PUT OPTIONS

King Saud University
Mathematics Department | ACTU461
Exercise's Lecture (7)
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PUT OPTION

A put option is a financial contract which gives the owner the right, but not the obligation, to sell a specified amount of a given security at a specified price at a specified time.

TO CLARIFY THE CONCEPT OF PUT OPTIONS :

Suppose Sara own a truck cost 40K dollars she is afraid her truck may be damaged or stolen so she entered a put option (Zero deductible insurance policy) from ABC insurance company on her truck for the full amount (40K=Strike price) with a premium of 5000 for one year.

Scenario 1: her truck is not damaged after a year. So, she will get nothing from the insurance company but she has been happy all the year for the protection. (Not Exercised)

Scenario 2: her truck has an accident and the repairing cost is 10k , her truck price now is 30K so ABC pay back 10K to Sara. (K-St)

Scenario 3: her truck is stolen (St=0) so ABC has to pay back 40K to sara. (K-St)

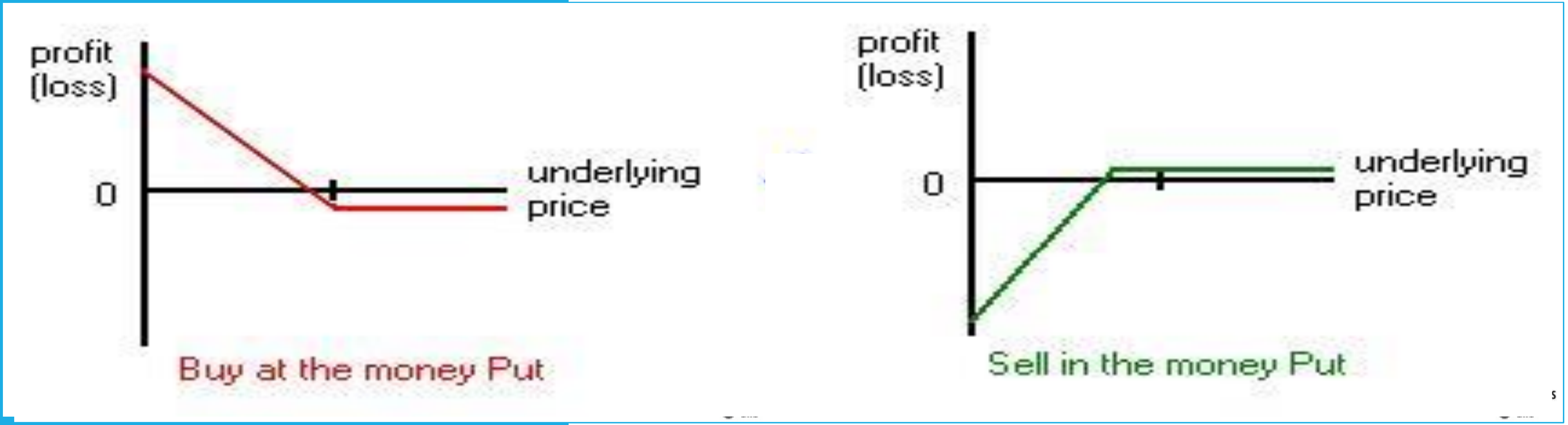
As the Call options Put options could exercised on Stocks , Assets..

Since, A put–option is a zero–sum game. The seller of a put option or the option put writer has a payoff equals the opposite of the holder's payoff. So, The sum of the two payoffs is zero

The put option holder's profit per unit:

$$\max(K - S_t, 0) - P(K,T)(1+i)^T$$

$$\begin{cases} K - S_t - P(K,T)(1+i)^T & if\ S_T < K \\ - P(K,T)(1+i)^T & if\ S_T \geq K \end{cases}$$



1-No Arbitrage

$$max\left((1+i)^{-T}K - S_0, 0\right) < Put(K,T) < K(1+i)^{-T}$$

Arbitrage could exist in two main cases:

CASE I

$$\left((1+i)^{-T}K - S_0\right) > Put(K,T)$$
$$(1+i)^{-T}K > Put(K,T) + S_0$$

$$\boldsymbol{K > (Put(K,T) + S_0)(1+i)^T}$$

CASE II

$$Put(K,T) > (1+i)^{-T}K$$

$$\boldsymbol{Put(K,T)(1+i)^T > K}$$

2-No Arbitrage

$$max\left((1+i)^{-T}(K-F_{0,T}),0\right) \leq Put(K,T) \leq K(1+i)^{-T}$$

Arbitrage could exist in two main cases:

Contracts could appear as prepaid forward contracts. [$\boldsymbol{PV(F_{0,T}) = F^{P}_{0.T}}$]

CASE I

$$(1+i)^{-T}\left(K-F_{0,T}\right) > Put(K,T)$$

$$\left(K-F_{0,T}\right) > Put(K,T)\ (1+i)^{-T}$$

$$\mathbf{K} > \boldsymbol{Put(K,T)\ (1+i)^{T} + F_{0,T}}$$

CASE II

$$Put(K,T) > K(1+i)^{-T}$$

$$\boldsymbol{Put(K,T)K(1+i)^{T} > K}$$

3-No Arbitrage

$$Put(K_1,T) \le Put(K_2,T) \le Put(K_1,T) + (K_2 - K_1)e^{-rT}$$

Given, $K_2 > K_1 > 0$

Arbitrage could exist in two main cases:

CASE I

$$Put(K_2,T) > Put(K_1,T) + (K_2 - K_1)e^{-rT}$$
$$Put(K_2,T) - Put(K_1,T) > (K_2 - K_1)e^{-rT}$$

$$\boldsymbol{(Put(K_2,T) - Put(K_1,T))e^{rT} > (K_2 - K_1)}$$

CASE II

$$\boldsymbol{Put(K_1,T) > Put(K_2,T)}$$

The purchased of the put option is:

Out-the-money

$S_t >$ K

At-the-money

S_T = K

In-the-money

$S_T <$ K

An investor purchased Option A and Option B for a certain stock today. With strike prices 70 and 80, respectively. Both options are European one year put options.

Determine which statements is true about the moneyness of these options, based on a particular stock price.

A) If Option A is in-the-money, then Option B is in-the-money.
B) If Option A is at-the-money, then Option B is out-of-the-money.
C) If Option A is in-the-money, then Option B is out-of-the-money.
D) If Option A is out-of-the-money, then Option B is in-the-money.
E) If Option A is out-of-the-money, then Option B is out-of-the-money.

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The current price of a forward of corn is \$3.3 per bushel. The annual effective interest rate is 7.5%. The price of a one–year European 3.5–strike put option for corn is \$0.18 per bushel. Find an arbitrage strategy and its minimum profit per bushel.

PAST EXAMS

Example 5 The current price of XYZ stock is 160 per share. The annual effective interest rate is 7%. The price of a one–year European 200–strike put option for XYZ stock is \$190 per share. Find an arbitrage strategy and the minimum profit per share.

The price of a one–year European 3.5–strike put option for corn is \$0.18 per bushel. The price of a one–year European 3.75–strike put option for corn is \$0.15 per bushel. The annual effective interest rate is 7.5%. Find an arbitrage strategy and it minimum profit.

Consider two European put options on a stock, both with expiration date exactly two years from now. One put option has strike price \$85 and the other one \$95. The price of the 85–strike put is 8. The price of the 95–strike put option is 20. The risk–free annual rate of interest compounded continuously is 5%. Find an arbitrage portfolio and its minimum profit