Math 111 – Quiz 1

Name: ____________________ ID: _______________ Marks: _____ (5)

Question 1 [2 marks]

Choose the correct answer:

(a) $\sum_{i=1}^{4}(i^2+5)$ equals

i. 35

ii. 80

iii. 50

iv. None

Solution: THE ANSWER IS (iii)

$$\begin{aligned}\sum_{i=1}^{4}(i^2+5) &= \left[\sum_{i=1}^{4} i^2 + \sum_{i=1}^{4} 5\right] \qquad (n=4)\\ &= \left[\frac{(4)(5)(2(4)+1)}{6} + 5(4)\right]\\ &= [30+20] = 50\end{aligned}$$

(b) $\int \frac{\sec^2 x}{\tan x} dx$ equals

i. $\frac{1}{3}\tan^2 x + c$

ii. $\ln|\tan x| + c$

iii. $\frac{1}{2}\cos^2 x + c$

iv. None

Solution: THE ANSWER IS (ii)

Let $f(x) = \tan x \implies f'(x) = \sec^2 x$, substitute in the integral we get

$$\int \frac{f'(x)}{f(x)} dx = \int \frac{\sec^2 x}{\tan x} dx = \ln|\tan x| + c.$$

Question 2 [3 marks]

Find the value of c that satisfies the conclusion of the Integral Mean Value Theorem of $f(x) = x^2 - 1$ on $I = [0, 3]$ where $\int_0^3 f(x)dx = 6$.

Solution:

The conclusion of the Integral Mean Value Theorem is

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx. \qquad (1\ mark)$$

We have $a = 0$, $b = 3$ and $\int_0^3 f(x)dx = 6$

$$f(c) = \frac{1}{3-0}(6) = \frac{6}{3} = 2$$

that implies

$$f(c) = c^2 - 1 = 2 \implies c^2 = 2 + 1 = 3 \implies c = \pm\sqrt{3} \qquad (1\ mark)$$

We pick c that lies in our interval

$$c = \sqrt{3} \in [0, 3] \qquad (1\ mark)$$