Virtual Black Holes from Generalized Uncertainty Principle and Proton Decay

Salwa Alsaleh *et al*

Department of Physics and Astronomy,
King Saud University, Riyadh 11451, Saudi Arabia

Email : salwams@ksu.edu.sa

عمادة البحث العلمي

Introduction

The existence of virtual black holes at the scale of quantum gravity is a possible observation of proton decay [1], a process that is forbidden by the standard model due to conservation of baryon number B. The proton half life is estimated to be larger than $\sim 10^{34}$ years, by experimental observations [25]. In Quantum gravity approaches, the proton decay is considered as mediator for the virtual black hole. Simply, consider the proton as a spherical object of radius $r_{proton} \sim 10^{-15}$ m, and virtual black holes form inside of that space, two of the three point-like quarks could fall into the black hole, to evaporate away It is observed that the proton half-life depends on the quantum gravity mass of the black hole. In our analysis , the mass M_{qg} is equal to the Planck mass M_p. Making the proton half life due to this process in the order of $\sim 10^{45}$ years. Proton decay via virtual black holes was also studied in large extra dimensions existed, like in Randal-Strudum model. In this case the half life is changed, because of these extra dimensions, and the probability will change because there is more ' space' for the quarks to move in.

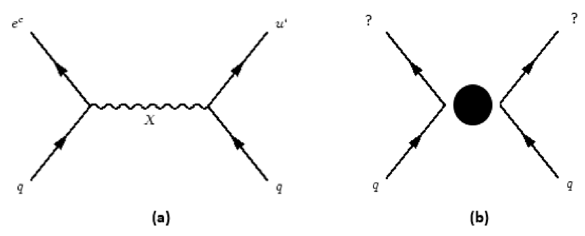

Figure 1: Proton decay Feynman diagram (a) Via the X-boson exchange particle, a decay predicted by GUP models. (b) Via two quarks falling into a virtual black hole.

The quantum gravity mass M_{qg} here may not be the Plank mass, but it is bounded to be $M_{qg} > (M_p^D \Lambda^4)^{1/D}$, where Λ is the energy scale defined by experimental bound $\Lambda \sim 10^{16} GeV$ [13].
However, some theories beyond the standard model, like grand unified theories (GUTs), supersymmetry (SUSY), electroweak sphaleron anomaly [5] and magnetic monopoles break the baryon number conservation.

Theory	Proton half life in years (τ_p)
Quantum gravity in D=4	$\sim 10^{45}$
Quantum gravity in $D > 4$	$\sim 10^{33} 10^{64} (\frac{M_{qg}}{\Lambda})^4$
Georgi-Glashow SU(5)	$\sim 10^{30} - 10^{31}$
Mimimal SUSY SU(5)	$\sim 10^{28} - 10^{32}$
SUSY (MSSM) SU(5)	$\sim 10^{34}$
SUSY (D=5) SU(5)	$\sim 10^{35}$
SO(10) GUT	$\lesssim 10^{35}$
Mimimal SUSY (MSSM) SO(10)	$\sim 10^{34}$
SUSY SO(10)	$\sim 10^{32} - 10^{35}$
Supergravity (SUGRA) SU(5)	$\sim 10^{32} - 10^{34}$
Superstring (Flipped SU(5))	$\sim 10^{35} - 10^{36}$

Figure 2: Proton half life (τ_p) in various models [1, 18, 24].

The existence of a minimal length scale that commonly predicted by various approaches to quantum gravity is manifested phenomenologically by deformation of the standard momentum dispersion relations [11] to incorporate a cut-off length ℓ_p - or equivalently- energy E_p scales [17]. This can be achieved by deformation of Heisenberg algebra [19]. This is known as the Generalized uncertainty principle (GUP) [20, 21], which is based on deforming the commutation relation between momentum and position operators in quantum mechanics, but keeping the associative structure. The most general type of deformation is [3]

$$[x^\mu, p_\nu] = \frac{\hbar}{2}\eta^\mu_\nu \left\{1 - \frac{\alpha}{E_p}\left(|p|\eta^\mu_\nu + \frac{p^\mu p_\nu}{|p|}\right) + \frac{\alpha}{E_p}\left(p^2 \eta^\mu_\nu + 3p^\mu p_\nu\right)\right\}. \tag{1}$$

Considering the GUP deformation as a phenomenological model of quantum gravity, it would be interesting to investigate the GUP deformation on the quantum gravity mass M_{qg} and hence the proton half life.

Virtual black holes and GUP

In order to explicitly calculate the value M_{GUP}, we follow a similar argument made in [2, 8, 23] in order to compute the minimal mass for black holes with minimal length. We make the argument very general and consider D dimensional spacetime with all the $D-1$ momenta p_i being equal and the quadratic GUP is given by [9, 23]

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left(1 + 14.9\left(\frac{D-3}{\pi}\right)^2 \alpha^2 \frac{\Delta p^2}{M_p^2}\right) \tag{2}$$

Which leads to expression for Δp

$$\Delta p \geq \frac{\Delta x}{\xi\,\alpha^2}\left\{1 - \sqrt{1 - \frac{\xi \hbar \alpha^2}{\Delta x^2}}\right\}, \tag{3}$$

where $\xi = 14.9\left(\frac{D-3}{\pi}\right)^2$. Now, let a particle be bounded at the black hole event horizon $\Delta x \sim r_s$, this particle resembles a particle emitted by Hawking radiation from the horizon at Temperature associated with the resulting uncertainty in the momentum/energy of that particle localized at the horizon. Therefore the modified Hawking temperature is calculated using this argument in [2].

$$T_{GUP} = 2T_H\left[1 + \sqrt{1 - \frac{\xi\alpha_0^2}{4\lambda_D^2 m^{\frac{2}{D-3}}}}\right] \tag{4}$$

In which

$$\lambda_D = \left(\frac{16\pi}{(D-2)\Omega_{D-2}}\right)^{\frac{1}{D-3}}$$
$$\Omega_D = \frac{2\pi^{\frac{D-1}{2}}}{\Gamma(\frac{D-1}{2})} \tag{5}$$
$$m = \frac{M}{M_p}$$

The GUP modified temperature leads to the existence of a minimal mass M_{min} which is the mass of GUP virtual black holes, and it is given by,

$$M_{GUP}(\alpha) = M_p\left(\sqrt{\frac{\pi\xi}{4}}\right)^{D-3}\frac{D-2}{8\Gamma(\frac{D-1}{2})}\alpha^{D-3} \tag{6}$$

Since what is multiplied with α is a pure numerical factor that depends on the dimension of spacetime, we denote such factor by $f(D)$

$$M_{GUP}^{(D)} = f(D) M_p^{(D)} \alpha^{D-3} \tag{7}$$

The proton half life is given by the expression :

$$\tau_p \sim M_{proton}^{-1}\left(\frac{M_p^{(D)}}{M_{proton}}\right)^D f(D)^D \alpha^{D(D-3)} \tag{8}$$

Another possible, and more general generalized uncertainty relation is [3, 2]

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left\{1 - \alpha\frac{3.76}{M_p}\left(\frac{D-3}{\pi}\right)\Delta p + \alpha^2\frac{7.64}{M_p^2}\left(\frac{D-3}{\pi}\right)^2 \Delta p^2\right\} \tag{9}$$

This uncertainty relation can be used similar to (2) to find the mass of GUP-deformed virtual black holes and study proton decay using the same argument as before
Solving (9) for Δp.

$$\Delta p \geq \frac{2\Delta x + \alpha M_p \gamma_1}{\alpha^2 \gamma_2}\left(1 - \sqrt{1 - \frac{2\gamma_2 \alpha^2 M_p^2}{2\Delta x + \alpha M_p \gamma_1}}\right) \tag{10}$$

Such that $\gamma_1 = 3.76\left(\frac{D-3}{\pi}\right)$ and $\gamma_2 = 15.28\left(\frac{D-3}{\pi}\right)^2$ Just like the quadratic GUP, we denote the numerical factor for the minimal mass by $g(D)$. Writing the minimal mass as

$$M_{GUP}'^{(D)} = g(D) M_p^{(D)} \alpha^{D-3} \tag{11}$$

The proton half life is given by the expression :

$$\tau_p \sim M_{proton}^{-1}\left(\frac{M_p^{(D)}}{M_{proton}}\right)^D g(D)^D \alpha^{D(D-3)} \tag{12}$$

We can use the data for Planck masses in different spacetime dimensions $M_p^{(D)}$ [10] and the relations (8)(12) to estimate the bounds on the GUP deformation parameter. The bound on α is given by:

$$\alpha' > 10^{-\frac{65+3D}{D(D-1)}} M_p^{(D)\frac{1}{D-1}} f/g(D)^{\frac{1}{D-1}} \tag{13}$$

Table 2 contains the bounds on α and α' in quadratic and linear-quadratic GUP deformations in different spacetime dimensions, relevant to physical models.

D	Bound on quadratic α	Bound on Linear-quadratic α'
4	$> 4.37 \times 10^{-3}$	$> 3.64 \times 10^{-3}$
6	> 101.01	> 84.95
9	> 0.87	> 0.74
10	> 0.51	> 0.42

Figure 3: Bounds on the GUP deformation parameters α and α'. From the experimentally measured half-life of the proton.

Conclusion

) In this work, we investigated the production of virtual black holes in higher dimensions in the context of generalized uncertainty principle. We use this black hole production to study the proton decay process that is considered as a mediator for these virtual black holes. We calculate the proton half life in higher dimensions and we set bound on the GUP deformation parameter α and α'. We found that the bounds on GUP parameters are around 100, .87, and 0.51 for 6, 9 and 10 dimensions, respectively. These values are stringent and consistent with the bound set by electroweak scale [4]. In fact, this is an improvement for various studies on phenomenological aspects of GUP, if the GUP parameter $\alpha \sim 1$, it appears to be a new and interesting result and relevant to be studied at low energy systems[4, 12]. This indicate that GUP could be useful to explain the proton decay process beyond the standard model and could open an interesting phenomenological window for studying quantum gravity effects for low energy systems. We hope to report on these issues in the future.

Acknowledgements This research project was supported by a grant from the " Research Center of the Female Scientific and Medical Colleges ", Deanship of Scientific Research, King Saud University.

References

[1] Fred C Adams, Gordon L Kane, Manasse Mbonye, and Malcolm J Perry. Proton decay, black holes, and large extra dimensions. *International Journal of Modern Physics A*, 16(13):2399–2410, 2001.

[2] Ahmed Farag Ali. No existence of black holes at lhc due to minimal length in quantum gravity. *Journal of High Energy Physics*, 2012(9):1–14, 2012.

[3] Ahmed Farag Ali, Saurya Das, and Elias C Vagenas. Discreteness of space from the generalized uncertainty principle. *Physics Letters B*, 678(5):497–499, 2009.

[4] Ahmed Farag Ali, Saurya Das, and Elias C. Vagenas. A proposal for testing Quantum Gravity in the lab. *Phys. Rev.*, D84:044013, 2011.

[5] Peter Arnold and Larry McLerran. Sphalerons, small fluctuations, and baryon-number violation in electroweak theory. *Physical Review D*, 36(2):581, 1987.

[6] John Baez and John Huerta. The algebra of grand unified theories. *Bulletin of the American Mathematical Society*, 47(3):483–552, 2010.

[7] Borut Bajc, Pavel Fileviez Perez, and Goran Senjanović. Proton decay in minimal supersymmetric su (5). *Physical Review D*, 66(7):075005, 2002.

[8] Marco Cavaglia and Saurya Das. How classical are tev-scale black holes? *Classical and Quantum Gravity*, 21(19):4511, 2004.

[9] Marco Cavaglia, Saurya Das, and Roy Maartens. Will we observe black holes at the lhc? *Classical and Quantum Gravity*, 20(15):L205, 2003.

[10] Serguei Chatrchyan, Vardan Khachatryan, Albert M Sirunyan, A Tumasyan, W Adam, T Bergauer, M Dragicevic, J Erö, C Fabjan, M Friedl, et al. Search for new physics with a monojet and missing transverse energy in p p collisions at s= 7 tev. *Physical review letters*, 107(20):201804, 2011.

[11] JL Cortes and J Gamboa. Quantum uncertainty in doubly special relativity. *Physical Review D*, 71(6):065015, 2005.

[12] Saurya Das and Elias C. Vagenas. Universality of Quantum Gravity Corrections. *Phys. Rev. Lett.*, 101:221301, 2008.

[13] Savas Dimopoulos and Greg Landsberg. Black holes at the large hadron collider. *Physical Review Letters*, 87(16):161602, 2001.

[14] Savas Dimopoulos, Stuart Raby, and Frank Wilczek. Proton decay in supersymmetric models. *Physics Letters B*, 112(2):133–136, 1982.

[15] Paul H Frampton and Thomas W Kephart. Higgs sector and proton decay in su (15) grand unification. *Physical Review D*, 42(11):3892, 1990.

[16] Howard Georgi and Sheldon L Glashow. Unity of all elementary-particle forces. *Physical Review Letters*, 32(8):438, 1974.

[17] Jerzy Kowalski-Glikman. Introduction to doubly special relativity. In *Planck Scale Effects in Astrophysics and Cosmology*, pages 131–159. Springer, 2005.

[18] Paul Langacker. Grand unified theories and proton decay. *Physics Reports*, 72(4):185–385, 1981.

[19] Michele Maggiore. The algebraic structure of the generalized uncertainty principle. *Physics Letters B*, 319(1-3):83–86, 1993.

[20] Michele Maggiore. A generalized uncertainty principle in quantum gravity. *Physics Letters B*, 304(1-2):65–69, 1993.

[21] Michele Maggiore. Quantum groups, gravity, and the generalized uncertainty principle. *Physical Review D*, 49(10):5182, 1994.

[22] A Masiero, DV Nanopoulos, Kyriakos Tamvakis, and T Yanagida. Naturally massless higgs doublets in supersymmetric su (5). *Physics Letters B*, 115(5):380–384, 1982.

[23] AJM Medved and Elias C Vagenas. When conceptual worlds collide: The generalized uncertainty principle and the bekenstein-hawking entropy. *Physical Review D*, 70(12):124021, 2004.

[24] Pran Nath and Pavel Fileviez Perez. Proton stability in grand unified theories, in strings and in branes. *Physics reports*, 441(5):191–317, 2007.

[25] H Nishino and others. Search for proton decay via $p \to e^+\pi^0$ and $p \to \mu^+\pi^0$ in a large water cherenkov detector. *Phys. Rev. Lett.*, 102:141801, Apr 2009.

[26] N Sakai and Tsutomu Yanagida. Proton decay in a class of supersymmetric grand unified models. *Nuclear Physics B*, 197(3):533–542, 1982.

[27] BV Sreekantan. Searches for proton decay and superheavy magnetic monopoles. *Journal of Astrophysics and Astronomy*, 5(3):251–271, 1984.