred ordinary temporary annuity policy:* Using equations (3, 3 and 20), the net annual premium

$$AP\,(m/a_{x:n|}) = \frac{N_{x+m+1} - N_{x+m+n+1}}{D_x} \times 1 \qquad (22)$$

II) *The deferred temporary annuity policy*

Using equations (3, 3 and 21), the net annual premium

$$AP\left(m/\ddot{a}_{x:\overline{n|}}\right) = \frac{N_{x+m} - N_{x+m+n}}{D_x} \times 1 \qquad (23)$$

Solved problems

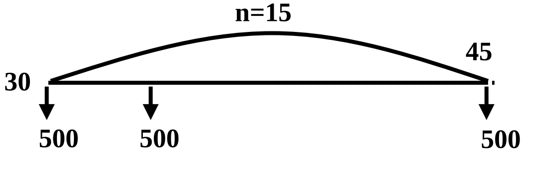
Example 9:

Calculate the net cost of 500 L.E a year temporary life annuity for 15 years for a 30- year old man, if the first payment is to be made when he is a) 30 b) 31 c) 42

Solution

Since the net cost = the single premium, so it can be calculated as follows:-

a) If the first payment is made when the management is 30



The net cost = single premium =

$$= 500\,\frac{N_{30} - N_{40}}{D_{30}} = 500\,\frac{91698461.9 - 57719347.6}{3905782}$$

$$= 4349.845 \text{L.E}$$

b) If the first payment is made when the management is 31

The net cost = single premium =

$$= 500\,\frac{N_{31} - N_{41}}{D_{30}} = 500\,\frac{87792679.9 - 54886345.7}{3905782}$$

$$= 4212.515 \text{L.E}$$

c) If the first payment is made when the management is 42

The net cost = single premium =

$$= 500\,\frac{N_{42} - N_{52}}{D_{30}} = 500\,\frac{52145567.7 - 29371723.9}{3905782}$$

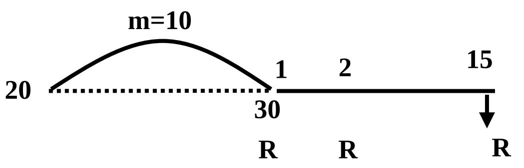
$$= 2915.388 \text{LE}$$

Example 10:

Osama aged 20 paid 1000 L.E to purchase a 15 year temporary life annuity with the first payment to be made to him at age 30, what is the size of each annual payment?.

Solution

This annuity is a deferred temporary life annuity due as indicated in the following diagram



Given that R = the size of annual payment

So:-

$$1000 = R\frac{N_{30} - N_{45}}{D_{20}} = R \times \frac{91698461.9 - 44455164.3}{53512721.8}$$

$$1000 = 8.828R$$

$$R = 113.276L.E$$

برواز

Training

Try to solve this problem again, given that the annuity is ordinary

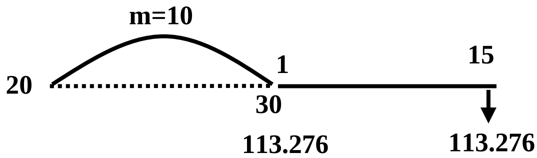
Example 11:

Refer to example 10, find the net annual premium if:

a) it is paid over the deferred period
b) It is paid over the first year after purchasing date.

Solution

a) If the annual premium is paid over the deferred period



The net annual premium

$$= 113.276 \frac{91698461.9 - 44455164.3}{138342809.3 - 91698461.9} = 114.730 \text{LE}$$

b) If the annual premium is paid over 5 years

The net annual premium

$$= 113.276 \frac{N_{30} - N_{45}}{N_{20} - N_{25}} = 113.276 \frac{91698461.9 - 44455164.3}{138342809.3 - 113189602.3}$$

$$= 212.757 \text{L.E}$$

برواز

Training:

Darling reader, try to solve example 10 again if the first payment to be made at age 31

Exercise (3.5)

1- A 25 year old woman was awarded 300,000 in a suit against a fault driver as a result of an accident by her car with his car. If the woman used 1/3 this compensation to purchase a temporary life annuity policy of 30 annual payment, what would be the size of each payment if the first payment is made when she is a) 25 , b) 26 , c) 30

2- Find the present value of a 15 year ordinary temporary life annuity of 1000 L.E yearly for a

woman aged 40, if the first payment is made on the date of purchase.

3- A man sold an apartment at 400,000 L.E. If he used 1/4 of this money to buy a temporary life annuity policy of 20 annual payments when his age 50 years old, what will be the size of each payment, if the first payment is made when he is a) 50, b) 51 and c) 61. Also, calculate the annual premium if it is paid over the deferment period and the first payment is made when he is 65.

In conclusion

Darling reader, in order to remember the foregoing equations for calculating the net single premium whether for whole life annuities or temporary life annuities, the are two general formulas as shown below:

i) For whole life annuities:

The net single premium (PV) of an annuity of 1 per year =

$$= \frac{N_{\text{age on the first payment date}}}{D_{\text{age on purchase date}}}$$

ii)For temporary life annuities:

The net single premiums (PV) of an annuity of 1 per year =

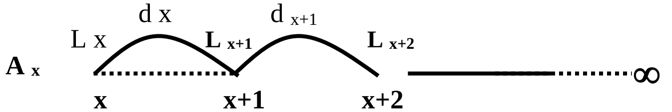
$$\frac{N_{\text{age on the first payment date}} - N_{\text{age on the first payment date} + \text{number of payment}}}{D_{\text{age on purchase date}}}$$

7.3-The net single premium and the net annual premium of the policies that cover the death risk

As pointed out earlier, in particular, in chapter five (fundamentals insurance), the policies that cover the death risk are two main types, they are:
1-Whole life insurance policies
2-Term insurance policies

7.3.1-Whole life insurance policy

By virtue of this policy insurance company pay the sum insured of the policy to the beneficiary upon the death of insured in any time as indicated in the following diagram:



A-Finding the net single premium
Assumptions:
Let the single premium of this policy is $\mathbf{A_x}$, and the sum insured is 1 L.E
Required: Finding the net single premium (A_x)
Prove:
Using equation (1) and the preceding diagram, we can determine both insureds' obligation and insurance company's obligations as follows:

- Insureds' obligations on the date of purchase at age x equal the number of living at age x times the net single premium paid by each ,that is
 $L_x \cdot A_x$

- Insurance company's obligations may be determined as follows:
  - At the end of the first year, the company will have to pay d_x L.E in death benefits.
  - At the end of the second year, the company will pay
    d_{x+1} L.E and so … on out to d_{99} L.E

Using equation (1) we find
PV of all net single premium to be paid = PV of all benefits to be received.
That is,
$L_x . A_x = d_x V^1 + d_{x+1} V^2 + d_{x+2} V^3 + \ldots\ldots.. + d_{99} V^{100-x}$
Solving for A_x and multiplying numerator and denominator by V^x, we obtain
$A_x = d_x V^{x+1} + d_{x+1} V^{x+2} + d_{x+2} V^{x+3} + \ldots\ldots + d_{99} V^{100}$
By using commutation functions, the preceding equation will become

$$\mathbf{A_x = \frac{C_x + C_{x+1} + C_{x+2} + \ldots\ldots + C_{98} + C_{99}}{D_x}}$$

Consequently, the net single premium (A_x) will be:

$$\mathbf{A_x = \frac{M_x}{D_x} \times 1} \qquad (24)$$

B-Finding the next annual premiums
The net annual premium for this policy can be calculated depending upon the form of the policy and equations (3, 3) as follows:
i) If the policy is straight life policy (premiums are payable periodically until death)

The net annual premium $\mathbf{AP_x = \frac{M_x}{N_x} \times 1}$ (25)

__ii) If the policy is limited payment whole life policy__
(premiums are payable periodically for a specified number of years)

The net annual premium $\mathbf{A^{n}P_{x} = \frac{M_{x}}{N_{x}N_{x+n}} \times 1}$ (26)

(Where n = period of paying the annual premium)

__Solved problems__

__Example 12:__

What is the net single premium of 500 L.E whole life policy issued to a person aged 50. If the policy is a 15 payment life policy, what is the net annual premium?

__Solution__

The net single premium (A_{50}) = $\mathbf{5000\frac{M_{50}}{D_{50}} = 5000\frac{102988}{1498744}}$

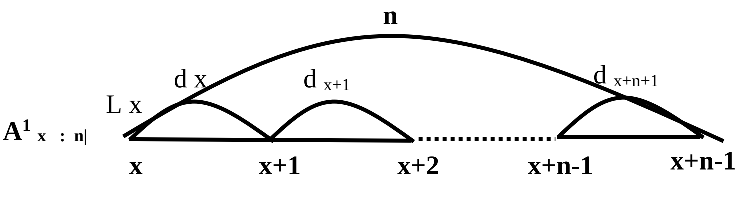
= 2574.086 L.E

The net annual premium =

$$\mathbf{5000\frac{M_{50}}{N_{50} - N_{65}} = 5000\frac{1028988}{33294951 - 10606827.5} = 226.767 L.E}$$

__7.3.2-Term insurance policy__

By means of this policy, insurance company pays the sum insured of the policy, if the insured dies during the term covered (n) by the policy as shown in the following diagram.



Let $\mathbf{A^{1}_{x}:_{\overline{n}|}}$ = the net single premium for 1 L.E, n year term policy to a person aged x

A-Finding the net single premium

The premiums and deaths benefits for the period of policy from x to x +n can be setup the following equation

$L_x.\ A^{\ 1}_{x:n|} = d_x V^1 + d_{x+1} V^2 + d_{x+2} V^3 + \ldots.. + d_{x+n-1} V^n$

Solving this equation by the same previous procedure we obtain

$$A^{1}_{x:n|} = \frac{C_x + C_{x+1} + C_{x+2} + \ldots\ldots + C_{x+n-1}}{D_x}$$

And since

$M_x = C_x + C_{x+1} + C_{x+2} + \ldots c_{x+n-1} + c_{x+n} \ldots\ldots \qquad + C_{99}$

And

$M_{x+n} = \qquad\qquad\qquad + \ C_{x+n} + C_{x+n+1} + \ldots. + C_{99}$

So the numerator can be written as $(Mx - Mx+n)$ so that

$$A^{\ 1}_{x:n|} = \frac{M_x - M_{x+n}}{D_x} \times 1 \qquad (27)$$

B-Finding the net annual premium

The net annual premium for this policy can be calculated as follows:

a) If the premium is paid over the period of the policy (n)

$$AP^{\ 1}_{x:n|} = \frac{M_x - M_{x+n}}{N_x - N_{x+n}} \times 1 \qquad (28)$$

b) If the premium is paid for a period m (where m < n)

$$A\ {}^{m}P^{\ 1}_{x:n|} = \frac{M_x - M_{x+n}}{N_x - N_{x+m}} \times 1 \qquad (29)$$

Solved problems

Example 13:

A 500, 40 year term policy is issued to Aaya Osman aged 25 find a) The net single premium

b) The net annual premium

Solution

a) The net single premium

$$= 5000 A^{\;1}_{25}:\overline{40|} = 5000\,\frac{M_{25} - M_{65}}{D25}$$

$$= 5000\,\frac{1276593 - 686751.2}{4573377.1}$$

$$= 644.865 \text{ LE}$$

b)The net single premium

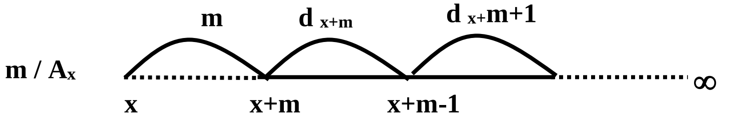
$$= 5000 AP^{1}_{25}:\overline{40|} = 5000\,\frac{M_{25} - M_{65}}{N_{25} - N_{65}}$$

$$= 5000\,\frac{1276593 - 686751.2}{113189602.3 - 10606827.5}$$

$$= 28.749 \text{ L.E}$$

7.3.3-Deferred whole life Insurance policy

By means of this policy, insurance company provide insurance protection by paying the sum insured upon the death of insured in anytime , but only after m years have passed as indicated in the following diagram.



Finding the net single

Let, the net single premium of this policy = m / A_x and sum insured is 1 L.E

Hence, the net single premium can be calculated by the same manner that had been followed in the preceding policies as show below:

$m / A_x \,.\, L_x = d_{x+m}V^{m+1} + d_{x+m+1}V^{m+2} + d_{x+m+2}V^{m+3} + \ldots + d_{99}V^{100-x}$

Solving this equation we obtain

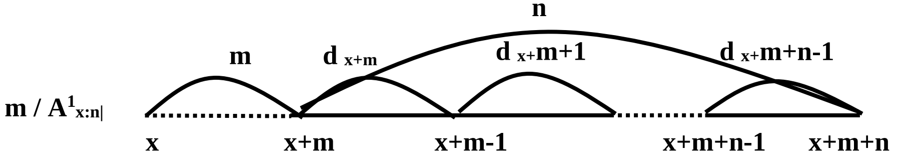
$$m/A_x = \frac{M_{x+m}}{D_x} \times 1 \qquad (30)$$

If the annual premium AP (m / A$_X$) of this policy paid over the deferment period it, can be deduced using equations (3 and 3) by the following equation:

$$AP(m/A_x = \frac{M_{x+n}}{N_x - N_{x+m}} \times 1 \qquad (31)$$

<u>7.3.4-Deferred term life insurance policy</u>

By virtue of this policy, insurance company provide insurance protection by paying the sum insured, if the insured dies during n years but only after ***m*** years have passed as indicated in the following diagram.



Let, the net single premium of this policy = $m/A^{1}_{X:n|}$ and the sum insured is 1 L.E.

Then, the value of $m/A^{1}_{X:n|}$ can be calculated by deducing the following equation from the preceding diagram

$m/A^{1}_{X:n|}$. $L_X = d_{x+m}V^{m+1} + d_{x+m+1}V^{m+2} + \dots + d_{x+m+n-1}V^{m+n}$

Solving this equation by the same previous manner we get:

$$m/A^{1}_{x:n|} = \frac{M_{x+m} - M_{x+m+n}}{D_x} \times 1 \qquad (32)$$

<u>برواز</u>

A notice

Equation (31) can be concluded by other method using the previous diagram as follows:

$$m/A^1_{x}:\overline{n|} = A^1_{x}:_{\overline{m+n|}} - A^1_{x}:_{\overline{m|}}$$

$$= \frac{M_x - M_{x+m+n}}{D_x} - \frac{M_x - M_{x+m}}{D_x}$$

$$m/A^1_{x}:_{\overline{n|}} = \frac{M_{x+m} - M_{x+m+n}}{D_x} \qquad (32)$$

Finding the net annual premium

Moreover, the net annual premium of this policy (7.3.4) can be deduced using equations (3 and 3) if it is paid over a period K (for example) by the following formula

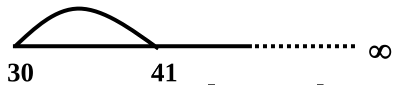
$$AP\left(K/P^1_X:\overline{n|}\right) = \frac{M_{X+m} - M_{X+m+n}}{N_X - N_{X+k}} \qquad (33)$$

Solved problems

Example (14)

What is the present value of a whole life insurance policy of 5000 L.E, deferred 11 years, issued to woman now aged 30. Find the net annual premium of this policy, if it is paid over 6 years.

Solution



a) Since the present value = the net single premium

So, the net single premium =

$$5000\frac{M_{41}}{D_{30}} = 5000\frac{1142147}{3905782} = 1462.123 \text{ L.E}$$

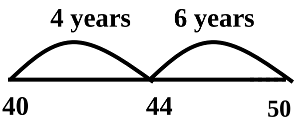
b) The net annual premium that is paid for 6 years

$$5000\frac{M_{41}}{N_{30}-N_{36}} = 5000\frac{1142147}{91698461.9-70021352.8} = 263.445 \text{ LE}$$

Example 15

Find the net single premium for a six year term insurance policy of 3000 L.E deferred four years, issued to a man who is 40 years old. What is the annual premium that is paid for 5 years?

Solution



- The net single premium

$$4/A^{1}_{40:\overline{6|}} = 6000\frac{M_{44}-M_{50}}{D_{40}}$$

$$= 6000\frac{1109925-1028988}{2833001.8}$$

$$= 857.983 \text{ L.E}$$

- The annual premium

$$AP\left(6/AP^{1}_{40:\overline{6|}}\right) = 3000\frac{M_{44}-M_{50}}{N_{40}-N_{46}}$$

$$= 3000\frac{1109925-1028988}{57719347.6-42062259.5}$$

$$= 15.508 \text{ L.E}$$

Exercise 3.6

1- What is the net single premium of a 10,000 L.E whole life policy issued to a man aged 50. If, the policy is a 10- payment life policy, what is the net annual premium?

2- Find the present value and the net annual premium for a 15- year , 10,000 L.E term policy issued to a woman aged a) 30.., b) 35., c)50
3- What is the net single premium for a 10- year term policy of 4000 L.E deferred 10 years , issued to a woman who is 45 years old.

<u>In Conclusion</u>

Darling reader, in order to remember the above equations for calculating the net single premium or the net annual premium, (whole life policy – deferred whole life policy – term life policy – deferred term life policy) there are *<u>two</u>* general formulas as shown below:

<u>i) The net single premium of a 1 L.E whole life or term insurance policy.</u>

The net <u>single</u> premium =

$$\frac{M_{\substack{\text{age when insurance}\\ \text{protection begins}}} - M_{\substack{\text{age when insurance}\\ \text{protection ends}}}}{D_{\text{age on purchase date}}}$$

<u>ii) The net annual premium of a 1 L.E whole life or term insurance policy</u>

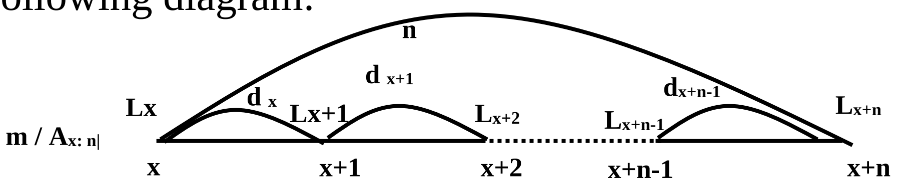
The net <u>annual</u> premium =

$$\frac{M_{\substack{\text{age when insurance}\\ \text{protection begins}}} - M_{\substack{\text{age when insurance}\\ \text{protection ends}}}}{N_{\text{age on purchase date}} - N_{\substack{\text{age when annual}\\ \text{premium ends}}}}$$

7.4-The net single premium and the net annual premium of the policies that cover <u>*a combination* of death risk and survival risk</u>

<u>*7.4.1-Endowment insurance policy*</u>

In addition to, the policies that have already studied in sections (7.2 and 7.3), insurance companies might issued a combination policies (i.e. complex policies) for covering two or more risks, for example, the <u>*endowment insurance policy*</u>. This policy combines the features of a <u>*pure endowment policy*</u> and <u>*term insurance policy*</u>. By virtue of this policy, insurance company will pay the sum insured whether the insured dies during the period of the policy or he is living at the end of the period of the policy as indicated in the following diagram:



<u>*A-Finding the net single premium*</u>

Let the single premium of the endowment policy of 1 L.E = $\mathbf{A}_{x:\overline{n|}}$

Then the single premium $\mathbf{A}_{x:\overline{n|}}$ can be calculated using equation (1) and the preceding diagram by determining both <u>insurds' obligations</u> and <u>insurance company's obligations</u> as follow:-

- <u>Insureds obligations</u> on the date of purchase at age x = $\mathbf{L_X A_{X:\overline{n|}}}$

- Insurance company's obligations maybe determined as follows:

At the end of the first year, the company will pay d_x L.E in death benefits

At the end of the second year, the company will pay d_{x+1} L.E and so on.. up to d_{x+n-1} L.E

At the end of the policy (n period) will pay L_{x+n} L.E

Consequently, by using equation (1), we find PV of all net single premiums to be paid = PV of all benefits to be received

That is,

$L_x \,.\; A_{x:\overline{n|}} = d_x V^1 + d_{x+1} V^2 + \ldots\ldots + d_{x+n-1} V^n + L_{x+n} V^n$

Solving this equation for $A_{x:\overline{n|}}$ by the same previous manner, we obtain:

$$A_{x:\overline{n|}} = \frac{(M_x - M_{x+n}) + D_{x+n}}{D_x} \times 1 \qquad (34)$$

B-Finding the net annual premium

Moreover, the net annual premium for this policy can be calculated by equations (3, and 3) as follows:

i) If the premium is paid over the period of the policy (n)

$$AP(P_{x:\overline{n|}}) = \frac{(M_x - M_{x+n}) + D_{x+n}}{N_x - N_{x+n}} \times 1 \qquad (35)$$

ii) If the premium is paid for a period K (where m < n)

$$AP^k(P_{x:\overline{n|}}) = \frac{(M_x - M_{x+n}) + D_{x+n}}{N_x - N_{x+k}} \times 1 \qquad (36)$$

7.4.2-Deferred endowment insurance policy

By virtue of this policy, insurance company provides insurance protection whether in the case of death of

insured or if he is still living at the end of policy, but only after m years have passed as indicated in the following diagram.

Let, the net single premium of this policy = $m/A_{x}:_{\overline{n|}}$ and the sum insured 1 L.E.

Then, the value of $m/A_{x}:_{\overline{n|}}$ can be calculated by deducing the following equation from the preceding diagram

$m/A_{x}:_{\overline{n|}} . L_{x} = d_{x+m}V^{m+1} + d_{x+m+1}V^{m+2} + .. + d_{x+m+n-1}V^{m+n} + L_{x+m+n}V^{m+n}$

Solving this equation we get:

$$m/A_{x}:_{\overline{n|}} = \frac{(M_{x+m} - M_{x+m+n}) + D_{x+m+n}}{D_{x}} \times 1 \qquad (37)$$

Also, ***the net annual premium*** for deferred endowment insurance policy may be determined by equations (3, and 3) as follows:-

i) If the premium is paid over period of the policy (n)

$$AP\left(m/P_{x}:_{\overline{n|}}\right) = \frac{(M_{x+m} - M_{x+m+n}) + D_{x+m+n}}{N_{x} - N_{x+n}} \times 1 \qquad (38)$$

ii)If the premium is paid for a period K (where m < n)

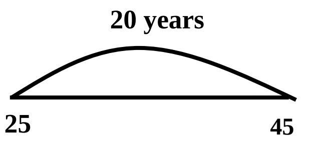
$$AP^{k}\left(m/P_{x}:_{\overline{n|}}\right) = \frac{(M_{x+m} - M_{x+m+n}) + D_{x+m}}{N_{x} - N_{x+k}} \times 1 \qquad (39)$$

Solved problems

Example 16:

Find the net single premium and the net annual premium for 20,000 L.E, 20 -year endowment policy issued to a man 25 years -old.

Solution



a) The net single premium =

$$\mathbf{A}_{25:\overline{20|}} = 20000\,\frac{(\mathbf{M}_{25}-\mathbf{M}_{45})+\mathbf{D}_{45}}{\mathbf{D}_{25}} \times 1$$

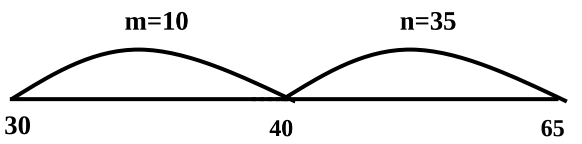
b) The net annual premium =

$$\mathbf{AP}\left(\mathbf{P}_{25:\overline{20|}}\right) = 20000\,\frac{(\mathbf{M}_{25}-\mathbf{M}_{45})+\mathbf{D}_{45}}{\mathbf{N}_{25}-\mathbf{N}_{45}} \times 1$$

Example 17:

What is the present value for a 25 year endowment insurance policy of 7000 L.E deferred 10 years, issued to a person now aged 30.

Solution



Since the present value = the single premium.

So, the single premium = $7000\,\frac{(\mathbf{M}_{40}-\mathbf{M}_{65})+\mathbf{D}_{65}}{\mathbf{D}_{30}}$

Exercise 3.7:

1- Find the present value for a 22 year endowment insurance policy of 4500 L.E deferred 8 years, issued to a woman now aged 30.
2- A husband 35 years- old and his wife 30 years- old take out a 60,000 L.E endowment at age 65

policy. Find the net single premium and the net annual premium.

3- Find the net annual premium for 20,000 LE , 25 year endowment policy issued to a) a 30 year - old man b) a 55 year old man

4- Find the net single premium for a 25 year endowment insurance policy of 5000 L.E deferred 15 years, issued to a woman who is now 25.

<u>In conclusion:</u>

To remember all equations of endowment insurance policy (equation 34 - equation 39) there are ***<u>two</u>*** general formulas as shown below:

<u>i)The net single premium of a 1 L.E</u>

The net <u>single</u> premium =

$$\frac{\left(\mathbf{M}_{\substack{\textbf{age when insurance}\\ \textbf{protection begins}}} - \mathbf{M.....ends} + \mathbf{D....ends}\right)}{\mathbf{D}_{\textbf{age on purchase date}}}$$

ii)The net annual premium of a 1 L.E

The net <u>annual</u> premium =

$$\frac{\left(\mathbf{M}_{\substack{\textbf{age when insurance}\\ \textbf{protection begins}}} - \mathbf{M.....ends} + \mathbf{D....ends}\right)}{\mathbf{N}_{\textbf{age on purchase date}} - \mathbf{N}_{\textbf{age on purchase date}}}$$

7.5-The Mathematical Reserve of Life Insurance Policies

<u>7.5.1-Definition of reserve</u>

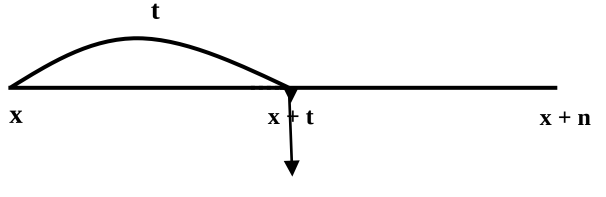
As pointed out earlier (see 5.4 characteristics of life insurance policies – chapter one) the premiums of most life insurance policies are *level premiums*, in spite of the probability of death risk is increasing from year to year. Consequently the level premiums are higher than mortality rates in the early years of the life policy and much less in the latter year (see figure 5.1). Hence, the *excess* payments during the early years of a level premium policy build ***a reserve***. Insurance company creates a fund from the reserve and invests it. This fund is held by insurance company for the benefits and credit of the policyholders.

Now, the following question may be raised.

What is the reserve and what are the methods for calculating it?

The answer

The reserve is "*The value accumulated from the excess premiums and its interest*". In other words, the reserve is "The value of a life insurance policy at the end of any policy year. *For example* at the end of (x + t) years as indicated in the following diagram:



بروز

A notice:

The net annual premium for the $(x + t)$ year is not included

7.5.2-Calculation Methods of Mathematical Reserve

In effect, the value of mathematical reserve at the end of a particular policy may be calculated by various methods. The two common methods are called *retrospective method*, and *prospective method*.

However, study these two methods are demanded the following *assumptions*:

1- The net annual premiums should be paid by policyholders at the beginning of each policy year and are invested at a given interest rate.
2- The insurance company will pay the death benefits for the beneficiary at the end of year of the death.
3- The mathematical reserve is invested by the same interest rate that had been taken into consideration during calculation of level premium.

Hence, the previous two methods (retrospective and prospective) can be illustrated as follows:

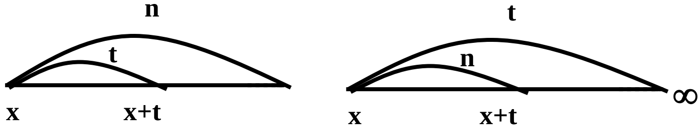
7.5.2.1-Retrospective method

The retrospective method is based on the premiums collected in *the past* and the death benefits paid in the past. Taking the foregoing assumptions into

consideration the difference between summation net premiums and their interest less death benefits payments is the ***reserve***.
Consequently, the reserve for a life insurance policy of 1 L.E and its annual premiums are paid over n years, can be determined at age $x + t$ by the following formula

برواز للمعادلة

$$\textbf{The reserve} = \begin{matrix}\textbf{Amount of net premium in} \\ \textbf{the past at age } x+t\end{matrix} - \begin{matrix}\textbf{Amount of death benefits in} \\ \textbf{the past at age } x+t\end{matrix} \qquad (40)$$

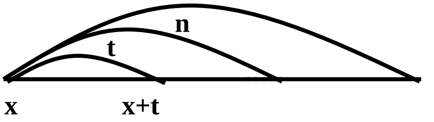


The preceding formula may be expressed _symbolically_, as shown in the following formula

$$\mathbf{V} = \frac{\mathbf{AP}(\mathbf{N}_x - \mathbf{N}_{x+t}) - (\mathbf{M}_x - \mathbf{M}_{x+t})}{\mathbf{D}_{x+t}} \ (\mathbf{t \le n}) \quad (41)$$

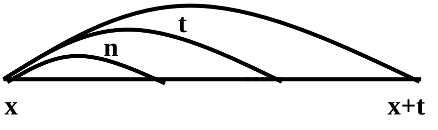
Where:
Ap = the net annual premium for a policy of 1 L.E
t = the reserve is desired at the end of the t^{th} policy year
V = t^{th} reserve per surviving policyholder for a 1 policy
V' = v $\times$ face value of the policy = the total t^{th} reserve per surviving policyholder
n = period of annual premiums

Formula (41) may be understood by looking at the following diagram:-



Where the date of reserve before the date of last payment of annual premiums

However, if the date of reserve is after the date of last payment of annual premiums as shown in the following diagram



The formula of V will be as follows:

$$\mathbf{V = \frac{AP\left(N_x - N_{x+n}\right) - \left(M_x - M_{x+t}\right)}{D_{x+t}} \quad (t > n)} \qquad (42)$$

Solved problems

Example 18:

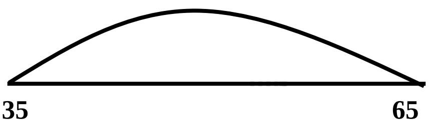
An employee at TANTA University aged 35 has 30 year term insurance policy for 5000 L.E

a) Find the annual premium if the policy is a 10 -payment policy

b) What is the reserve at the end of

i) 15 years ii) 8 years

Solution



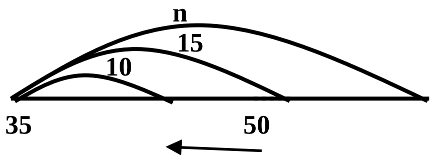
a) The annual premium =

$$AP\left(P_{35}:_{35|}\right) = 5000\,\frac{\left(M_{35}-M_{65}\right)}{N_{35}-N_{45}}$$

$$= 5000\,\frac{1194810-686751.2}{73352648.1-44455164.1}$$

$$= 0.01758$$

<u>b) i) Reserve at the end of 15 years</u>



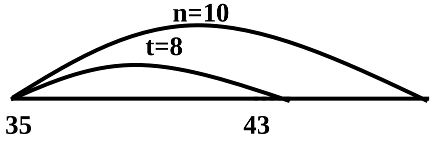
Reserve $= 0.01758\,\dfrac{\left(N_{35}-N_{45}\right)}{D_{50}}$

$$= \frac{\left(73352648.1-44455164.1\right)-\left(1194810-1028988\right)}{1998744}$$

$$= 0.17120$$

So, *<u>the fifteenth reserve</u>* of policy= 5000 x 0.17210 = 856.027

<u>b) ii) Reserve at the end of 8 years</u>



Reserve = $0.01758\,\dfrac{\left(N_{35}-N_{43}\right)-\left(M_{35}-M_{43}\right)}{D_{43}}$

$$= \frac{\left(73352648.1-49494836.4\right)-\left(1194810-1121197\right)}{2562793.9}$$

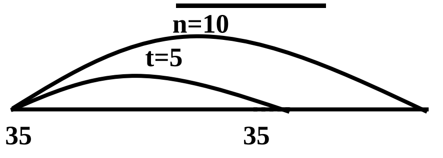
$$= 0.1349$$

So, *<u>the eighth reserve</u>* of the policy = 5000 x 0.1349 = 674.69

<u>Example 19:</u>

Find the fifth reserve for a 10- year endowment policy of 10000 L.E to a man aged 30 if you know the annual premium over the period of the policy is 858.203 L.E

Solution



$$\text{Reserve for 1 L.E} = \frac{(N_{30} - N_{35}) - (M_{30} - M_{35})}{D_{35}}$$

$$= 0.08582\,\frac{(91698461.9 - 73352648.1) - (1234953 - 1144810)}{3331295.4}$$

$$= 0.460569$$

So, <u>*the fifth reserve*</u> of the policy = 10000×0.460569

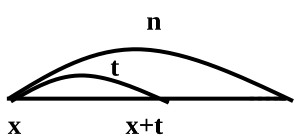
$= 4605.694$ L.E

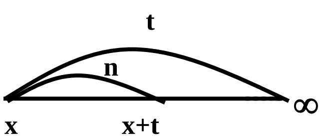
<u>*7.5.2.2-Prospective method*</u>

This method is based on the annual premium to be collected in <u>*the future*</u> and the annual death benefits to be paid in the future.

Consequently, the reserve for a life insurance policy of 1 L.E and its annual premium are paid over n year, can be determined at age x + t by the following formula:

$$t^{th}\ \text{reserve}(V^{-}) = \begin{matrix}\textbf{PV of future death}\\ \textbf{benefit at age }(x+t)\end{matrix} - \begin{matrix}\textbf{PV of future net annual}\\ \textbf{premiums at age }(x+t)\end{matrix} \qquad (43)$$





Since at age x + t the present value of future death benefits is the net single premium for the benefits.
Also, the present value of future net annual premiums is the present value (i.e. the single premium) of a life annuity due.
So, the formula (43) may be rewritten as follows:

***t*^th^ reserve (V^-) =** **sum unsired** **The net single premium for the benefit** **-** **Annual x the net single ppremium of a life annuity at age x + t** **(44)**

برواز

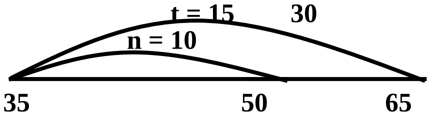
A notice: It is worthwhile to mentioned that , finding the reserve of any policy, whether by *retrospective method* or *prospective method*, will results in the same result (i.e. they give identical values). However, finding the reserve by the prospective method is easier than retrospective method as shown below.

Solved problems

Example 20:

Use the prospective method to compute the reserve in example 18

Solution



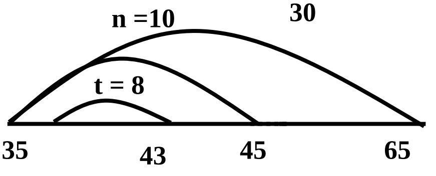
i)The reserve at the end of 15 years =

$$5000\frac{(M_{35}-M_{65})-0}{D_{50}}=5000\frac{1028988.1-686751.1}{1998144}=856.027$$

The same result in the *retrospective method*

(Where the net single premium PV of a life annuity at age 50 = zero)

ii) Reserve at the end of 8 years



$$\text{Reserve}=5000\frac{(M_{43}-M_{65})-87.9(M_{43}-M_{45})}{D_{43}}$$

$$=5000\frac{(1121197.7-686751.2)-87.9(49494836.4-44455164.3)}{2562794}$$

= 674.74 L.E (The same result by retrospective method)

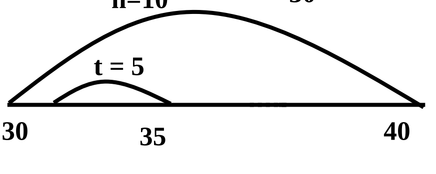
Where the annual premium of the policy =

$$5000\frac{(M_{35}-M_{65})-0}{N_{35}-N_{45}}=87.9$$

Example 21:

Use the prospective method to compute the reserve in example 19

Solution



$$\text{Reserve} = 10000\,\frac{(M_{35} - M_{40}) - 808.203(N_{35} - N_{40})}{D_{35}}$$

$$= 10000\,\frac{(1194810 - 1151856) + 2833001.8 - 808.203(73352648.1 - 57719347.6)}{3331295.4}$$

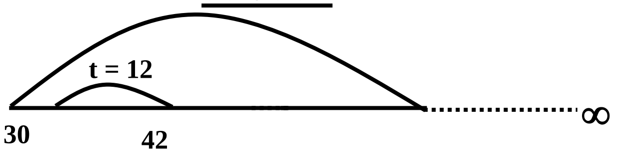
=4605.71 (The same result by retrospective method)

Example 22:

Find the twelfth reserve for 1000 L.E of straight life policy issued to a person aged 30

a) by prospective method
b) by retrospective method

Solution



a) Reserve (by prospective method) = 1000 A_{42} . Annual premium $x\,\ddot{a}_{42}$

Since annual premium = $1000\,\dfrac{M_{30}}{N_{30}} = 1000\,\dfrac{1234953}{91698461.9} = 13.467$

So the Reserve

$$= 1000\,\frac{M_{42}}{D_{42}} - 13.467\,\frac{N_{42}}{D_{42}} = 1000\,\frac{1131928}{2650731.3} - 13467\,\frac{52145567.7}{2650731.3} = 162.10$$

b) Reserve (by retrospective method) =

$$0.013467\,\frac{(N_{30} - N_{42}) - (M_{30} - M_{42})}{D_{42}}$$

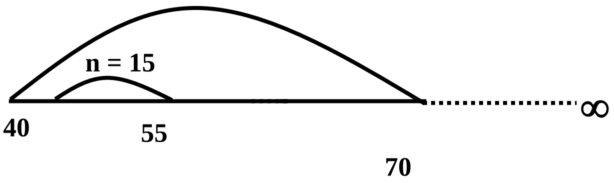
$$0.013467\,\frac{(91698461.9 - 521455677) - (1234953 - 1131928)}{2650731.3} = 162.081$$

By comparison, we find the same result roughly.

Example 23

Find the 30th reserve for A 10,000 whole life policy is issued to a woman aged 40 if the policy is 15 payment life policy by prospective method.

Solution



$${}^{-}V_{30} = 10000 \times A_{70} - 0 = 10000\frac{M_{70}}{D_{70}} = 10000\frac{525191.7}{706256.3} = 7436.27$$

Exercise 3.8

Solve the following problems by

a) retrospective method b) prospective method

1- A 20 year term life insurance policy of 5000 L.E was issued to a woman aged 35. Find the 10th and the 20th reserve
2- Find a) the 12th and b) the 25th reserve for 25 year endowment policy of 5000 L.E issued to a man aged 25
3- A 20,000 L.E whole life policy is issued to a person aged 25 if the policy is 10- payment life policy. Find a) the 3rd b) the 5th and c) the 20th reserve
4- A life insurance policy was issued to a woman aged 30. The value of net single premium of benefits at the end of 15th policy year is 1150.5

L.E. If the net annual premium 39.75 L.E payable for a life. Find the 15^{th} reserve by retrospective method only.

5- What is the reserve per survivor at the end of 9^{th} policy year for a 5000 L.E, 20 -year endowment policy issued to a 30 year old woman?

Review Exercise for Chapter Seven

1- Insurance policy issued to a person aged 32 by which the insurance company will pay a) 4000 L.E if he dies between age 32 and 40, b) 6000 L.E if he is a live at age 40, c) 5000 L.E if he dies after reaching age 40 but before 60 and d) 2000 if he dies after reaching age 60 but before 70, or if he is a live at age 70. Find the present value of this policy, then, Find the net annual premium, if the policy is a 30- payment policy.

2- An insurance policy issued to a businessman aged 45 provides the following benefits a) 50,000 L.E if he dies with 15 years b) 100,000 L.E if he dies after 15 years and c) 70,000 L.E if he is a live at age 60. Find the net annual premium for a life.

3- A Businessman purchased insurance policy for his child aged 10. By this policy insurance company promises to pay a)20,000 L.E if the insured dies before age 26 b) 8000 if death occurs after 25 but before 35 c) 100,000 if he still living at age 40 and d) 10,000 if death occurs after age 30 but before 60 or if the insured

is living at age 60. Find a) the present value of this policy b) the net annual premium if the policy is a 20 -payment policy.

4- Insurance policy issued to a person at age 30 promises that the insurance company will pay 2000 L.E if the insured dies before reaching age 45 , 5000 if he dies after reaching 45 but before 50, 10,000 if he dies after reaching 50, but before 70 or if he is a live at 70. ***Required***:
   a. Finding the net single premium
   b. Finding the net annual premium if a) the premium of the policy is payable annually for a life, and b) the premium is payable in 10 equal payments.

5- A businesswoman her age 30 years- old inherits one million dollars. She wants to purchase as much paid up life insurance policy as possible with 10% from her heritance. This amount (10%) will cover the net single premium of the policy. Find the size of policy if it is
   a. 15 year term policy
   b. 20 year term policy
   c. 20 year endowment policy
   d. whole life policy

6- A 40 year old man has 5000 L.E a year to use for purchasing life insurance policy. If this amount will cover the net annual premium. What is the size of policy if the policy is:
   a. 20 payment life
   b. 15 year term

c. paid up at age 50

7- *Ayman Abdel Mawla*, purchased a life insurance policy when his age 25. This policy provides the following benefits 3000 L.E if he is a live at age 50, 6000 L.E if he dies before reaching age 50, 4000 L.E if he dies between age 50 and age 60, 3000 L.E whole life annuity at the beginning age 60. ***Required*** :

Find the net single premium of this policy.

8- A household aged 40 purchased a life insurance policy from Misr insurance company. This policy provides for him, his son, his daughter and his wife the following benefits 10,000 L.E if he is still living at age 60, 5000 L.E pays for his wife if he dies before reaching 60, 4000 L.E pays for his son at the beginning of University (18 years) , 3000 L.E pays for his daughter at the beginning of secondary school (at age 15), 5000 L.E a whole life annuity starts at age 60, 2000 L.E pays for his wife if she is reaching age 50. ***Required*** Finding the net annual premium if the policy is 10 -payment policy and the ages of the wife, the son, the daughter are 35, 6, 5 years respectively.

9- Find the 11^{th} reserve for a 5000 L.E policy issued to businessman aged 35. Given that, the policy is

a. a straight life policy,
b. a 20 year term insurance policy
c. a 20 payment life policy
d. 27- year endowment insurance policy.

Solve each point two times, one by retrospective method and the second by prospective method. Compare your solutions by the two methods and what is your comment?

10- Compute the tenth year reserve for a 5000 L.E 15- payment whole life policy issued at age 40

11- Calculate the twelfth reserve for 10000 L.E policy issued to a person aged 30 on each of the following policies a) sixteen- payment whole life b) ten- payment whole life policy and c) ordinary life policy.

Important concepts and terminologies to remember:

Deferred endowment insurance policy
Deferred ordinary whole life annuity policy
Deferred temporary life annuity due policy
Deferred term insurance policy
Deferred whole life annuity due policy
Deferred whole life policy
Endowment insurance policy
Immediate whole life annuity
Mathematical reserve
Ordinary temporary life annuity policy
Ordinary whole life annuity
Prospective method
Pure endowment policy
Retrospective method
Temporary life annuity due
Term insurance policy
The net annual premium

The net single premium
Whole life annuities
Whole life annuity due
Whole life policy
Whole life policy