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Question 1. SYSTEMS OF LINEAR EQUATIONS

Determine the values of k such that the linear system

$$\begin{array}{rcrcr} 9x_1 & + & kx_2 & = & 9 \\ kx_1 & + & x_2 & = & -3 \end{array}$$

is consistent.

Answer

We apply row-reduction algorithm to the augmented matrix corresponding to the system given above: Assume that $k \neq 0$, then we get

$$\left[\begin{array}{ccc} 9 & k & 9 \\ k & 1 & -3 \end{array}\right] \xrightarrow{(-k/9)R_1+R_2 \to R_2} \left[\begin{array}{ccc} 9 & k & 9 \\ 0 & 1-\frac{k^2}{9} & -3-k \end{array}\right].$$

By Theorem 2, we know that the system above is consistent if and only if there is no row of the form [0 0 1] which implies that either we must have $1 - \frac{k^2}{9} \neq 0$ or we must have $1 - \frac{k^2}{9} = 0$ and $-3 - k = 0$.

We need to examine the case $k = 0$. If $k = 0$, then we have $9x_1 = 9$ or $x_1 = 1$ and $x_2 = -3$. So, the system is consistent. Note that if $k = -3$ the given system is still consistent. Finally, we conclude that the system above is consistent if and only if $k \neq 3$.

Question 2. ROW REDUCTION AND ECHELON FORMS

Determine when the augmented matrix below represents a consistent linear system.

$$\begin{bmatrix} 1 & 0 & 2 & a \\ 2 & 1 & 5 & b \\ 1 & -1 & 1 & c \end{bmatrix}$$

Answer

We apply row-reduction algorithm to the augmented matrix corresponding to the system given above:

$$\begin{bmatrix} 1 & 0 & 2 & a \\ 2 & 1 & 5 & b \\ 1 & -1 & 1 & c \end{bmatrix} \xrightarrow[-1R_1+R_3\leftrightarrow R_3]{-2R_1+R_2\to R_2} \begin{bmatrix} 1 & 0 & 2 & a \\ 0 & 1 & 1 & b-2a \\ 0 & -1 & -1 & c-a \end{bmatrix} \xrightarrow{R_1+R_2\to R_2} \begin{bmatrix} 1 & 0 & 2 & a \\ 0 & 1 & 1 & b-2a \\ 0 & 0 & 0 & b-3a+c \end{bmatrix}.$$

By Theorem 2, we know that the system above is consistent if and only if $b - 3a + c = 0$.

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Question 4. THE MATRIX EQUATION A**x**=**b**

A. Solve the matrix equation $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 0 \\ 1 & 1 & 2 \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Answer

We need to reduce the augmented matrix

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 1 & 3 & 0 & 0 \\ 1 & 1 & 2 & 0 \end{bmatrix} \xrightarrow[-R_1+R_3 \to R_3]{-R_1+R_2 \to R_2} \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix} \xrightarrow[-2R_2+R_1 \to R_1]{R_2+R_3 \to R_3} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

We see that x_1 and x_2 are basic variables and x_3 is a free variable. We rewrite the system, i.e., we get $x_1 + 3x_3 = 0$ and $x_2 - x_3 = 0$ OR $x_1 = -3x_3$ and $x_2 = x_3$.

$$G.S. = \begin{cases} x_1 = -3x_3 \\ x_2 = x_3 \\ x_3 \text{ is free.} \end{cases}$$

B. Is it possible to solve $A\mathbf{x} = \mathbf{b}$ for **any given b** $= \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ where A is the matrix given in part **A**? Explain.

Answer

NO. By Theorem 6, $A\mathbf{x} = \mathbf{b}$ has a solution FOR ANY GIVEN $\mathbf{b}$ if and only if A has 3 pivot positions. As you can see above, A has only 2 pivot positions. As a conclusion, it is not possible to solve $A\mathbf{x} = \mathbf{b}$ for any given $\mathbf{b}$.

C. Describe the set of all $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ for which $A\mathbf{x} = \mathbf{b}$ does have a solution.

Answer

We need to reduce the augmented matrix

$$\begin{bmatrix} 1 & 2 & 1 & b_1 \\ 1 & 3 & 0 & b_2 \\ 1 & 1 & 2 & b_3 \end{bmatrix} \xrightarrow[-R_1+R_3 \to R_3]{-R_1+R_2 \to R_2} \begin{bmatrix} 1 & 2 & 1 & b_1 \\ 0 & 1 & -1 & b_2 - b_1 \\ 0 & -1 & 1 & b_3 - b_1 \end{bmatrix} \xrightarrow[-2R_2+R_1 \to R_1]{R_2+R_3 \to R_3} \begin{bmatrix} 1 & 0 & 3 & -2b_2 + 3b_1 \\ 0 & 1 & -1 & b_2 - b_1 \\ 0 & 0 & 0 & b_3 + b_2 - 2b_1 \end{bmatrix}$$

By Theorem 2, the equation $A\mathbf{x} = \mathbf{b}$ has a solution if and only if $b_3 + b_2 - 2b_1 = 0$.

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6-1 Mar 30, 2015, 12:25 PM
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