Quasilinear First-Order PDEs

Exercises

Notations: $z = z(x, y)$, $p = z_x$ and $q = z_y$

1. Obtain the general solution of the following PDEs.

   a. $zp + yq = x$

   b. $xp + yq = nz$ where n is a constant

   c. $(x + y)(p - q) = z$

   d. $(y + x)p + (y - x)q = z$

   e. $x(y^2 - z^2)p - y(x^2 + z^2)q = (x^2 + y^2)z$.

2. Make the change of independent variables $\xi = \log x,\ \eta = \log y$ and reduce the differential equation to one with constant coefficients. Obtain the general solution

   a. $4xp - 2yq = 0$

   b. $2xp + 3yq = \log x$.