

Ch 14 TERMINAL RESERVE

❖ METHODS OF CALCULATING THE RESERVE

There are 2 methods:

1-The Retrospective method: we look @ the past.

The Value of the Reserve =

*{ Collected Prem's plus Interest at age $x+t$ } (Amount)
- { Paid Benefits plus Interest at age $x+t$ } (Amount)*

or: Reserve =

*{ Accumulated Value of Paid Prem's @ age $x+t$
- Accumulated Value of Paid Benefits @ age $x+t$ }*

Ch 14 TERMINAL RESERVE

2-The Prospective Method:

The value of the Reserve =

{PV of Future Benefits at age $x+t$ }

minus: {PV of Future Premiums at age $x+t$ }

Ex 1: *Find the reserve per survivor using the prospective method at the end of the 5th policy yr for a \$100,000, 10-yr endowment policy issued to a 20- yr- old male if the policy issued for:*

a)an annual prem.

b)a single prem.

✧ Ch 14 TERMINAL RESERVE

Solution:

a)an annual prem: @ the end of the 5th policy yr, the insured is 25 yrs old.

At that time the value of the reserve =
the PV of the future benefits (NSP for a 5-yr, \$100,000 endowment policy issued to a 25-yr-old male)

minus the PV of the future prems (the remaining 5 prems which represent an Annuity).

This value can be calculated as follow:

Ch 14 TERMINAL RESERVE

$$V = \left\{ \frac{(M_{x+t} - M_{x+n} + D_{x+n}) - P(N_{x+t} - N_{x+n})}{D_{x+t}} \right\}$$
$$V = \left\{ \frac{(M_{25} - M_{30} + D_{30}) - P(N_{25} - N_{30})}{D_{25}} \right\} \times 100,000$$

Ch 14 TERMINAL RESERVE

$$\text{where: } P = \frac{(M_{20} - M_{30} + D_{30})}{(N_{20} - N_{30})}$$

$$P = \frac{(9477.262 - 8408.291 + 475485)}{(15258554 - 9089224)} = \frac{48617.471}{616933} = 0.0788$$

$$V = \frac{(8859.475 - 8408.291 + 47548.5) - 0.0788(1182196.8 - 908922.4)}{59767.4} \times 100,000$$

$$V = \frac{(47999684 - 21534027)}{59767.4} \times 100,000$$

$$V = \frac{26465661}{59767.4} \times 100,000 = 44,281$$

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a) a single prem: @ the end of the 5th policy yr, the insured is 25 yrs old.

At that time the value of the reserve =
the value of the future benefits & equal NSP
for a 5-yr, \$100,000 endowment policy
issued to a 25-yr-old male.

From the formula this value is:

Ch 14 TERMINAL RESERVE

where: $V = A_{25 : \overline{5}|} \times 100,000$

$$V = \frac{M_{25} - M_{30} + D_{30}}{D_{25}} \times 100,000$$

$$V = \left(\frac{8859.475 - 8408.291 + 47548.5}{59767.4} \right) \times 100,000 = \$80,311$$

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Ex 2: Find the reserve per survivor @ the end of the 10th policy yr using the prospective method for a \$70,000, whole life policy issued to a 20- yr- old male if the policy issued for:

- a) a single prem.***
- b) an annual prem.***
- c) a limited annual prem for 15 yrs.***

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a) a single prem: @ the end of the 10th policy yr, the insured is 30 yrs old.

At that time, there are future benefits from age 30 & up & there are no future prems from age 30 & up (he paid NSP).

Then, the value of the Reserve =

$$\begin{aligned} & \{PV \text{ of Future Benefits @ age 30} \\ & - PV \text{ of Future Prems @ age 30} \\ & \text{which} = 0 \text{ (he paid NSP)} \end{aligned}$$

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a) a single prem:

$$V = \frac{M_{30}}{D_{30}} \times 70,000$$

$$V = \frac{8408291}{475485} \times 70,000 = 12,378,527$$

Ch 14 TERMINAL RESERVE

b)an annual prem: @ the end of the 10th policy yr, the insured is 30 yrs old.

At that time, there are future benefits from age 30 & up & there are future prems from age 30 & up.

Then, the value of the Reserve =

*{PV of Future Benefits @ age 30
- PV of Future Prems @ age 30
(which represent an Annuity).*

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b) an annual prem:

$$V = \left\{ \frac{M_{x+t} - P(N_{x+t})}{D_{x+t}} \right\}$$

$$V = \left\{ \frac{(M_{30} - P(N_{30}))}{D_{30}} \right\} \times 70,000$$

where: $P = \frac{M_{20}}{N_{20}}$

$$P = \frac{9477.262}{15258554} = 0.00621$$

$$V = \frac{(8408291 - (0.00621 \times 9089224))}{475485} \times 70,000 = 4,068,936$$

Ch 14 TERMINAL RESERVE

c) a limited annual prem: @ the end of the 10th policy yr, the insured is 30 yrs old.

At that time, there are future benefits from age 30 & up & there are future premiums from age 30 & up to age 35.

Then, the value of the Reserve =

***{PV of Future Benefits @ age 30
- PV of Future Premiums @ age 30 to 35
(which represent an Annuity for 5 yr)}***

✳ Ch 14 TERMINAL RESERVE

c) a limited annual premium for 15 years:

$$V = \left\{ \frac{M_{x+t} - P(N_{x+t} - P(N_{x+n}))}{D_{x+t}} \right\}$$

$$V = \left\{ \frac{(M_{30} - P(N_{30} - N_{35}))}{D_{30}} \right\} \times 70,000$$

where: $P = \frac{M_{20}}{N_{20} - N_{35}}$

$$P = \frac{9477.262}{15258554 - 691534} = 0.011359$$

$$V = \frac{(8408.291 - 0.011359(9089224 - 691534))}{475485} \times 70,000 = 8,743.249$$

Ch 14 TERMINAL RESERVE

Ex 3: Find the reserve per survivor at the end of the 10th policy yr using the retrospective method for a \$70,000, whole life policy issued to a 20- yr- old male if the policy issued for:

- a) a single prem.***
- b) an annual prem.***
- c) a limited annual prem for 15 yrs.***

Ch 14 TERMINAL RESERVE

a) a single prem: At the end of the 10th policy yr, the insured is 30 yrs old.

At that time, the insurer collected the whole prems (NSP) & paid 10 yrs benefits.

Then, the value of the Reserve =

{The amount of collected prem @ age 30

- The amount of paid benefits @ age 30

Or: = The net single prem @ age 30

Ch 14 TERMINAL RESERVE

a) a single prem:

$$V = \frac{M_x - (M_x - M_{x+t})}{D_{x+t}}$$

$$V = \frac{M_{20} - (M_{20} - M_{30})}{D_{30}} \times 70,000$$

$$V = \frac{M_{30}}{D_{30}} \times 70,000 = \frac{8408291}{475485} \times 70,000 = 12,378,527$$

Ch 14 TERMINAL RESERVE

b)an annual prem: *@t the end of the 10th policy yr, the insured is 30 yrs old.*

At that time, the insurer collected 10 prems & paid 10 yr benefits.

Then, the value of the Reserve =

*{The amount of collected prem @ age 30
- The amount of paid benefits @ age 30*

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b) an annual prem:

$$V = \frac{P(N_{20} - N_{30}) - (M_{20} - M_{30})}{D_{30}}$$

where: $P = \frac{M_{20}}{N_{20}}$

$$P = \frac{9477.262}{15258554} = 0.000621$$

$$V = \frac{0.000621(15258554 - 9089224) - (9477.262 - 8408291)}{475485} \times 70,000$$

$$V = \frac{(3831.154 - 1068971)}{475485} \times 70,000 = 4,066,433$$

Ch 14 TERMINAL RESERVE

c)a limited annual prem: @ the end of the 10th policy yr, the insured is 30 yrs old.

At that time, the insurer collected 10 prems & there are 5 remaining prems & paid 10 yr benefits.

Then, the value of the Reserve =

*{The amount of collected prem @ age 30
- The amount of paid benefits @ age 30
(which represent an Annuity for 10 yr)*

✧ Ch 14 TERMINAL RESERVE

c) a limited annual prem:

$$V = \frac{P(N_{20} - N_{30}) - (M_{20} - M_{30})}{D_{30}}$$

where: $P = \frac{M_{20}}{N_{20} - N_{35}}$

$$P = \frac{9477.262}{15258554 - 691534} = 0.011359$$

$$V = \frac{0.011359(15258554 - 9089224) - (9477.262 - 8408291)}{475485} \times 70,000$$

$$V = 8,743.249$$

Ch 14 TERMINAL RESERVE

Ex 4: An ordinary life ins. policy of \$70,000 has been issued on the life of a person now aged 30 for the last 10 yrs.

He now wishes to change this policy to a 15–payment whole life policy as of his original age at issue.

How much must he pays to the ins. Co.

Ch 14 TERMINAL RESERVE

a)an annual prem: @t the end of the 10th policy yr, the insured is 30 yrs old.

At that time, the insurer collected 10 prems & paid 10 yr benefits.

Then, the value of the Reserve =

*{The amount of collected prem @ age 30
- The amount of paid benefits @ age 30*

✳ Ch 14 TERMINAL RESERVE

a) an annual prem:

$$V = \frac{P(N_{20} - N_{30}) - (M_{20} - M_{30})}{D_{30}}$$

where: $P = \frac{M_{20}}{N_{20}}$

$$P = \frac{9477.262}{15258554} = 0.000621$$

$$V = \frac{0.000621(15258554 - 9089224) - (9477.262 - 8408291)}{475485} \times 70,000$$

$$V = \frac{(3831.154 - 1068971)}{475485} \times 70,000 = 4,066,433$$

Ch 14 TERMINAL RESERVE

b)a limited annual prem: @ the end of the 10th policy yr, the insured is 30 yrs old.

At that time, the insurer collected 10 prems & there are 5 remaining prems & paid 10 yr benefits.

Then, the value of the Reserve =

**{The amount of collected prem @ age 30
- The amount of paid benefits @ age 30
(which represent an Annuity for 10 yr)}**

✧ Ch 14 TERMINAL RESERVE

b) a limited annual prem:

$$V = \frac{P(N_{20} - N_{30}) - (M_{20} - M_{30})}{D_{30}}$$

where: $P = \frac{M_{20}}{N_{20} - N_{35}}$

$$P = \frac{9477.262}{15258554 - 691534} = 0.011359$$

$$V = \frac{0.011359(15258554 - 9089224) - (9477.262 - 8408291)}{475485} \times 70,000$$

$$V = 8,743.249$$

Ch 14 TERMINAL RESERVE

Then the insured should pay the difference between the two reserves, i.e.:

$$***8743,249 - 4066.433 = 4676.816***$$

Ch 14 TERMINAL RESERVE

Ex 5: An ordinary life ins policy of \$70,000 issued to a person aged 20 is discontinued at age 30.

He now wishes to use the reserve for the same policy without paying any more prems.

How much would be the amount of ins of the new policy.

Ch 14 TERMINAL RESERVE

The reserve of the first policy after 10 yrs:

@t the end of the 10th policy yr, the insured is 30 yrs old.

At that time, the insurer collected 10 prems & paid 10 yr benefits.

Then, the value of the Reserve =

**{The amount of collected prem @ age 30
- The amount of paid benefits @ age 30**

✧ Ch 14 TERMINAL RESERVE

a) The reserve of the whole life policy after 10 yrs:

$$V = \frac{P(N_{20} - N_{30}) - (M_{20} - M_{30})}{D_{30}}$$

where: $P = \frac{M_{20}}{N_{20}}$

$$P = \frac{9477.262}{15258554} = 0.000621$$

$$V = \frac{0.000621(15258554 - 9089224) - (9477.262 - 8408291)}{475485} \times 70,000$$

$$V = \frac{(3831.154 - 1068971)}{475485} \times 70,000 = 4,066,433$$

Ch 14 TERMINAL RESERVE

This reserve represents the net single prem for a whole life policy issued at age 30:

$$A_{30} = \frac{M_{30}}{D_{30}} \times Y$$

$$4066.433 = \frac{8408.291}{47548.5} \times Y$$

$$4066.433 = 0.1768361 Y$$

$$Y = 22995.491$$

Ch 14 TERMINAL RESERVE

Ex 6: An ordinary life ins policy of \$70,000 issued to a person aged 20 is discontinued at age 30.

He now wishes to use the reserve to buy a 20 yr term insurance at age 30.

How much would be the amount of ins?

Ch 14 TERMINAL RESERVE

The reserve of the first policy after 10 yrs:

@t the end of the 10th policy yr, the insured is 30 yrs old.

At that time, the insurer collected 10 prems & paid 10 yr benefits.

Then, the value of the Reserve =

**{The amount of collected prem @ age 30
- The amount of paid benefits @ age 30**

✳ Ch 14 TERMINAL RESERVE

a) The reserve of the whole life policy after 10 yrs:

$$V = \frac{P(N_{20} - N_{30}) - (M_{20} - M_{30})}{D_{30}}$$

$$\text{where: } P = \frac{M_{20}}{N_{20}}$$

$$P = \frac{9477.262}{15258554} = 0.000621$$

$$V = \frac{0.000621(15258554 - 9089224) - (9477.262 - 8408291)}{475485} \times 70,000$$

$$V = \frac{(3831.154 - 1068971)}{475485} \times 70,000 = 4,066,433$$

Ch 14 TERMINAL RESERVE

This reserve represents the net single prem for a 20 yr term ins policy issued at age 30:

$$A^1_{30:20} = \frac{M_{30} - M_{50}}{D_{30}}$$

$$4,066.433 = \frac{8408.291 - 6616.417}{475485} \times Y$$

$$4066.433 = \frac{8408.291}{475485} \times Y$$

$$4066.433 = 0.037685 Y$$

$$Y = 107,905.878$$