

II) [4 marks] Let $f(x) = 2x + 1$ and let R be the region under the graph of f , from 0 to 5. Let P be the regular partition of $[0, 5]$ into n subintervals.

A) Find, in terms of n , the Riemann sum R_P , if $w_k = \frac{5k}{n}$, for $k = 1, 2, \dots, n$.

$$R_P = \sum_{k=1}^n f(w_k) \Delta x, \quad \Delta x = \frac{5-0}{n} = \frac{5}{n}$$

$$= \sum_{k=1}^n f\left(\frac{5k}{n}\right) \frac{5}{n} = \frac{5}{n} \sum_{k=1}^n f\left(\frac{5k}{n}\right) = \frac{5}{n} \sum_{k=1}^n 2\left(\frac{5k}{n}\right) + 1$$

$$= \frac{5}{n} \sum_{k=1}^n \frac{10k}{n} + 1 = \frac{5}{n} \left(\sum_{k=1}^n \frac{10k}{n} + \sum_{k=1}^n 1 \right) = \frac{5}{n} \left(\frac{10}{n} \sum_{k=1}^n k + n \right)$$

$$= \frac{50}{n^2} \sum_{k=1}^n k + \frac{5}{n} \cdot n = \frac{50}{n^2} \frac{n(n+1)}{2} + 5 = \frac{25(n+1)}{n} + 5$$

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B) Use Riemann sum in (A) to compute the area of the region R .

$$\text{Area} = \lim_{n \rightarrow \infty} R_P = \lim_{n \rightarrow \infty} \sum_{k=1}^n f(w_k) \Delta x = \lim_{n \rightarrow \infty} \frac{25(n+1)}{n} + 5 = 25 + 5 = 30 \text{ unit}^2$$

(2)