Chapter 1: Review Exercises

1 - 34 ■ Evaluate the following integrals:

1. $\int 2x\,dx$
2. $\int (3x^2+1)\,dx$
3. $\int \left(\frac{1}{2}x^3+x\right)\,dx$
4. $\int (x^4+x^3)\,dx$
5. $\int (x^2+3x-1)\,dx$
6. $\int (1-2x-5x^3)\,dx$
7. $\int \frac{1}{x^2}\,dx$
8. $\int \sqrt{x^5}\,dx$
9. $\int \frac{1}{\sqrt{x^3}}\,dx$
10. $\int (x-1)(x+1)\,dx$
11. $\int (2x^3-3\sqrt{x}+\frac{4}{x^5})\,dx$
12. $\int \sqrt[5]{1+x}\,dx$
13. $\int (x^3+1)(x-1)\,dx$
14. $\int \frac{x^2-1}{x-1}\,dx$
15. $\int \frac{x-3}{\sqrt{x}}\,dx$
16. $\int \frac{x+1}{\sqrt[3]{x}}\,dx$
17. $\int (x^3-1)^2\,dx$
18. $\int \sin(x+1)\,dx$
19. $\int (\cos x - x)\,dx$
20. $\int (\sec^2 x - 4)\,dx$
21. $\int (\sec x \tan x + x)\,dx$
22. $\int (\csc^2 x + x^2 + 1)\,dx$
23. $\int \frac{1}{\cos^2 x}\,dx$
24. $\int \frac{1}{\sin^2 x}\,dx$
25. $\int \frac{\tan x}{\cos x}\,dx$
26. $\int \sec x\,(\tan x - \sec x)\,dx$
27. $\int (2+\tan^2 x)\,dx$ (Hint: $\tan^2 x = \sec^2 x - 1$)
28. $\int \frac{\cos x}{\sin^2 x}\,dx$
29. $\int \frac{\tan x}{\cos^2 x}\,dx$
30. $\int \sin x \sec^2 x\,dx$
31. $\int \cos x \csc^2 x\,dx$
32. $\int \sec x\,(\sec x + 2\tan x)\,dx$
33. $\int \csc x\,(\csc x + 3\cot x)\,dx$
34. $\int \sin x \sqrt{\cos^3 x}\,dx$

35 - 64 ■ Evaluate the following integrals:

35. $\int x^4(3x^5+1)^{10}\,dx$

36. $\int x\sqrt{x^2+1}\,dx$

37. $\int (2x+1)\sqrt{x^2+x+2}\,dx$

38. $\int (x^2-1)\sqrt[3]{x^3-3x+2}\,dx$

39. $\int (5x+1)(5x^2+2x-5)^3\,dx$

40. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

41. $\int \frac{\sin^2\sqrt{x}}{\sqrt{x}\cos^2\sqrt{x}}\,dx$

42. $\int \frac{\cos^2\sqrt{x}}{\sqrt{x}\sin^2\sqrt{x}}\,dx$

43. $\int \frac{\sin 2x}{\cos^2 2x}\,dx$

44. $\int \frac{\cos\sqrt{x}}{\sqrt{x}\sin^2\sqrt{x}}\,dx$

45. $\int x\sin x^2\,dx$

46. $\int \frac{x}{\cos^2 x^2}\,dx$

47. $\int \frac{x+1}{\sin^2(x^2+2x-1)}\,dx$

48. $\int \frac{\csc^2\sqrt[3]{x}}{\sqrt[3]{x^2}}\,dx$

49. $\int \frac{\sec^2(\sqrt[5]{x}+1)}{\sqrt[5]{x^4}}\,dx$

50. $\int \frac{x}{\sqrt{x^2+9}}\,dx$

51. $\int \frac{x}{\sqrt[3]{x^2-1}}\,dx$

52. $\int \cos^2 x\,\sin x\,dx$

53. $\int \frac{\sin\sqrt{x}\cos\sqrt{x}}{\sqrt{x}}\,dx$

54. $\int \frac{\cos^3\sqrt{x}\sin\sqrt{x}}{\sqrt{x}}\,dx$

55. $\int \frac{2+\cos x}{\sin^2 x}\,dx$

56. $\int \frac{x}{\sqrt[5]{x+1}}\,dx$

57. $\int x\sqrt{x-3}\,dx$

58. $\int \frac{1}{\sqrt{x}(\sqrt{x}+1)^3}\,dx$

59. $\int \frac{2-x}{\sqrt{x}\sqrt{4-x}}\,dx$

60. $\int \sin x\left(\cos^2(x+1)\right)\,dx$

61. $\int \frac{\sin 2x}{(5+\cos 2x)^3}\,dx$

62. $\int \frac{x^3}{\sqrt{x^4-1}}\,dx$

63. $\int \frac{x^{\frac{3}{4}}}{\sqrt[4]{x}}\,dx$

64. $\int \frac{\sec\sqrt[3]{x}\tan\sqrt[3]{x}}{x^{\frac{2}{3}}}\,dx$

65 - 70 ■ Choose the correct answer:

65. The value of the integral $\int \frac{\sin x}{\sqrt{2+\cos x}}\,dx$ is equal to

(a) $-2\sqrt{2+\cos x}+c$
(b) $\sqrt{2+\cos x}+c$
(c) $-\sqrt{2+\cos x}+c$
(d) $2\sqrt{2+\cos x}+c$

66. The value of the integral $\int \frac{\sin(\tan x)}{\cos^2 x}\,dx$ is equal to

(a) $\cos(\tan x)+c$
(b) $\sin(\tan x)+c$
(c) $-\cos(\tan x)+c$
(d) $-\sin(\tan x)+c$

67. The integral $\int x\sqrt{x^2+1}\,dx$ is equal to

(a) $\frac{1}{2}x^2\sqrt{x^2+1}+c$
(b) $\frac{2}{3}(x^2+1)^{\frac{3}{2}}+c$
(c) $-\frac{2}{3}(x^2+1)^{\frac{3}{2}}+c$
(d) $\frac{1}{3}(x^2+1)^{\frac{3}{2}}+c$

68. The integral $\int \frac{x}{\cos^2 x^2}\,dx$ is equal to

(a) $\frac{1}{2}\tan x^2+c$
(b) $\tan x^2+c$
(c) $\frac{1}{2}\tan x+c$
(d) $-\frac{1}{\cos x^2}+c$

69. The value of the integral $\int \frac{\sec^2 x}{\cot^2 x}\,dx$ is equal to

(a) $\frac{1+\cos^2 x}{3\cos^3 x}+c$
(b) $\frac{1-3\cos^2 x}{3\cos^3 x}+c$
(c) $\frac{\cot^4 x}{4}+c$
(d) $\frac{\tan^3 x}{3}+c$

70. The value of the integral $\int \frac{\cos x}{\sqrt{4+\sin x}}\,dx$

(a) $\frac{1}{2}\sqrt{\sin x+4}+c$
(b) $\sqrt{\sin x+4}+c$
(c) $2\sqrt{\sin x+4}+c$
(d) $-2\sqrt{\sin x+4}+c$

Chapter 2: Review Exercises

1 - 4 ■ Express the following sums in terms of n:

1. $\sum_{k=1}^{n}(k-1)$
2. $\sum_{k=1}^{n}(2k+1)$
3. $\sum_{k=1}^{n}(k^2-k+1)$
4. $\sum_{k=1}^{n}(k^3+2k+1)$

5 - 8 ■ Find the following sums:

5. $\sum_{k=1}^{4}(2k+1)$

6. $\sum_{j=1}^{5}\frac{1}{j+1}$

7. $\sum_{k=1}^{3}(k^2+2k)$

8. $\sum_{i=1}^{4}(i-1)^2$

9 - 12 ■ For the partition P, find the norm $\| P \|$.

9. $P=\{0,1.01,1.1,2.5,3.6,4,6\}$

10. $P=\{1,2.5,3,4,5.1,6\}$

11. $P=\{-3,-2.5,-1,0.5,1.2,2\}$

12. $P=\{0,1.04,1.09,2.15,3.7,4,5\}$

13 - 16 ■ Find a Riemann sum R_P for the given function f by choosing the mark ω as follows:

(a) the left-hand endpoint,

(b) the right-hand endpoint,

(c) the midpoint,

13. $f(x)=x+1,\ \ \{1,2.5,3,3.5,4,5,6\}$

14. $f(x)=2x-1,\ \ \{-1,0,1,1.5,2,3,3.5\}$

15. $f(x)=x^2+1,\ \ \{1,1.5,2,2.5,3,3.5,4\}$

16. $f(x)=1-x^3,\ \ \{-2,-1,0,1,3,5,6\}$

17 - 22 ■ Find the area under the graph of f from a to b by taking the limit of a Riemann sum.

17. $f(x)=x+3,\ \ a=1,\ b=3$

18. $f(x)=3-x,\ \ a=0,\ b=1$

19. $f(x)=x^2,\ \ a=-1,\ b=1$

20. $f(x)=x^2-x+1,\ \ a=-1,\ b=3$

21. $f(x)=\frac{x}{2},\ \ a=2,\ b=4$

22. $f(x)=x^3+x+1,\ \ a=0,\ b=2$

23 - 28 ■ Let A be the area under the graph of the given function f from a to b. Approximate A by dividing $[a,b]$ into subintervals of equal length Δx and using inscribed polygons (A_{IP}) and circumscribed polygons (A_{CP}) for $\Delta x=1/2$ and by taking the limit of a Riemann sum.

23. $f(x)=x+3,\ \ a=1,\ b=3$

24. $f(x)=3-x,\ \ a=0,\ b=1$

25. $f(x)=x^2,\ \ a=-1,\ b=1$

26. $f(x)=x^2-x+1,\ \ a=-1,\ b=3$

27. $f(x)=\frac{x}{2},\ \ a=2,\ b=4$

28. $f(x)=x^3+x+1,\ \ a=0,\ b=2$

29 - 42 ■ Evaluate the following integrals:

29. $\int_{-2}^{4} 2\,dx$

30. $\int_{0}^{5} (3-x)\,dx$

31. $\int_{-1}^{4} (2x^2+x-1)\,dx$

32. $\int_{2}^{2} (6x^2+3)\,dx$

33. $\int_{0}^{1} (x^3-4x^4)\,dx$

34. $\int_{-1}^{1} x\sqrt{x^2+1}\,dx$

35. $\int_{0}^{5} |x-1|\,dx$

36. $\int_{0}^{3} |2x-3|\,dx$

37. $\int_{1}^{3} (x-2)(x+3)\,dx$

38. $\int_{0}^{\pi} \cos x\,dx$

39. $\int_{0}^{\frac{\pi}{2}} \sin x\,dx$

40. $\int_{0}^{\pi} \sec x\,(\tan x-\sec x)\,dx$

41. $\int_{0}^{\pi} x\cos x^2\,dx$

42. $\int_{\pi/4}^{\pi} \dfrac{\csc^2\sqrt{x}}{\sqrt{x}}\,dx$

43 - 48 ■ If $\int_a^b f(x)\,dx=2$, $\int_b^c f(x)\,dx=2$ and $\int_a^b g(x)\,dx=3$ where $c\in(a,b)$, evaluate the following integrals:

43. $\int_{b}^{a} f(x)\,dx$

44. $\int_{a}^{c} f(x)\,dx$

45. $\int_{a}^{b} \big(2f(x)+g(x)\big)\,dx$

46. $\int_{b}^{a} \big(5f(x)-3g(x)\big)\,dx$

47. $\int_{a}^{b} \big(\frac{1}{3}f(x)+7g(x)\big)\,dx$

48. $\int_{a}^{a} \big(4f(x)+g(x)\big)\,dx$

49 - 54 ■ Use the properties of the definite integrals to prove the following inequalities without evaluating the integrals:

49. $\int_{0}^{1} x\,dx \geq \int_{0}^{1} x^2\,dx$

50. $\int_{0}^{3} \dfrac{x}{x^3+2}\,dx \geq \int_{0}^{3} x\,dx$

51. $\int_{1}^{4} (2x+2)\,dx \geq \int_{1}^{4} (3x+1)\,dx$

52. $\int_{0}^{3} (x^2-3x+4)\,dx \geq 0$

53. $\int_{1}^{2} \sqrt{5-x}\,dx \geq \int_{1}^{2} \sqrt{x+1}\,dx$

54. $2 < \int_{-1}^{2} \sqrt{1+x^2}\,dx$

55 - 59 ■ Find the average value of the function f on the given interval.

55. $f(x)=x^2,\quad [1,4]$

56. $f(x)=9-x^2,\quad [0,3]$

57. $f(x)=x-x^2,\quad [0,2]$

58. $f(x)=x^3+1,\quad [-1,2]$

59. $f(x) = 6x^2 - 2x + 4, \quad [-1, 3]$

60 - 63 ■ Find the number z that satisfies the Mean Value Theorem for the function f on the given interval.

60. $f(x) = 2 + x^2, \quad [0, 4]$

61. $f(x) = x^3, \quad [-1, 3]$

62. $f(x) = \sqrt{x}, \quad [0, 9]$

63. $f(x) = 4x^3 - 1, \quad [1, 2]$

64 - 71 ■ Find the derivative of the following functions:

64. $\displaystyle\int_0^x \sin\sqrt{t}\, dt$

65. $\displaystyle\int_1^x \frac{1}{t}\, dt$

66. $\displaystyle\int_{3x}^{x^3} \sin(t^3+1)^{10}\, dt$

67. $\displaystyle\int_2^{x+1} \frac{1}{t^2+1}\, dt$

68. $\displaystyle\int_{\cos x}^{x} \cos t^2\, dt$

69. $\displaystyle\int_0^x \sqrt{t^2+1}\, dt$

70. $\displaystyle\int_{6x-1}^{0} \sqrt[3]{2t+4}\, dt$

71. $\displaystyle\int_3^{\sqrt{x}} \tan t^2\, dt$

72 - 75 ■ By using the trapezoidal rule, approximate the definite integral for the given n, then estimate the error.

72. $\displaystyle\int_1^5 x^3\, dx, \qquad n = 4$

73. $\displaystyle\int_1^2 \frac{1}{x}\, dx, \qquad n = 10$

74. $\displaystyle\int_0^1 e^x\, dx, \qquad n = 4$

75. $\displaystyle\int_1^3 \sqrt{1+x^3}\, dx, \qquad n = 6$

76 - 79 ■ By using the Simpson's rule, approximate the definite integral for the given n, then estimate the error.

76. $\displaystyle\int_0^{\pi} \frac{1}{2-\sin x}\, dx, \qquad n = 4$

77. $\displaystyle\int_0^1 \ln(1+e^x)\, dx, \qquad n = 6$

78. $\displaystyle\int_1^2 e^x\, dx, \qquad n = 6$

79. $\int_0^{\pi} \cos\, x^2\, dx, \qquad n = 4$

80 - 81 ■ Find the minimum number of subintervals to approximate the integral $\int_1^3 x^5 + 1\, dx$ by using the trapezoidal rule such that the error is less than

80. 10^{-2} 81. 10^{-4}

82 - 83 ■ Find the minimum number of subintervals to approximate the integral $\int_1^3 x^5 + 1\, dx$ by using the Simpson's rule such that the error is less than

82. 10^{-2} 83. 10^{-3}

84 - 106 ■ Choose the correct answer:

84. The sum $\sum_{k=1}^{n^2}(k-1)$ is equal to
(a) $\frac{n^2(n-1)}{2}$ (b) $\frac{n(n-1)}{2}$ (c) $\frac{n^2(n^2+1)}{2}$ (d) $\frac{n^2(n^2-1)}{2}$

85. The sum $\lim_{n\to\infty} \sum_{k=1}^{n}(\frac{k}{n^2})$ is equal to
(a) 0 (b) ∞ (c) 2 (d) $\frac{1}{2}$

86. If $\sum_{k=1}^{n}(k+\alpha) = \frac{n^2}{2}(n \geq 1)$, then the value of α is equal to
(a) $-\frac{n}{2}$ (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 1

87. If $\sum_{k=1}^{4}(k+a) = 14$, then the value of a is equal to
(a) 1 (b) 4 (c) -4 (d) -1

88. If $\sum_{k=1}^{5}(\alpha k^2 + k - 1) = 20$, then the value of α is equal to
(a) $\frac{2}{11}$ (b) $\frac{-2}{11}$ (c) $\frac{1}{11}$ (d) $\frac{-1}{11}$

89. If $\sum_{k=1}^{6}(k^2 + 3k + 2\alpha) = 130$, then the value of α is equal to
(a) 2 (b) -2 (c) 1 (d) 3

90. The average value of the function $f(x) = \sqrt[3]{x+1}$ on $[-2,0]$ is equal to
(a) 3 (b) 0 (c) -1 (d) -3

91. The average value of the function $f(x) = \sin\, x \cos\, x$ on $[0, \frac{\pi}{4}]$ is equal to
(a) $-\frac{1}{\pi}$ (b) $\frac{1}{4}$ (c) $\frac{1}{\pi}$ (d) $-\frac{1}{4}$

92. The average value of $f(x) = \mid x - 1 \mid$ on $[0,1]$ is equal to
(a) $-\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 0 (d) $\frac{1}{2}$

93. The average value of $f(x) = \sin\, x \cos\, x$ on $[-\pi, \pi]$ is equal to
(a) $\frac{1}{2\pi}$ (b) $\frac{1}{\pi}$ (c) 1 (d) 0

94. If $F(x) = \int_1^{x^2} \sqrt[3]{t^4+1}\, dt$, the $F'(x)$ is equal to
(a) $\sqrt[3]{x^8+1}$ (b) $x^2\sqrt[3]{x^8+1}$ (c) $2x\sqrt[3]{x^8+1}$ (d) $2x\sqrt[3]{x^4+1}$

95. The value of the integral $\int_0^2 \mid x - 1 \mid\, dx$ is equal to

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) 2

96. If $f(1)=3$, $f(4)=7$, $f(2)=4$ and $f(14)=23$, the value of the integral $\int_1^2 (x^2+1)f'(x^3+3x)\,dx$ is equal to
(a) $\frac{1}{3}$ (b) 16 (c) 1 (d) $\frac{16}{3}$

97. If $F(x)=x\int_{\sqrt{\pi}}^{x}\cos\ t^2\,dt$, then $F'(\sqrt{\pi})$ is equal to
(a) 0 (b) $\sqrt{\pi}$ (c) $-\sqrt{\pi}$ (d) 1

98. If $F(x)=\int_{2x}^{x^2}\sin\ t^3\,dt$, then $F'(x)$ is equal to
(a) $2x\sin\ x^6-\sin\ 8x^3$ (c) $2x\sin\ x^6-2\sin\ 6x^3$
(b) $2x\sin\ x^6-2\sin\ 8x^3$ (d) $2x\sin\ x^6+2\sin\ 8x^3$

99. The number z that satisfies the Mean Value Theorem for $f(x)=x^2$ on $[0,2]$ is
(a) $\sqrt{\frac{8}{3}}$ (b) $\frac{8}{\sqrt{3}}$ (c) $\sqrt{\frac{2}{3}}$ (d) $\frac{2}{\sqrt{3}}$

100. The number z that satisfies the Mean Value Theorem for $f(x)=1+x^2$ on $[-3,0]$ is
(a) $-\sqrt{3}$ (b) $\sqrt{3}$ (c) $\sqrt{2}$ (d) $-\sqrt{2}$

101. If $F(x)=\int_{x-1}^{x+1}\tan(t^2)\,dt$, then $F'(x)$ is equal to
(a) $\tan\ (x^2+2x+1)+\tan\ (x^2-2x+1)$ (c) $\tan\ (x^2+1)-\tan\ (x^2-1)$
(b) $\tan\ (x^2+2x+1)-\tan\ (x^2-2x+1)$ (d) 0

102. If $F(x)=\int_1^{x^3}\sqrt{5+t^2}\,dt$, then $F'(1)$ is equal to
(a) 0 (b) $3\sqrt{6}$ (c) $\sqrt{6}$ (d) $\frac{2}{\sqrt{6}}$

103. If $\int_0^{x^2} f(\sqrt{t})\,dt=x$, then $f(x)$ is equal to
(a) 1 (b) $\frac{1}{2x}$ (c) $\frac{1}{x^2}$ (d) $\frac{1}{2}$

104. The value of the integral $\int_{-1}^{1} 2\,|\,x\,|^3\ dx$
(a) 2 (b) 1 (c) 0 (d) -1

105. The derivative of the integral $\int_0^x \left(1+\frac{d\tan\ t}{dt}\right)dt$ is equal to
(a) $1+\tan\ x$ (b) $1-\tan\ x$ (c) $1-\sec^2\ x$ (d) $1+\sec^2\ x$

106. If $G(x)=\int_e^{x^2}\frac{\ln t}{4}\,dt$, then $G'(e)$ is equal to
(a) $2e$ (b) 1 (c) e (d) $4e$

Chapter 3 : Review Exercises

1 - 6 ■ Solve for x in the following equations:

1. $x = e^{\ln 2}$
2. $\ln x = 1$
3. $\ln x = \ln 3 - 2\ln 8$
4. $\ln x^2 = \ln 4 + \ln 2$
5. $\ln x = \ln(x+1) + \ln(x-1)$
6. $e^{2x} + 2e^x - 8 = 0$

7 - 12 ■ Find the following limits:

7. $\lim\limits_{x\to 0} \ln\cos\ x$
8. $\lim\limits_{x\to\infty} \frac{1}{1+\ln x}$
9. $\lim\limits_{x\to\infty} e^{-x} + 1$
10. $\lim\limits_{x\to\infty} \ln e^x$
11. $\lim\limits_{x\to\infty} \log_2 x + e^x$
12. $\lim\limits_{x\to 0^+} \ln\sin\ x$

13 - 44 ■ Find the derivatives of the following functions:

13. $f(x) = \ln x^2$
14. $f(x) = \ln(x^2 + 3x + 1)$
15. $f(x) = \ln\cos^3\ x$
16. $f(x) = \ln\sin\ x^2$
17. $f(x) = \ln\sqrt{x^3 + x - 1}$
18. $f(x) = \ln(\sqrt{x} - \sqrt{x-1})$
19. $f(x) = \sin\ x\ \ln\cos\ x$
20. $f(x) = \ln(\frac{x^2 \sin\ x}{\sqrt{x+1}})$
21. $f(x) = \frac{1}{\ln x} + \ln(\frac{1}{x})$
22. $f(x) = (\ln x^3)^2$
23. $f(x) = \sqrt{x}\ \ln(x^2 + x - 2)$
24. $f(x) = e^x \sec\ x$
25. $f(x) = e^{\ln x^2 + x - 1}$
26. $f(x) = e^{x+1} \sin^3\ x$
27. $f(x) = e^{\frac{x}{x+1}}$
28. $f(x) = \ln(\sin\ e^x)$
29. $f(x) = e^{2x+1}$
30. $f(x) = e^{\sin\ x}$
31. $f(x) = e^{\sec^2\ x}$
32. $f(x) = \sin(e^{2x^3 + x - 1})$
33. $f(x) = e^{2x+1}$
34. $f(x) = \frac{e^x}{x+1}$
35. $f(x) = \frac{e^x}{\ln x}$
36. $f(x) = e^{x\tan\ x}$
37. $f(x) = e^x \ln x$
38. $f(x) = x^2 e^{\sqrt{x}}$
39. $f(x) = \pi^{\cos\ x}$
40. $f(x) = 2^{\sin^2\ x}$
41. $f(x) = 10^{3x}$
42. $f(x) = \tan(2^{\sin\ x})$
43. $f(x) = \log_3(\frac{6x+1}{2x-1})$
44. $f(x) = \log(\ln x)$

45 - 50 ■ Find the derivatives of the following functions:

45. $y = (\tan\ x)^{\tan\ x}$

46. $y = x^x$

47. $y = x^{\sqrt{x}}$

48. $y = x^{4x}$

49. $y = x^{\sin\ x}$

50. $y = (\ln x)^{\tan\ x}$

51 - 72 ■ Evaluate the following integrals:

51. $\int \frac{x^2}{x^3+2}\,dx$

52. $\int \frac{\sin\ x}{\cos\ x}\,dx$

53. $\int \frac{x+1}{x^2+2x}\,dx$

54. $\int \frac{\sqrt{\ln x}}{x}\,dx$

55. $\int_0^1 \frac{x}{x^2+1}\,dx$

56. $\int_{-2}^0 \frac{x}{x^2+3}\,dx$

57. $\int \frac{\cos\ (\ln x)}{x}\,dx$

58. $\int (\sqrt{x}+\frac{1}{\sqrt{x}})^2\,dx$

59. $\int \frac{1}{x(\ln x)^2}\,dx$

60. $\int \frac{\sin\ x-\cos\ x}{\sin\ x+\cos\ x}\,dx$

61. $\int x3^{-x^2}\,dx$

62. $\int 2xe^{x^2}\,dx$

63. $\int \frac{e^x+e^{-x}}{e^x-e^{-x}}\,dx$

64. $\int \frac{\cos\ x\ e^{\ln(\sin\ x)}}{\sin\ x}\,dx$

65. $\int e^{\tan\ x}\sec^2\ x\,dx$

66. $\int \frac{5^{\sqrt{x}}}{\sqrt{x}}\,dx$

67. $\int_0^{\ln 2} e^x(2-3e^x)\,dx$

68. $\int \frac{x^3}{x^4+1}\,dx$

69. $\int 4^{3x}\,dx$

70. $\int_0^3 x3^{-x^2}\,dx$

71. $\int x10^{x^2+1}\,dx$

72. $\int \frac{a^{\sqrt{x+1}}}{\sqrt{x+1}}\,dx$ where $a>0$

73 - 89 ■ Choose the correct answer:

73. If $f(x) = \log_2 \frac{x}{x-1} = 1$, then x is equal to
(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) -1

74. The value of the integral $\int_0^1 5^x\,dx$ is equal to
(a) $\frac{4\ln 5}{5}$ (b) $\frac{\ln 5}{4}$ (c) $\frac{4}{\ln 5}$ (d) $\frac{5\ln 5}{4}$

75. If $f(x) = x^{x+1}$, then $f'(x)$ is equal to
(a) $(1+\frac{1}{x}+\ln x)x^{x+1}$ (b) $(\ln x+\frac{1}{x})x^{x+1}$ (c) $(1+\ln x)x^{x+1}$ (d) $(1+\frac{1}{x}+\ln x)x^x$

76. $\lim\limits_{x\to\infty} \frac{e^x+e^{2x}}{1+e^{2x}}$ is equal to
(a) ∞ (b) 1 (c) 0 (d) None of these

77. The integral $\int \tan\ 2x\ dx$ is equal to
(a) $\frac{-1}{2}\ln|\sec\ 2x|+c$ (b) $\frac{1}{2}\sec^2\ 2x+c$ (c) $\frac{-1}{2}\ln|\cos\ 2x|+c$ (d) $2\sec^2\ 2x+c$

78. The integral $\int \ln(2^{\sin\ x})\ dx$ is equal to
(a) $\frac{1}{2}\ln(2)\sin\ x+c$ (b) $2^{-\sin\ x}\cos\ x+c$ (c) $-\sin\ x+c$ (d) $-\ln 2\cos\ x+c$

79. The integral $\int_0^1 \frac{e^x}{(e^x+1)^2}\ dx$ is equal to
(a) $\frac{e-1}{2(1+e)}$ (b) 0 (c) -1 (d) $\frac{1}{(1+e)^2}$

80. If $f(x)=x^{\ln x}$ then $f'(e)$ is equal to
(a) 2 (b) $2e$ (c) 0 (d) e

81. $\lim\limits_{x\to 0^+} \frac{\sin x}{\ln x}$ is equal to
(a) ∞ (b) 0 (c) 1 (d) $-\infty$

82. If $f(x)=\ln(\ln x)$ then $f'(x)$ is equal to
(a) $\frac{1}{\ln x}$ (b) $\frac{1}{x\ln x}$ (c) $-\frac{1}{(\ln x)^2}$ (d) $-\frac{1}{x\ln x}$

83. The integral $\int 2^{\sin\ x}\cos\ x\ dx$ is equal to
(a) $2^{\sin\ x}+c$ (b) $(\ln 2)2^{\sin\ x}+c$ (c) $\frac{2^{\sin\ x}}{\ln 2}+c$ (d) $-\frac{2^{\sin\ x}}{\ln 2}+c$

84. The integral $\int \frac{\tan^2\ x}{\sec\ x}\ dx$ is equal to
(a) $\ln|\sec\ x+\tan\ x|+\sin\ x+c$ (b) $\ln|\sec\ x+\tan\ x|-\cos\ x+c$
(c) $\ln|\sec\ x+\tan\ x|-\sin\ x+c$ (d) $\ln|\sec\ x|-\sin\ x+c$

85. The value of the integral $\int_0^1 3^x\ dx$ is equal to
(a) $\frac{2}{\ln 3}$ (b) $\frac{3}{\ln 3}$ (c) 3 (d) 2

86. If $f(x)=x^x$, then $f'(1)$ is equal to
(a) 0 (b) e (c) 1 (d) $\frac{1}{e}$

87. The value of the integral $\int_0^1 (7x)7^{x^2}\ dx$ is equal to
(a) $\frac{21}{\ln 7}$ (b) $21\ln 7$ (c) $\frac{49}{\ln 7}$ (d) $\frac{7}{\ln 7}$

88. If $F(x)=x^{\frac{1}{x}}$, the $F'(e)$ is equal to
(a) 0 (b) e (c) $e^{\frac{1}{e}}$ (d) $\frac{e^{\frac{1}{e}}}{e^2}$

89. If $\log_2 \frac{x-1}{x}=2$, then x is equal to
(a) -1 (b) $\frac{1}{3}$ (c) $-\frac{1}{3}$ (d) 1

Chapter 4 : Review Exercises

1 - 18 ■ Find y' in the following:

1. $y = \sin^{-1}(3x+1)$
2. $y = \cos^{-1}\sqrt{x}$
3. $y = \tan^{-1}\frac{2}{3}x$
4. $y = \sec^{-1} 3x$
5. $y = \sinh(4x+1)$
6. $y = \cosh e^x$
7. $y = \sqrt{x}\tanh\sqrt{x}$
8. $y = e^{3x}\cosh 2x$
9. $y = \sqrt{\sinh 3x + \cosh 5x}$
10. $y = \tan^{-1}(\sinh x)$
11. $y = e^{\operatorname{sech} x}\cosh(\cosh x)$
12. $y = \frac{\sinh x}{\cosh x}$
13. $y = \operatorname{sech}^{-1} 3x$
14. $y = \coth^{-1}\sqrt{x}$
15. $y = x^4\cosh^{-1} x$
16. $y = e^x\tanh^{-1}\sqrt[3]{x}$
17. $y = \sinh^{-1}(\tanh x)$
18. $y = \tanh^{-1}\left(\frac{1-x}{1+x}\right)$

19 - 22 ■ Find the following limits:

19. $\lim_{x\to\infty}\frac{1}{\sinh x}$
20. $\lim_{x\to-\infty}\cosh x$
21. $\lim_{x\to\infty} e^x\tanh x$
22. $\lim_{x\to\infty} e^{\operatorname{sech} x}$

23 - 42 ■ Evaluate the following integrals:

23. $\int \sinh^3 x\cosh x\,dx$
24. $\int \tanh^4 x\operatorname{sech}^2 x\,dx$
25. $\int e^{\sinh x}\cosh x\,dx$
26. $\int e^x\operatorname{csch} x\,dx$
27. $\int \frac{\cosh\sqrt{x}}{\sqrt{x}}\,dx$
28. $\int x\operatorname{sech} x^2\tanh x^2\,dx$
29. $\int \frac{1}{\operatorname{sech} 3x}\,dx$
30. $\int \tanh x\,dx$
31. $\int \frac{1}{3+x^2}\,dx$
32. $\int \frac{1}{x\sqrt{x^4-4}}\,dx$
33. $\int \frac{1}{\sqrt{e^{2x}-1}}\,dx$
34. $\int \frac{x-1}{\sqrt{4-x^2}}\,dx$
35. $\int \frac{x+1}{x\sqrt{25-x^2}}\,dx$
36. $\int \frac{1}{x\sqrt{x^8-16}}\,dx$
37. $\int \frac{1}{\sqrt{1+4x^2}}\,dx$
38. $\int \frac{1}{4-9x^2}\,dx$
39. $\int_4^8 \frac{x}{x^4-16}\,dx$
40. $\int_2^3 \frac{1}{\sqrt{x^2-1}}\,dx$
41. $\int \frac{1}{\sqrt{25+9x^2}}\,dx$
42. $\int \frac{1}{\sqrt{e^{2x}-16}}\,dx$

43 - 55 ■ Choose the correct answer:

43. The derivative of the function $f(x) = \tan^{-1}(\sinh x)$ is equal to
(a) $\operatorname{sech} x$ (b) $\operatorname{csch} x$ (c) $\tanh x$ (d) $-\operatorname{sech} x$

44. The value of the integral $\int_{-1}^{1} \sinh x\, dx$ is equal to
(a) 0 (b) $2e$ (c) $2e^{-1}$ (d) $\frac{1}{2}e$

45. If $f(x) = \cosh^{-1}\sqrt{x}$, then $f'(x)$ is equal to
(a) $\frac{1}{2\sqrt{x^2-x}}$ (b) $\frac{1}{2\sqrt{x-x^2}}$ (c) $\frac{1}{2\sqrt{x^2+x}}$ (d) None of these

46. The integral $\int \frac{x-2}{x\sqrt{x^2-25}}\, dx$, is equal to
(a) $\cosh^{-1}\frac{x}{5} - 2\sec^{-1}\frac{x}{5} + c$ (b) $\cosh^{-1}\frac{x}{5} - \frac{2}{5}\sec^{-1} x + c$
(c) $\cosh^{-1}\frac{x}{5} - \frac{2}{5}\sec^{-1}\frac{x}{5} + c$ (d) None of these

47. If $f(x) = \tanh^{-1}(\cos 3x)$, then $f'(x)$ is equal to
(a) $3\csc 3x$ (b) $-3\csc 3x$ (c) $\frac{-3\sin 3x}{1+\cos^2 3x}$ (d) 0

48. The integral $\int \frac{\cos x}{1+\sin^2 x}\, dx$, is equal to
(a) $\frac{1}{1+\sin x} + c$ (b) $\tan^{-1}(\sin x) + c$ (c) $\frac{1}{1+\cos x} + c$ (d) $\tanh^{-1}(\sin x) + c$

49. The value of the integral $\int \frac{dx}{\sqrt{16-25x^2}}$ is
(a) $-\frac{\cos^{-1}\frac{x}{16}}{25} + c$ (b) $\frac{\cos^{-1}\frac{x}{16}}{25} + c$ (c) $\frac{\sin^{-1}\frac{5x}{4}}{5} + c$ (d) $-\frac{\sin^{-1}\frac{5x}{4}}{5} + c$

50. The value of the integral $\int \frac{1}{\sqrt{x^2+2}}\, dx$ is
(a) $\sin^{-1} x + c$ (b) $\sinh^{-1} x + c$ (c) $\sinh^{-1}\frac{x}{\sqrt{2}} + c$ (d) $\sin^{-1}\frac{x}{\sqrt{2}} + c$

51. The integral $\int \frac{\cosh x}{1-\sinh^2 x}\, dx$ is equal to
(a) $-\tan^{-1}(\sinh x) + c$ (b) $\tan^{-1}(\sinh x) + c$
(c) $\frac{1}{1+\cosh x} + c$ (d) $\tanh^{-1}(\sinh x) + c$

52. If $F(x) = \tan^{-1} x + \tan^{-1}(\frac{1}{x})$ where $x \neq 0$, then $F'(x)$ is equal to
(a) $\frac{2}{1+x^2}$ (b) $\frac{-1}{1+x^2}$ (c) 0 (d) $\frac{x^2}{1+x^2}$

53. The derivative of the function $f(x) = \tan^{-1}(\sinh x)$ is equal to
(a) $\frac{1}{1+\sinh^2 x}$ (b) $\sec^2(\sinh x)$ (c) $\frac{1}{\cosh x}$ (d) $\frac{\cosh x}{1-\sinh^2 x}$

54. The value of the integral $\int \frac{1}{\sqrt{4+x^2}}\, dx$ is equal to
(a) $\sinh^{-1}\frac{x}{2} + c$ (b) $\sin^{-1}\frac{x}{2} + c$ (c) $\frac{1}{2}\sinh^{-1}\frac{x}{2} + c$ (d) $\frac{1}{2}\sin^{-1}\frac{x}{2} + c$

55. The value of the integral $\int_{-1}^{1} \cosh x\, dx$ is equal to
(a) 0 (b) $2e$ (c) $2e^{-1}$ (d) $e - e^{-1}$