لا يكتب في
هذا الهامش

"Probability, conditional probability"
and independent

Q1 $S = \{H,T\} \times \{H,T\} \times \{H,T\}$
$= \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$

a) $n(S) = 2 \times 2 \times 2 = 8$

b) A : exactly two heads

$A = \{HHT, HTH, THH\} \Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{3}{8} = .375$

c) B : exactly three heads, $B = \{HHH\}$

$\because$ A and B $= A \cap B = \emptyset \Rightarrow \therefore$ A and B are disjoint

d) C : the first coin is heads, $C = \{HHH, HHT, HTH, HTT\} \Rightarrow P(C) = \frac{4}{8} = \frac{1}{2}$

D : the second and third coin are tails

$D = \{HTT, TTT\} \Rightarrow P(D) = \frac{2}{8} = \frac{1}{4}$

$C \cap D = \{HTT\} \Rightarrow P(C \cap D) = \frac{1}{8}$

as $P(C \cap D) \neq 0$ $\therefore$ C and D are not disjoint
and as $P(C) \neq P(D)$ $\therefore$ C and D are not equally likely
and as $P(C \cap D) = P(C) \cdot P(D) = \frac{1}{8}$ $\therefore$ C and D are independent

لا يكتب في هذا الهامش

Q2 $S = \{1,2,3,4,5,6\} \times \{1,2,3,4,5,6\}$

$= \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$

$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),$

$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),$

$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$

$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),$

$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\} \Rightarrow n(S) = 6 \times 6 = 36$

1) A : the sum of numbers of two dice ≤ 4

$A = \{(1,1),(1,2),(1,3),(2,1),(2,2),(3,1)\} \Rightarrow P(A) = \frac{6}{36} = \frac{1}{6} = .1667$

2) B : at least one of the die shows 4

$B = \{(1,4),(2,4),(3,4),(4,4),(4,5),(4,6),$

$(4,1),(4,2),(4,3),(5,4),(6,4)\} \Rightarrow P(B) = \frac{11}{36} = .3056$

3) C : the sum of numbers of two dice = 4 and one die shows 1

$C = \{(1,3),(3,1)\} \Rightarrow P(C) = \frac{2}{36} = .0556$

4)

D : the sum of two dice = 4, $D = \{(1,3),(3,1),(2,2)\} \Rightarrow P(C) = \frac{3}{36} = .0833$

E : one die shows 2

$E = \{(1,2),(2,1),(2,3),(2,4),(2,5),(2,6),$

$(3,2),(4,2),(5,2),(6,2)\} \Rightarrow P(E) = \frac{10}{36} = .2778$

$D \cap E = \{\ \} = \phi \Rightarrow P(D \cap E) = 0$ $\therefore$ D and E are disjoint

$P(E \cap D) = 0 \neq P(E)P(D) \Rightarrow$ D and E dep.

Q3 $P(A) = .3$, $P(B) = .4$, $P(A \cap B \cap C) = .03$, $P(\overline{A \cap B}) = P(A \cap B)^c = .88$

1) $P(A \cap B) = 1 - P(A \cap B)^c = 1 - .88 = .12 \neq 0$

$\therefore$ A and B are not disjoint

$P(A) \cdot P(B) = .3 \times .4 = .12 = P(A \cap B)$

$\therefore$ A and B are independent

2) $P(C \mid A \cap B) = \frac{P(C \cap (A \cap B))}{P(A \cap B)} = \frac{P(C \cap A \cap B)}{P(A \cap B)} = \frac{P(A \cap B \cap C)}{P(A \cap B)} = \frac{.03}{.12} = .25$

Q4 A : it will rain tomorrow $\Rightarrow P(A) = .23$

$\therefore P(A^c) = P(\bar{A}) = 1 - P(A) = 1 - .23 = .77$

$P(A \cup B) = \begin{cases} P(A) + P(B) - P(A \cap B) \\ P(A^c \cap B^c)^c = 1 - P(A^c \cap B^c) \end{cases}$, $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$, $P(B \mid A) = \frac{P(A \cap B)}{P(A)}$

$P(A \cap B) = P(A^c \cup B^c)^c = 1 - P(A^c \cup B^c) = 1 - [P(A^c) + P(B^c) - P(A^c \cap B^c)]$

$P(A) = P(A \cap B) + P(A \cap B^c)$, $P(B) = P(A \cap B) + P(A^c \cap B)$

$P(A) = 1 - P(A^c)$, $P(B) = 1 - P(B^c)$

Q5 A : factory open a branch in Riyadh $\Rightarrow P(A) = .7$

B : factory open a branch in Jeddah $\Rightarrow P(B) = .4$

$P(A \cup B) = .8$

	A	A^c	Sum
B	$P(A \cap B) = .3$	$P(A^c \cap B) = .1$	$P(B) = .4$
B^c	$P(A \cap B^c) = .4$	$P(A^c \cap B^c) = .2$	$P(B^c) = .6$
Sum	$P(A) = .7$	$P(A^c) = .3$	1

where $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow .8 = .7 + .4 - P(A \cap B) \Rightarrow P(A \cap B) = .3$ (ثم نكمل القيم المتبقية لتكملة الجدول)

1) $P(A \cap B) = .3$

2) $P(A^c \cap B^c) = .2$ or $P(A^c \cap B^c) = P(A \cup B)^c = 1 - P(A \cup B) = 1 - .8 = .2$

Q6 A : the lab specimen is contaminated $\Rightarrow P(A) = .1$

$\therefore P(A^c) = 1 - P(A) = 1 - .1 = .9$

and we have three samples of lab specimen checked independent

sample space $S = \{A, A^c\} \times \{A, A^c\} \times \{A, A^c\}$

$= \{AAA, AAA^c, AA^cA, A^cAA, A^cAA^c, A^cA^cA, AA^cA^c, A^cA^cA^c\}$

1)

B : none of lab specimen is contaminated

$B = \{A^cA^cA^c\} \Rightarrow P(B) = P(\{A^cA^cA^c\}) = P(A^c)P(A^c)P(A^c) = (.9)(.9)(.9) = .729$

2) C : exactly one of lab specimen is contaminated

$C = \{A^cAA^c, A^cA^cA, AA^cA^c\}$

$\Rightarrow P(C) = P(\{A^cAA^c\}) + P(\{A^cA^cA\}) + P(\{AA^cA^c\})$

$= P(A^c)P(A)P(A^c) + P(A^c)P(A^c)P(A) + P(A)P(A^c)P(A^c)$

$= 3[P(A^c)P(A^c)P(A)] = 3[(.9)(.9)(.1)] = .243$

لا يكتب في
هذا الهامش

Q7

	M	$M^c = F$	Sum
E	$n(M\cap E) = 28$	$n(F\cap E) = 50$	$n(E) = 78$
S	$n(M\cap S) = 38$	$n(F\cap S) = 45$	$n(S) = 83$
C	$n(M\cap C) = 22$	$n(F\cap C) = 17$	$n(C) = 39$
Sum	$n(M) = 88$	$n(F) = 112$	200

1) $p(M) = \frac{n(M)}{200} = \frac{88}{200} = .44$

2) $p(M|S) = \frac{P(M\cap S)}{P(S)} = \frac{n(M\cap S)}{n(S)} = \frac{38}{83} = .4578$

3) $p(C^c|F) = \frac{P(C^c\cap F)}{P(F)} = \frac{n(C^c\cap F)}{n(F)} = \frac{n(C^c\cap F)}{112}$ to find it

	F	F^c	sum
C	$n(F\cap C) = 17$*	$n(F^c\cap C) = 22$	$n(C) = 39$*
C^c	$n(F\cap C^c) = 95$	$n(F^c\cap C^c) = 66$	$n(C^c) = 161$
Sum	$n(F) = 112$*	$n(F^c) = 88$	200*

$\Rightarrow \therefore n(C^c\cap F) = 95$

$\therefore p(C^c|F) = \frac{95}{112} = .8482$ or $\boxed{P(C^c|F) = 1 - P(C|F) = 1 - \frac{n(C\cap F)}{n(F)}}$

4) we want to see that $p(M\cap E) \overset{?}{=} p(E)\,p(M)$

$L.H.S = p(M\cap E) = 28/200 = .14$

$R.H.S = p(M)\,p(E) = \frac{88}{200} \times \frac{78}{200} = .1716$ $\therefore L.H.S \neq R.H.S$

$\therefore$ M and E are not independent (dependent)

تعريفات زيادة

* $P(C^c\cap F) = \frac{95}{200}$
* $P(C^c\cup F) = P(C^c) + P(F) - P(C^c\cap F) = \frac{161+112-95}{200}$
* $P(C^c\cap F^c) = P(C\cup F)^c = 1 - P(C\cup F) = 1 - [P(C) + P(F) - P(C\cap F)]$

$= 1 - \left[\frac{39+112-17}{200}\right]$

* $P(C^c\cup F^c) = P(C\cap F)^c = 1 - P(C\cap F) = 1 - \frac{17}{200}$

لا يكتب هذا الهامش

Q8

	M	$M^c = F$	Sum
D	$n(M\cap D) = 300$	$n(F\cap D) = 50$	$n(D) = 350$
O	$n(M\cap O) = 200$	$n(F\cap O) = 50$	$n(O) = 250$
N	$n(M\cap N) = 100$	$n(F\cap N) = 300$	$n(N) = 400$
Sum	$n(M) = 600$	$n(F) = 400$	1000

1) $P(F) = \frac{n(F)}{1000} = \frac{400}{1000} = .4$

2) $P(F\cap D) = \frac{n(F\cap D)}{1000} = \frac{50}{1000} = .05$

3) $P(F\mid D) = \frac{P(F\cap D)}{P(D)} = \frac{n(F\cap D)}{n(D)} = \frac{50}{350} = .1429$

4) we want to see that $P(F\cap D) \overset{?}{=} P(F)\,P(D)$

$L.H.S = P(F\cap D) = .05$

$R.H.S = P(F)\,P(D) = \frac{400}{1000} \times \frac{350}{1000} = .14$

$\therefore L.H.S \neq R.H.S$ $\therefore$ F and D are dependent

Q9 A: the first engine start $\Rightarrow P(A) = .4$

B: the second engine start $\Rightarrow P(B) = .6$

and $\because$ the two engine operate independent

$\therefore P(A\cap B) = P(A)\,P(B) = (.4)(.6) = .24$

Q10 $p(B) = .3$, $p(A|B) = .4$

Then $p(A \cap B) = p(A|B)\, p(B)$ $\left[\text{from } p(A|B) = \frac{p(A \cap B)}{p(B)} \right]$

$= (.4)(.3) = .12$

Q11 A: the computer system has electrical failure

B: the computer has virus

$\therefore p(A) = .15$, $p(B) = .25$, $p(A \cap B) = .1$

$\therefore$ then $p(A \cup B) = p(A) + p(B) - p(A \cap B) = .15 + .25 - .1 = .3$

Q12

4b
2g

3 balls (independently and with replaced)

$p(b) = \frac{4}{6} = \frac{2}{3}$

$p(g) = \frac{2}{6} = \frac{1}{3}$

$p(2g \text{ and } 1b) = p(1^{st} \text{ ball blake and } 2^{nd} \text{ and } 3^{th} \text{ ball are } g)$

$+ p(1^{st}\, g \text{ and } 2^{nd} \text{ ball blake and } 3^{th}\, g)$

$+ p(1^{st} \text{ and } 2^{nd} \text{ ball are } g \text{ and } 3^{th} \text{ ball is blake})$

$= p(1^{st}\, b)\, p(2^{nd}\, g)\, p(3^{th}\, g)$

$+ p(1^{st}\, g)\, p(2^{nd}\, b)\, p(3^{th}\, g)$

$+ p(1^{st}\, g)\, p(2^{nd}\, g)\, p(3^{th}\, b)$

we doing this because the indep.

$S = \{bbb, bbg, bgb, gbb, ggb, gbg, bgg, ggg\}$

$= \left[\left(\frac{4}{6}\right)\left(\frac{2}{6}\right)\left(\frac{2}{6}\right) \right](3) = \frac{8}{36} = \frac{2}{9} = \frac{6}{27}$

Q13 $P(A_1)=.4$, $P(A_1\cap A_2)=.2$, $P(A_3|A_1\cap A_2)=.75$

then $P(A_2|A_1)=\frac{P(A_2\cap A_1)}{P(A_1)}=\frac{.2}{.4}=.5$

and as given that

$P(A_3|A_1\cap A_2)=.75=\frac{P(A_3\cap A_1\cap A_2)}{P(A_1\cap A_2)} \Rightarrow P(A_3\cap A_1\cap A_2)=P(A_3|A_1\cap A_2)\,P(A_1\cap A_2)$

$=(.75)(.2)=.15$

Q14 $P(A)=.9$, $P(B)=.6$, $P(A\cap B)=.5$

then $\boxed{1}$ $P(A\cap B^c)=P(A)-P(A\cap B)$ [from $P(A)=P(A\cap B)+P(A\cap B^c)$]

$=.4$

$\boxed{2}$ $P(A^c\cap B^c)=P(A\cup B)^c=1-P(A\cup B)=1-[P(A)+P(B)-P(A\cap B)]$

$=1-[.9+.6-.5]=1-1=0$

$\boxed{3}$ $P(B|A)=\frac{P(A\cap B)}{P(A)}=\frac{.5}{.9}=.556$

also we can use the table to solution the last.

	A	A^c	Σ
B	.5	.1	.6
B^c	.4	0	.4
Σ	.9	.1	1

$\boxed{4}$ and $\boxed{5}$

$P(A\cap B)=.5$

$P(A)P(B)=(.9)(.6)=.54$

so as $P(A\cap B)\neq P(A)P(B)$ ∴ A and B are dep.

and as $P(A\cap B)\neq 0$ ∴ A and B are joint

Q 15

choose

6 girls — 2 children with replacement (this given to me that are indep.)

2 boys

$n = 8 = \text{total}$

$\therefore p(G) = \frac{6}{8} = .75$ (G : denote for girl)

$p(B) = \frac{2}{8} = 1 - .75 = .25$ (B : denote for boy)

$\boxed{1}$

$S = \{G, B\} \times \{G, B\} = \{GG, GB, BG, BB\}$

$n(S) = 4 = 2 \times 2$

$\boxed{2}$ $A = \{\text{at most one boy}\} = \{GB, BG, GG\}$

$\boxed{3}$ $C = \{\text{no girls}\} = \{BB\}$

$\Rightarrow p(C) = p(\{BB\}) = p(\{B\})\, p(\{B\})$ [because the indep.]

$= (.25)(.25) = .0625$

$\boxed{4}$ $D = \{\text{exactly one boy}\} = \{GB, BG\}$

or $\cong \cup \cong +$

$\Rightarrow p(D) = p(\{GB, BG\}) = p(\{GB\}) + p(\{BG\})$

and $\cong \cap \cong \times$

$= p(\{G\})\, p(\{B\}) + p(\{B\})\, p(\{G\})$

$= (.75)(.25) + (.25)(.75) = (2)(.25)(.75) = .375$

$\boxed{5}$

$\rightarrow p(A) = p(\{GB, BG, GG\}) = p(\{GB\}) + p(\{BG\}) + p(\{GG\})$

$= p(\{G\})\, p(\{B\}) + p(\{B\})\, p(\{G\}) + p(\{G\})\, p(\{G\})$

$= (.75)(.25) + (.75)(.25) + (.75)(.75)$

$= 2(.75)(.25) + (.75)^2 = .4375$

16

T: the set of members that plays tennis

S: " " " " squash

B: " " " " badminton

$n(T)=36$, $n(T\cap S)=22$, $n(T\cap S\cap B)=4$

$n(S)=28$, $n(T\cap B)=12$

$n(B)=18$, $n(S\cap B)=9$

N: the number of members of the club $\Rightarrow N=5$

Note: $p(E_1 \cup E_2 \cup \dots \cup E_n) = \sum_{i=1}^{n} p(E_i) - \sum_{i_1<i_2} p(E_{i_1} \cap E_{i_2}) + \dots$

$$+ (-1)^{r+1} \sum_{i_1<i_2<\dots<i_r} p(E_{i_1} \cap E_{i_2} \cap \dots \cap E_{i_r})$$

$$+ \dots + (-1)^{n+1} p(E_1 \cap E_2 \cap \dots \cap E_n)$$

The summation $\sum_{i_1<i_2<\dots<i_r} p(E_{i_1} \cap E_{i_2} \cap \dots \cap E_{i_r})$ is taken over all of the $\binom{n}{r}$ possible subset of size r of the set $\{1, 2, \dots, n\}$

we have

$$p(T\cup S\cup B) = p(T)+p(S)+p(B) - p(T\cap S) - p(T\cap B) - p(S\cap B) + p(T\cap S\cap B)$$

$$= \frac{36+28+18-22-12-9+4}{N} = \frac{43}{50}$$

21

D	N
5	15

2

Define the following events

A = { the first fuse is defective }

B = { the second fuse is defective }

$A \cap B$ = { the first fuse is defective and the second fuse is defective }

= { both fuses are defective }

$\therefore P(A \cap B) = P(A)\,P(B \mid A) = \frac{5}{20} \cdot \frac{4}{19} = .052632$

السحبة الأولى

السحبة الثانية

D	N
5	15

20

D	N
4	15

19

22

Playing Card (52) → hearts ♡ (13)
→ diamonds ◇ (13)
→ spedes ♤ (13)
→ clubs ♧ (13)

every set has : A, K, Q, J, 10, 9, 8, 7, 6, 5, 4, 3, 2

where A: Ace

K: kinge

Q: Queen

J: Jack

$n(A_1) = 2$, $n(A_2) = 8$, $n(A_3) = 12$ → بحيث لا توجد عناصر مشتركة بين الثلاث مجموعات يؤثر على عددها

$$P(A_1 \cap A_2 \cap A_3) = P(A_1)\, P(A_2 | A_1)\, P(A_3 | A_1 \cap A_2) = \frac{2}{52}\ \frac{8}{51}\ \frac{12}{50} = .0014479$$

السحبة الاولى

2	50
r.a	o.w

52

السحبة الثانية

8	43
10/J	o.w

51

43 → كانت 44

السحبة الثالثة

12	38
3<#<7	o.w

50

38 → كانت 40