Math 111 – Quiz 1

Name: ______________ ID: ______________ Marks: ______ (5)

Question 1 [2 marks]

Choose the correct answer:

(a) $\sum_{i=1}^{6} \frac{(i+3)}{3}$ equals

i. 13

ii. 27

iii. 20

iv. None

Solution: THE ANSWER IS (i)

$$\begin{aligned}
\sum_{i=1}^{6} \frac{(i+3)}{3} &= \frac{1}{3}\sum_{i=1}^{6}(i+3) \qquad\qquad (n=6)\\
&= \frac{1}{3}\Big[\sum_{i=1}^{6} i + \sum_{i=1}^{6} 3\Big]\\
&= \frac{1}{3}\Big[\frac{(6)(7)}{2} + 3(6)\Big]\\
&= \frac{1}{3}[21+18] = \frac{39}{3} = 13
\end{aligned}$$

(b) $\int \frac{x^2+1}{x} dx$ equals

i. $\frac{1}{2}\ln|x^2+1| + c$

ii. $\ln(x^2+1) + c$

iii. $\frac{1}{2}x^2 + \ln|x| + c$

iv. None

Solution: THE ANSWER IS (iii)

$$\begin{aligned}
\int \frac{x^2+1}{x} dx &= \int \Big[\frac{x^2}{x} + \frac{1}{x}\Big] dx\\
&= \int \Big[x + \frac{1}{x}\Big] dx\\
&= \int x^2 dx + \int \frac{1}{x} dx\\
&= \frac{1}{2}x^2 + \ln|x| + c.
\end{aligned}$$

Question 2 [3 marks]

Find the value of c that satisfies the conclusion of the Integral Mean Value Theorem of $f(x) = x^2$ on $I = [1, 4]$ where $\int_1^4 f(x)dx = 21$.

Solution:

The conclusion of the Integral Mean Value Theorem is

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx. \qquad (1\ mark)$$

We have $a = 1$, $b = 4$ and $\int_1^4 f(x)dx = 21$

$$f(c) = \frac{1}{4-1}(21) = \frac{21}{3} = 7$$

that implies

$$f(c) = c^2 = 7 \implies c = \pm\sqrt{7} \qquad (1\ mark)$$

We pick c that lies in our interval

$$c = \sqrt{7} \in [1, 4] \qquad (1\ mark)$$