Question: Convert from polar coordinates to Cartesian coordinates.

(1) $(1, \pi/4)$

(2) $(2, \pi)$

(3) $(2, -2\pi/3)$

(4) $(4, 3\pi/4)$

Solution:

(1) From the polar point $(1, \pi/4)$, we have $r = 1$ and $\theta = \frac{\pi}{4}$. Hence,

باستخدام الجدول في الأسفل

$$x = r\cos\ \theta = (1)\cos\ \frac{\pi}{4} = \frac{1}{\sqrt{2}},$$

$$y = r\sin\ \theta = (1)\sin\ \frac{\pi}{4} = \frac{1}{\sqrt{2}}.$$

Therefore, in the Cartesian coordinates, the point $(1, \pi/4)$ is represented by $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

(2) From the polar point $(2, \pi)$, we have $r = 2$ and $\theta = \pi$. Hence,

$$x = r\cos\ \theta = 2\cos\ \pi = -2\ ,$$

$$y = r\sin\ \theta = 2\sin\ \pi = 0.$$

Hence, the polar point $(2, \pi)$ is $(-2, 0)$ in the Cartesian coordinates.

(3) From the polar point $(2, -2\pi/3)$, we have $r = 2$ and $\theta = \frac{-2\pi}{3}$. Hence,

$$x = r\cos\ \theta = 2\cos\ \frac{-2\pi}{3} = -1\ ,$$

$$y = r\sin\theta = 2\sin\ \frac{-2\pi}{3} = -\sqrt{3}.$$

Therefore, the Cartesian coordinate $(-1, -\sqrt{3})$ is the point corresponding to the polar point $(2, -2\pi/3)$.

(4) From the polar point $(4, 3\pi/4)$, we have $r = 4$ and $\theta = \frac{3\pi}{4}$. Hence,

$$x = r\cos\ \theta = 4\cos\ \frac{3\pi}{4} = -2\sqrt{2}\ ,$$

$$y = r\sin\ \theta = 4\sin\ \frac{3\pi}{4} = 2\sqrt{2}.$$

In the Cartesian coordinates, the point $(4, 3\pi/4)$ is represented by $(-2\sqrt{2}, 2\sqrt{2})$.

Degrees	0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$\frac{-1}{2}$	$\frac{-1}{\sqrt{2}}$	$\frac{-\sqrt{3}}{2}$	-1	$\frac{-\sqrt{3}}{2}$	$\frac{-1}{\sqrt{2}}$	$\frac{-1}{2}$	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$\frac{-1}{2}$	$\frac{-1}{\sqrt{2}}$	$\frac{-\sqrt{3}}{2}$	-1	$\frac{-\sqrt{3}}{2}$	$\frac{-1}{\sqrt{2}}$	$\frac{-1}{2}$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1

على الطالب أن يحفظ هذا الجدول

Question: For the given Cartesian point, find one representation in the polar coordinates.

(1) $(1, -1)$

(2) $(2\sqrt{3}, -2)$

(3) $(-2, 2)$

(4) $(1, 1)$

Solution:

(1) From the given Cartesian point, we have $x = 1$ and $y = -1$. Hence,

باستخدام الجدول

$$x^2 + y^2 = r^2 \Rightarrow r = \sqrt{2},$$
$$\tan\theta = \frac{y}{x} = -1 \Rightarrow \theta = -\frac{\pi}{4}.$$

In the polar coordinates, the Cartesian point $(1, -1)$ can be represented by $(\sqrt{2}, -\frac{\pi}{4})$.

Remember , there are infinitely polar representations of the point (x, y) (see Note 4 on page **??**).

(2) From the Cartesian point, we have $x = 2\sqrt{3}$ and $y = -2$. Hence,

$$x^2 + y^2 = r^2 \Rightarrow r = 4,$$
$$\tan\theta = \frac{y}{x} = \frac{-1}{\sqrt{3}} \Rightarrow \theta = \frac{5\pi}{6}.$$

Therefore, the polar point $(4, \frac{5\pi}{6})$ is one representation of the Cartesian point $(2\sqrt{3}, -2)$.

(3) From the Cartesian point, we have $x = -2$ and $y = 2$. Hence,

$$x^2 + y^2 = r^2 \Rightarrow r = 2\sqrt{2},$$
$$\tan\theta = \frac{y}{x} = -1 \Rightarrow \theta = \frac{3\pi}{4}.$$

The polar point $(2\sqrt{2}, \frac{3\pi}{4})$ is one representation of the Cartesian point $(-2, 2)$.

(4) From the Cartesian point, we have $x = 1$ and $y = 1$. Hence,

$$x^2 + y^2 = r^2 \Rightarrow r = \sqrt{2},$$
$$\tan\theta = \frac{y}{x} = 1 \Rightarrow \theta = \frac{\pi}{4}.$$

The Cartesian point $(1, 1)$ can be represented by $(\sqrt{2}, \frac{\pi}{4})$ in the polar coordinates.