Resolution of image

How important is partial differential equation in photography or imeges ?

PDEs are used in many fundamental operations such as Corner Detection, Line Detection, Feature Extraction and Local Statistics Extraction (any many more). These operations give base to many commercial algorithms such as Panoramic Image stitching, Color Blending, HDR Imaging, GigaPan shots etc.

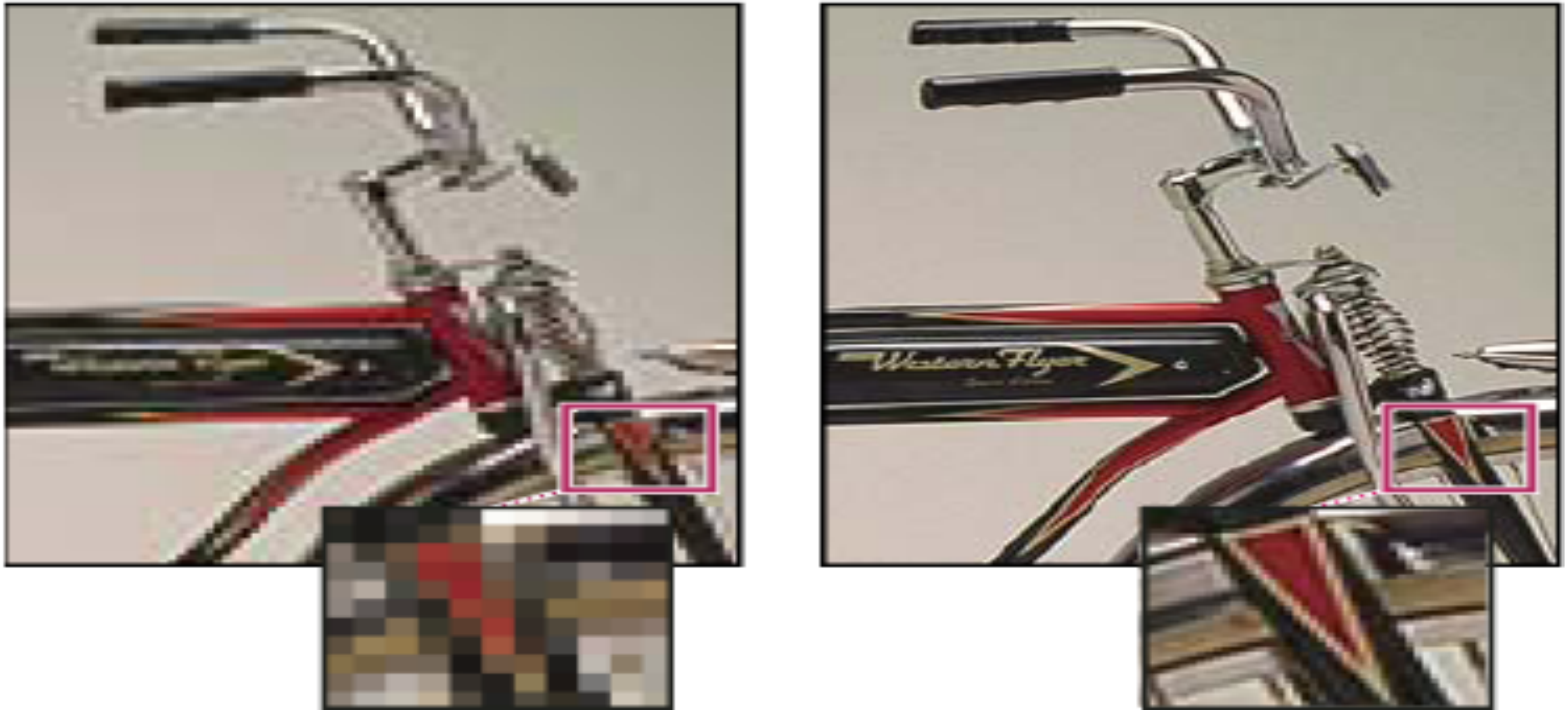
Why PDE?

Because Images, akin to thermodynamics and fluid mechanics and others, can be modeled as a continuous field which depend on variables. PDEs lets us play with these assumed variables and extract meaningful information from them.

Some of scientist use PDEs :

Gabor (1965) , Jain (1977) :
(Idea on the use of PDEs in image processing) .

Koenderink (1983) and Witkin (1984) :
(representation of images simultaneously at multiple scales)


Western Flyer

$\mathscr{R}$ $\mathscr{R}^2$ $\mathscr{R}^3$

(x) (x,y) (x,y,z)

For The present discussion an "image" is just a real function of two real variables:

$$L: \mathscr{R}^2 \longrightarrow \mathscr{R}$$

$$L(\,r\,) = L(x,y) = \boldsymbol{\lambda}$$

$r \in \mathscr{R}^2$

$\boldsymbol{\lambda} \in \mathscr{R}$

$\boldsymbol{\lambda}$ Will be called the (luminance)

Define a real function K of three variables

$$K: \mathscr{R}^3 \longrightarrow \mathscr{R}$$

$$K(R) = K(x,y,z) = A$$

$R \in \mathscr{R}^3$

$A \in \mathscr{R}$

A Is a constant.

In such a way that

$$K(x,y,0) = L(x,y)$$

$$K(x,y,z) = A$$

$$K_x = 0 \qquad K_y = 0 \qquad K_z = 0$$

. the principal curvatures are:

$$K_i = \frac{\lambda_i}{\sqrt{(K_x^2 + K_x^2 + K_x^2)}} \qquad i=1,2$$

When the λ_i are the roots of the (quadratic) equation

$$\mathbf{Det} = \begin{vmatrix} K_{xx} - \lambda & K_{xy} & K_{xz} & K_x \\ K_{yx} & K_{yy} - \lambda & K_{yz} & K_y \\ K_{zx} & K_{zy} & K_{zz} - \lambda & K_z \\ K_x & K_y & K_z & 0 \end{vmatrix} = 0$$

(because we consider the case $K_x = 0$ $K_y = 0$ $K_z = 0$)

We Get :

$$\lambda^2 - \lambda (K_{xx} + K_{yy}) + (K_{xx} K_{yy} - K^2_{xy}) = 0$$

* By hypothesis $(K_{xx} K_{yy} - K^2_{xy})$ is positive .
* This sign is given by the sign of $(K_{xx} + K_{yy}) = \Delta K$

The partial differential equation

$$\Delta K = \alpha^2 (x,y,z) \, K_z$$

Where α Denotes An Arbitrary But Nowhere Vanishing Real function.

Thank you..