# College of Sciences <br> Department of <br> Statistics and Operations <br> Research 

## First Midterm Exam

| Sunday, March 3, 2019 | STAT 105 | Academic year 2018-2019 |
| :---: | :---: | :---: |
| 7:00-8:30 pm |  | Second Semester |
| Student's Name |  |  |
| ID number |  |  |
| Section No. |  |  |
| Classroom No. |  |  |
| Teacher's Name |  |  |
| Roll Number |  |  |

## Instructions:


$\diamond$ Switch off your mobile and place it under your seat.
$\diamond$ Time allowed is 90 Minutes.
$\diamond$ Do not copy answers from your neighbors. They have different questions forms.
$\diamond$ Choose the nearest number to your answer.
$\diamond$ Do not use pencils or red pens.
$\diamond$ For each question, put the code (Capital Letters) of the correct answer in the following table beneath the question number.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C | B | C | D | B | C | C | C | B | A |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| D | C | B | C | D | B | D | B | D | A |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| D | B | C | D | A | B | D | C | B | A |

Questions (1): The battery failure time, measured in hours, has a probability density function:

$$
f(x)= \begin{cases}\frac{2}{(x+1)^{3}}, & x>0 \\ 0, & \text { otherwise }\end{cases}
$$

1) The probability that a battery lasts more than five hours is

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| $4 / 9$ | $1 / 2$ | $1 / 36$ | $2 / 5$ |

Questions (2-3): A random variable $X$ has a probability density function:

$$
f(x)= \begin{cases}\frac{1}{6}, & x \in\{1,2,3,4,5,6\} \\ 0, & \text { otherwise }\end{cases}
$$

2) The expected value of $X, E(X)$ is equal to

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 2 | 3 | 5 | 1 |

3) The variance of $X, V(X)$ is equal to

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 |

Questions (4): Let $X$ be a normal random variable with mean 30 and variance 36.
4) $\mathbf{P}(32<X<38)$ is equal to

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 0.485 | 0.342 | 0.156 | 0.279 |

Questions (5): Let $X$ be an exponential random variable with mean $\frac{1}{\lambda}=5$.
5) $\mathbf{P}(7<X)$ is equal to

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 0.524 | 0.246 | 0.178 | 0.425 |

Questions (6): Let $X$ be an exponential random variable with mean 10 and standard deviation 2.
6) The value of $x$ with $9 \%$ of the area to its left is

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 5.33 | 2.75 | 7.32 | 1.85 |

Questions (7-8): The random variable $T$ follows the exponential distribution with mean time to failure is 6 years.
7) The probability that it is still functioning at the end of 10 years is

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 0.211 | 0.167 | 0.189 | 0.231 |

8) If 4 of these components are installed in different systems. The probability that at least 1 is still functioning at the end of 10 years is

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 0.214 | 0.133 | 0.567 | 0.312 |

Questions (9): The television picture tubes of manufacturer A have a mean lifetime of 5.5 years and a standard deviation of two years, while those of manufacturer $B$ have a mean lifetime of 5 years and a standard deviation of 1.5 year.
9) The probability that a random sample of 40 tubes from manufacturer A will have a mean lifetime that is 1 year more than the mean lifetime of a sample of 50 tubes from manufacturer B is

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 0.024 | 0.0951 | 0.102 | 0.25 |

Questions (10): Let $T$ be a student random variable.
10) The probability $\mathbf{P}\left(-t_{0.25}<T<t_{0.5}\right)$ is

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 0.25 | 0.35 | 0.15 | 0.05 |

Questions (11-12): Let $X$ be a Chi-squared random variable.
11) Find $\mathcal{X}_{\alpha}^{2}$ such that, The probability $\mathbf{P}\left(\mathcal{X}_{\alpha}^{2}<X<2.204\right)=0.075$ with $\nu=6$

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 0.505 | 0.325 | 0.054 | 1.237 |

12) Find $\mathcal{X}_{\alpha}^{2}$ such that, The probability $\mathbf{P}\left(\mathcal{X}_{\alpha}^{2}<X\right)=0.075$ with $\nu=5$

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 5.125 | 6.532 | 2.675 | 4.521 |

Questions (13-14): Let $F$ be a Fisher random variable.
13) The $f_{0.95}$ with $\nu_{1}=15$ and $\nu_{2}=15$

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 0.3333 | 0.4167 | 0.1543 | 0.2578 |

14) The $f_{0.99}$ with $\nu_{1}=19$ and $\nu_{2}=3$

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 0.3212 | 0.4512 | 0.1996 | 0.2141 |

Questions (15-17): Suppose that 20 percent of adults of a male population were obese. In a simple random sample of size 200 from this population
15) The mean of the proportion $\widehat{P}$ of adults in this sample who were obese is

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 0.5 | 0.7 | 0.1 | 0.2 |

16) The variance of the proportion $\widehat{P}$ of adults in this sample is

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 0.0052 | 0.0008 | 0.0134 | 0.1022 |

17) The probability that the proportion $\widehat{P}$ in this sample who are obese is fewer than 15 percent is

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 0.0452 | 0.0848 | 0.0152 | 0.0384 |

Questions (18-20): Suppose that there are two high schools, in a certain town. At School 1, $55 \%$ of students did their homework last night. Besides, $65 \%$ of the students at School 2 did their homework last night. The counselor at School 1 takes a simple random sample of 100 students and records the proportion that did the homework $\widehat{P}_{1}$. School 2 counselor's takes a simple random sample of 150 students and records the proportion that did the homework $\widehat{P}_{2}$.
18) The standard deviation of the difference $\widehat{P}_{2}-\widehat{P}_{1}$ is

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 0.251 | 0.0632 | 0.1588 | 0.0025 |

19) The sampling distribution of the difference $\widehat{P}_{2}-\widehat{P}_{1}$ is

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| $t$ | $\mathcal{X}^{2}$ | $F$ | $N(0,1)$ |

20) The probability of getting a difference $\widehat{P}_{2}-\widehat{P}_{1} \geq 0.05$ is

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 0.785 | 0.251 | 0.512 | 0.315 |

Questions (21-26): Let $X$ be a random variable that has the following probability function

| $X$ | -1 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 0.1 | 0.2 | 0.4 | 0.3 |

21) $\mathbf{P}(-1<X<3)$ is equal to:

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 0.1 | 0.2 | 0.5 | 0.6 |

22) $\mathbf{P}(-1 \leq X \leq 2)$ is equal to:

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 0.9 | 0.7 | 0.3 | 0.4 |

23) $\mathbf{P}(0 \leq X<2)$ is equal to:

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 0.6 | 0.8 | 0.2 | 0.5 |

24) $\sum_{k \in\{-1,1,2,3\}} \mathbf{P}(X=k)$ is equal to:

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 2.5 | 1.1 | 1.3 | 1 |

25) The mean $\mu=E(X)$ is equal to:

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 1.8 | 2.1 | 1 | 1.5 |

26) The variance $\sigma^{2}$ is equal to:

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 2.11 | 1.36 | 1.22 | 3.16 |

Questions (27): A dice (a cube with six faces, on each face a number from 1 to
6 ) is tossed once.
27) The probability of getting a number less than 3 is

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| $2 / 3$ | $1 / 5$ | $3 / 5$ | $1 / 3$ |

Questions (28-29): The probability that a patient recovers from a rare blood disease is 0.4 . If 5 people are known to have contracted this disease.
28) The probability that at most 2 will survive this disease.

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 0.258 | 0.134 | 0.683 | 0.912 |

29) The expected number of survivors from this disease is

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 5 | 2 | 1 | 8 |

Questions (30): The traffic accidents in a city follows a Poisson distribution with rate of 2 accidents every hour.
30) The probability that in certain hour there will be four accidents is

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| 0.09 | 0.006 | 0.201 | 0.512 |

