**PREMIUMS**

|  |  |  |
| --- | --- | --- |
| Fully continuous | The premiums will be paid continuously | The benefit will be paid at the moment of death |
| Fully discrete | The premiums will be paid at the start of each year | The benefit will be paid at the end of the year of death |
| Semi-continuous | The premiums will be paid at the start of each year | The benefit will be paid at the moment of death |

When we have CFM:

* The premium (in case of cont. whole life) will be
* In case of discrete whole life
* Also,

When we have a De Moivre Law:

|  |  |  |
| --- | --- | --- |
|  | Continuous | Discrete |
| Whole life |  |  |
| n-payments whole life |  |  |
| n-year endowment |  |  |
| n-term |  |  |
| Pure endowment |  |  |
| m-payment  n-term |  |  |

**Loss-at-issue (, AT TIME 0)**

* If then the company is losing, because the death benefit is higher than the premium.
* If then the company is not winning but also not losing, (the EP)
* If then the company is winning, because the death benefit is lower than the premium.

When we take the expectation of L (which is the net premium reserve), we will have ., because of the EP:

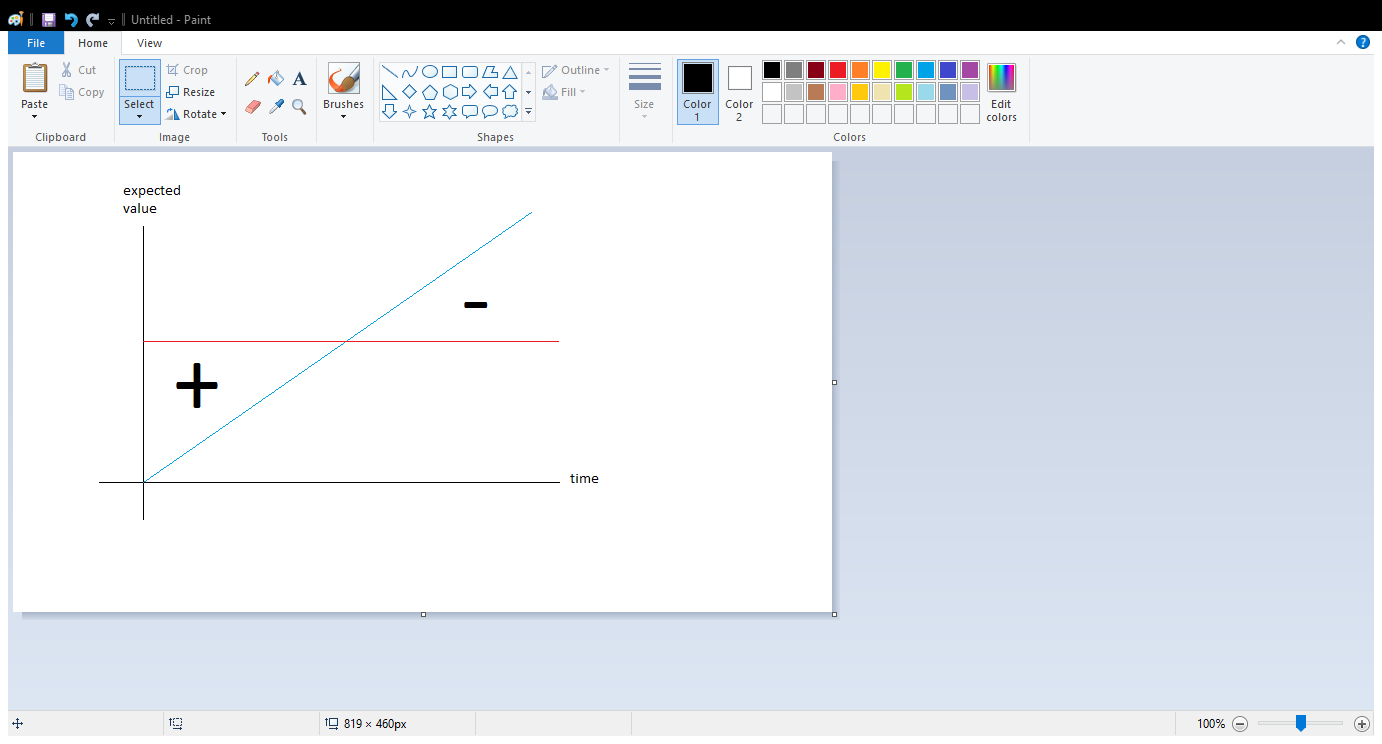
For example, for a cont. whole life insurance:

So we take the variance of L, which is .

**Percentiles**

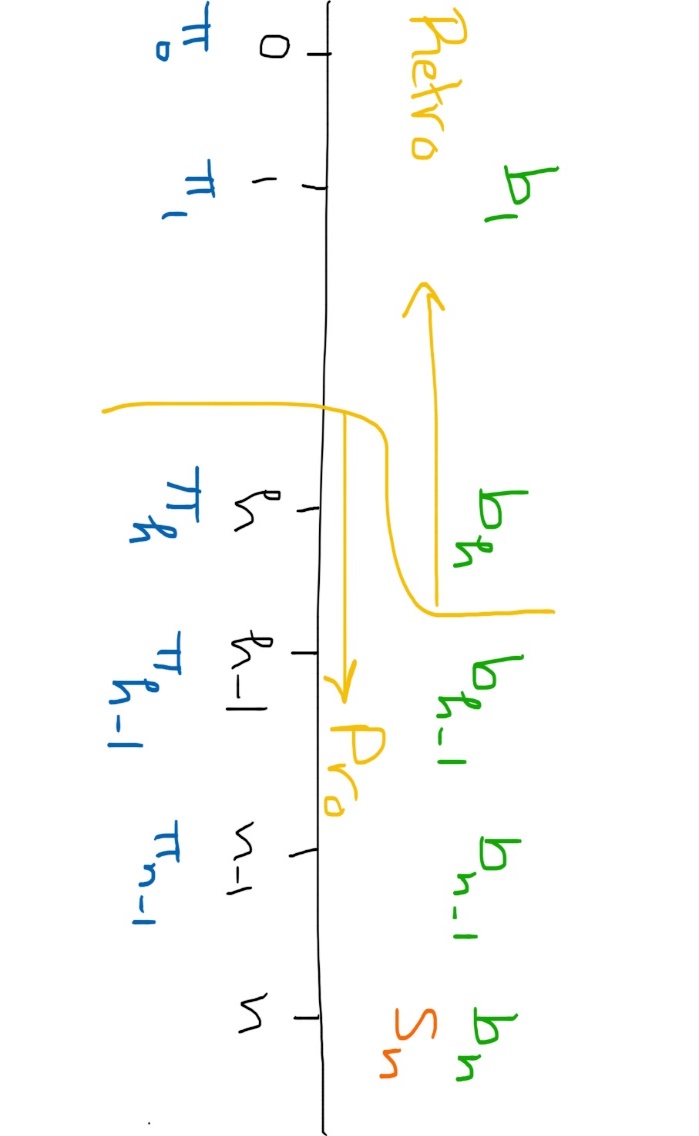
The given percent of the loss at issue is the survival probability, and the loss at issue can be found by 1st finding t, 2nd finding the premium. And then the Loss-at-issue will be found.

**NET PREMIUM RESERVES**



* The blue line represents the insurance, and its increasing because the probability of the policyholder’s death is increasing by time.
* The red line represents the premium, and its constant because the policy will pay premiums annual/mthly.
* When the two lines meets, expected value of the loss at issue will be zero (the EP), but before the meet the insurance is lower than the premium (the company is winning) and here we calculate the net premium reserve. How much we are winning. And same goes with after the meeting point, the death benefit is higher than the premium (company is losing), and by calculating the reserve we will know how much we are covering.

There’s 2 methods to calculate the reserve:

****

**The prospective approach:**

|  |  |  |  |
| --- | --- | --- | --- |
|  | | Continuous | Discrete |
| Whole life insurance on (x) | **The original formula** |  |  |
| **Using annuity only** |  |  |
| **Using insurance only** |  |  |
| **Using premiums only** |  |  |
| n-year endowment on (x) | **The original formula** |  |  |
| **Using annuity only** |  |  |
| **Using insurance only** |  |  |

**Notes:**

, where S is the death benefit.

Also if we have CFM we get because also , then we have

If we had a *semi-continuous* net premium, then the reserve would be:

**Variance of the loss at issue:**

|  |  |  |
| --- | --- | --- |
|  | | Assuming EP |
| Whole life insurance of S on (x) |  |  |
| n-year endowment life insurance of S on (x) |  |  |

**The retrospective approach:**

Continuous whole life:

Discrete whole life:

Where , ,

Note:

Finding the reserve by prospective approach, using the formulas above. Since the prospective is looking for the future, and we calculate after h year, so it’s like we take which gives us:

**The recursive approach:**

is called the **initial reserve** of h+1 year.

is the **terminal reserve** of h+1 year.

is the **net amount risk** of h+1 year.

We can also rearrange it:

What if h contains fractional numbers? (let t real number and u is fractional)

In case of n-term discrete insurance:

And if there was a survival benefit, we add to the APV(B)

**GROSS PREMIUMS: (expense-loaded premiums)**

covers the benefits and includes expenses, profits, and contingency margins

|  |  |  |
| --- | --- | --- |
| Year | Per policy | Percentage of premiums |
| Initial year (1st year) |  |  |
| Renewal year |  |  |

APV of initial and renewal expense:

And since ,

we got,

**the gross future loss at issue:**

PV of benefit outgo + PV of expense – PV of gross premium income

To find the gross premium we must first find the APV of the future benefit and the APV of the settlement, then we find the APV of the expense (shown above),

Under the EP:

APV of gross premium income = APV of benefits + APV of settlements + APV of expense

**MULTIPLE DECREMENT MODELS**

Having multiple ways of death, whether is it an accident, cancer, or just simply heart attack. Or it could be failure of something and it does have multiple reasons to that thing to fail. Suppose we have only 2 types of death, then:

is the number of deaths due to accidents between age x and x+1

is the number of deaths due to other causes between age x and x+1

is the total number of deaths between x and x+1.

**Formulas:**

Survival probability

The joint distribution of

Associated single decrement:

**(SUDD)**

The derivative of that is where

Where

For for 3 modes

For 2 modes:

For 1 mode:

Which is

**CFMD** and

Remember:

1. \*
2. Ratio property: \*
3. Partition property: \*

**MUDD**

1. and
2. Ratio property: \*
3. Partition property: \*