$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 \quad \propto J^\mu = 0$

$\frac{\partial \mathcal{L}}{\partial(\partial \phi)}$

$\phi \to \phi + \alpha \quad \leftarrow \Delta\phi = 1$

Find $j^\mu(x) \quad \leftarrow$

$j^\mu(x) = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \Delta\phi = \partial_\mu \phi$

EX2 $\mathcal{L} = |\partial_\mu \phi|^2 - m^2 |\phi|^2$

$\phi \to e^{i\alpha}\phi, \; \phi^* \to e^{-i\alpha}\phi^*$

Total $j^\mu(x)$ $\quad \Delta\phi \quad \Delta\phi^*$

$e^{i\alpha} \to \sim 1 + i\alpha$

$\phi \to e^{i\alpha}\phi \simeq (1+i\alpha)\phi = \phi + i\alpha\phi$

$\Delta\phi = i\phi$

$\phi^* \to \Delta\phi^* = -i\phi^*$

$|\partial_\mu \phi|^2 = \partial_\mu \phi \partial^\mu \phi^*$ 2 E.o.m

$m^2 |\phi|^2 = m^2 \phi \phi^*$

$\frac{\partial \mathcal{L}}{\partial \phi}, \frac{\partial \mathcal{L}}{\partial(\partial \phi)}$

$\frac{\partial \mathcal{L}}{\partial \phi^*}, \frac{\partial \mathcal{L}}{\partial(\partial \phi^*)}$

– From: $j_1^\mu(x) \to$ for ϕ

$j_2^\mu(x) \to$ for ϕ^*

$j^\mu(x) = j_1^\mu(x) + j_2^\mu(x)$

$= i[(\partial^\mu \phi^*)\phi - \phi^* \partial_\mu \phi]$

$\partial_\mu j^\mu(x) = 0 \quad \to$ K-G

$(\partial_\mu \partial^\mu \phi = m^2$

$x^\mu \to x^\mu - a^\mu$

K-G fields $\to$ Harmonic Oscilators

$\phi_x = \int \frac{d^3p}{(2\pi)^3} e^{ip\cdot x} \phi_p$

$\left(\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} + m^2\right)\phi_x = 0\right)$

$\left[\frac{\partial^2}{\partial t^2} + \underbrace{(|p|^2 + m^2)}_{w^2}\right]\phi_p = 0$

$\boxed{\ddot{\phi}(p) + w^2 \phi_p = 0}$ $\quad \ddot{x} + w^2 x$

$\phi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2w_p}} (a_p + a^\dagger_{-p}$

$|[\phi, \pi] = i\,\delta^3(x-y)|$

transition amplitude

||

propagator $\quad \langle f | i \rangle$

f

i

$\langle f | i \rangle = \sum_{\text{all path}} e^{i\cdot(\text{phas}}$

$f(x) = y \quad = \boxed{\int \mathcal{D}x(t)}\, e^{iS}$