

SAMPLING DISTRIBUTIONS

10.1. Single Mean:

Q1. A machine is producing metal pieces that are cylindrical in shape. A random sample of size 5 is taken and the diameters are 1.70, 2.11, 2.20, 2.31 and 2.28 centimeters. Then,

1) The sample mean is:

- (A) 2.12 (B) 2.32 (C) 2.90 (D) 2.20 (E) 2.22

2) The sample variance is:

- (A) 0.59757 (B) 0.28555 (C) 0.35633 (D) 0.06115 (E) 0.53400

Solution:

$$\bar{X} = \frac{\sum X_i}{n} = \frac{1.7 + 2.11 + 2.20 + 2.31 + 2.28}{5} = 2.12$$

$$S^2 = \frac{\sum (X_i - \bar{X})^2}{n - 1} = \frac{(1.7 - 2.12)^2 + (2.11 - 2.12)^2 + (2.2 - 2.12)^2 + (2.31 - 2.12)^2 + (2.28 - 2.12)^2}{4} = 0.06115$$

Q2. The average life of a certain battery is 5 years, with a standard deviation of 1 year. Assume that the live of the battery approximately follows a normal distribution.

1) The sample mean \bar{X} of a random sample of 5 batteries selected from this product has a mean $E(\bar{X}) = \mu_{\bar{X}}$ equal to:

- (A) 0.2 (B) 5 (C) 3 (D) None of these

2) The variance $Var(\bar{X}) = \sigma_{\bar{X}}^2$ of the sample mean \bar{X} of a random sample of 5 batteries selected from this product is equal to:

- (A) 0.2 (B) 5 (C) 3 (D) None of these

3) The probability that the average life of a random sample of size 16 of such batteries will be between 4.5 and 5.4 years is:

- (A) 0.1039 (B) 0.2135 (C) 0.7865 (D) 0.9224

4) The probability that the average life of a random sample of size 16 of such batteries will be less than 5.5 years is:

- (A) 0.9772 (B) 0.0228 (C) 0.9223 (D) None of these

5) The probability that the average life of a random sample of size 16 of such batteries will be more than 4.75 years is:

- (A) 0.8413 (B) 0.1587 (C) 0.9452 (D) None of these

6) Delete If $P(\bar{X} > a) = 0.1492$ where \bar{X} represents the sample mean for a random sample of size 9 of such batteries, then the numerical value of a is:

- (A) 4.653 (B) 6.5 (C) 5.347 (D) None of these

Solution:

1) $E(\bar{X}) = \mu_{\bar{X}} = \mu = 5$

2) $V(\bar{X}) = \sigma_{\bar{X}}^2 = \frac{\sigma^2}{n} = \frac{1}{5} = 0.2$

3) $P(4.5 < \bar{X} < 5.4) = P\left(\frac{4.5-5}{1/\sqrt{16}} < \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < \frac{5.4-5}{1/\sqrt{16}}\right) = P(-2 < Z < 1.6) = 0.9224$

4) $P(\bar{X} < 5.5) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < \frac{5.5-5}{1/4}\right) = P(Z < 2) = 0.9772$

5) $P(\bar{X} > 4.75) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{4.75-5}{1/4}\right) = 1 - P(Z < -1) = 0.8413$

$$6) P(\bar{X} > a) = 0.1492 \Rightarrow P(\bar{X} < a) = 0.8508 \Rightarrow \frac{a-5}{\frac{1}{\sqrt{9}}} = \frac{a-5}{\frac{1}{3}} = 1.04 \Rightarrow a = 5.347$$

Q3. The random variable X, representing the lifespan of a certain light bulb, is distributed normally with a mean of 400 hours and a standard deviation of 10 hours.

1. What is the probability that a particular light bulb will last for more than 380 hours?
2. Light bulbs with lifespan less than 380 hours are rejected. Find the percentage of light bulbs that will be rejected.
3. If 9 light bulbs are selected randomly, find the probability that their average lifespan will be less than 405.

Solution:

$$X \sim N(400, 10)$$

$$1) P(X > 380) = P\left(\frac{X-\mu}{\sigma} > \frac{380-400}{10}\right) = 1 - P(Z < -2) = 1 - 0.0288 = 0.9712$$

$$2) P(X < 380) * 100\% = 2.88\%$$

$$3) P(\bar{X} < 405) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < \frac{405-400}{10/3}\right) = P(Z < 1.5) = 0.9332$$

Q4. H.W Suppose that you take a random sample of size $n=64$ from a distribution with mean $\mu=55$ and standard deviation $\sigma=10$. Let $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$ be the sample mean.

- (a) What is the approximated sampling distribution of \bar{X} ?
- (b) What is the mean of \bar{X} ?
- (c) What is the standard error (standard deviation) of \bar{X} ?
- (d) Find the probability that the sample mean \bar{X} exceeds 52.

Q5. H.W The amount of time that customers using ATM (Automatic Teller Machine) is a random variable with the mean 3.0 minutes and the standard deviation of 1.4 minutes. If a random sample of 49 customers is observed, then

- (1) the probability that their mean time will be at least 2.8 minutes is
 (A) 1.0 (B) 0.8413 (C) 0.3274 (D) 0.4468
- (2) the probability that their mean time will be between 2.7 and 3.2 minutes is
 (A) 0.7745 (B) 0.2784 (C) 0.9973 (D) 0.0236

Q6. The average life of an industrial machine is 6 years, with a standard deviation of 1 year. Assume the life of such machines follows approximately a normal distribution. A random sample of 4 of such machines is selected. The sample mean life of the machines in the sample is \bar{x} .

- (1) The sample mean has a mean $\mu_{\bar{x}} = E(\bar{X})$ equals to:
 (A) 5 (B) 6 (C) 7 (D) 8
- (2) The sample mean has a variance $\sigma_{\bar{x}}^2 = Var(\bar{X})$ equals to:
 (A) 1 (B) 0.5 (C) 0.25 (D) 0.75
- (3) $P(\bar{X} < 5.5) =$
 (A) 0.4602 (B) 0.8413 (C) 0.1587 (D) 0.5398
- (4) Delete If $P(\bar{X} > a) = 0.1492$, then the numerical value of a is:
 (A) 0.8508 (B) 1.04 (C) 6.52 (D) 0.2

8.22 The heights of 1000 students are approximately normally distributed with a mean of 174.5 centimeters and a standard deviation of 6.9 centimeters. Suppose 200 random samples

of size 25 are drawn from this population and the means recorded to the nearest tenth of a centimeter. Determine

- (a) the mean and standard deviation of the sampling distribution of \bar{X} ;
- (b) the number of sample means that fall between 172.5 and 175.8 centimeters inclusive;
- (c) the number of sample means falling below 172.0 centimeters.

Solution:

$$a) E(\bar{X}) = \mu_{\bar{X}} = \mu = 174.5$$

$$b) P(172.5 < \bar{X} < 175.8) = P\left(\frac{172.5-174.5}{6.9/\sqrt{25}} < \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < \frac{175.8-174.5}{6.9/\sqrt{25}}\right) = P(-1.45 < Z < 0.94) = 0.8264 - 0.0735 = 0.7529$$

the number of sample means that fall between 172.5 and 175.8 centimeters inclusive = $200 * 0.7529 = 150.58 \approx 151$

$$c) P(\bar{X} < 172.0) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < \frac{172-174.5}{6.9/\sqrt{25}}\right) = P(Z < -1.81) = 0.0531$$

the number of sample means falling below 172.0 centimeters = $200 * 0.0531 = 10.62 \approx 11$

10.2. Two Means:

Q1. A random sample of size $n_1 = 36$ is taken from a normal population with a mean $\mu_1 = 70$ and a standard deviation $\sigma_1 = 4$. A second independent random sample of size $n_2 = 49$ is taken from a normal population with a mean $\mu_2 = 85$ and a standard deviation $\sigma_2 = 5$. Let \bar{X}_1 and \bar{X}_2 be the averages of the first and second samples, respectively.

- a) Find $E(\bar{X}_1)$ and $\text{Var}(\bar{X}_1)$.
- b) Find $E(\bar{X}_1 - \bar{X}_2)$ and $\text{Var}(\bar{X}_1 - \bar{X}_2)$.
- c) Find $P(70 < \bar{X}_1 < 71)$.
- d) Find $P(\bar{X}_1 - \bar{X}_2 > -16)$.

Solution:

$$a) E(\bar{X}_1) = \mu_1 = 70, V(\bar{X}_1) = \sigma_{\bar{X}_1}^2 = \frac{\sigma_1^2}{n_1} = \frac{16}{36} = 0.444$$

$$b) E(\bar{X}_1 - \bar{X}_2) = \mu_1 - \mu_2 = 70 - 85 = -15$$

$$\text{Var}(\bar{X}_1 - \bar{X}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} = \frac{16}{36} + \frac{25}{49} = 0.9546$$

$$c) P(70 < \bar{X}_1 < 71) = P\left(\frac{70-70}{4/\sqrt{36}} < Z < \frac{71-70}{4/\sqrt{36}}\right) = P(0 < Z < 1.5) = 0.4332$$

$$d) P(\bar{X}_1 - \bar{X}_2 > -16) = P\left(Z > \frac{-16+15}{\sqrt{0.9546}}\right) = 1 - P(Z < 1.02) = 0.1539$$

Q2.H.W A random sample of size 25 is taken from a normal population (first population) having a mean of 100 and a standard deviation of 6. A second random sample of size 36 is taken from a different normal population (second population) having a mean of 97 and a standard deviation of 5. Assume that these two samples are independent.

- (1) the probability that the sample mean of the first sample will exceed the sample mean of the second sample by at least 6 is
 (A) 0.0013 (B) 0.9147 (C) 0.0202 (D) 0.9832
- (2) the probability that the difference $\bar{X}_1 - \bar{X}_2$ between the two sample means will be less than 2 is
 (A) 0.099 (B) 0.2480 (C) 0.8499 (D)

8.28 A random sample of size 25 is taken from a normal population having a mean of 80 and a standard deviation of 5. A second random sample of size 36 is taken from a different normal population having a mean of 75 and a standard deviation of 3. Find the probability that the sample mean computed from the 25 measurements will exceed the sample mean computed from the 36 measurements by at least 3.4 but less than 5.9. Assume the difference of the means to be measured to the nearest tenth.

Solution:

$$n_1 = 25, \mu_1 = 80, \sigma_1 = 5$$

$$n_2 = 36, \mu_2 = 75, \sigma_2 = 3$$

$$\begin{aligned} P(3.4 < \bar{X}_1 - \bar{X}_2 < 5.9) &= P\left(\frac{3.4 - (80 - 75)}{\frac{5}{\sqrt{25}} + \frac{3}{\sqrt{36}}} < Z < \frac{5.9 - (80 - 75)}{\frac{5}{\sqrt{25}} + \frac{3}{\sqrt{36}}}\right) \\ &= P(-1.07 < Z < 0.6) = 0.7257 - 0.1432 = 0.5834 \approx 0.6 \end{aligned}$$

8.29 The distribution of heights of a certain breed of terrier has a mean of 72 centimeters and a standard deviation of 10 centimeters, whereas the distribution of heights of a certain breed of poodle has a mean of 28 centimeters with a standard deviation of 5 centimeters. Assuming that the sample means can be measured to any degree of accuracy, find the probability that the sample mean for a random sample of heights of 64 terriers exceeds the sample mean for a random sample of heights of 100 poodles by at most 44.2 centimeter

Solution:

$$n_1 = 64, \mu_1 = 72, \sigma_1 = 10$$

$$n_2 = 100, \mu_2 = 28, \sigma_2 = 5$$

$$P(\bar{X}_1 - \bar{X}_2 < 44.2)$$

Homework : 8.26 , 8.32

10.3. Single Proportion:

Q1. Suppose that 20% of the students in a certain university smoke cigarettes. A random sample of 5 students is taken from this university. Let \hat{p} be the proportion of smokers in the sample.

- (1) Find $E(\hat{p}) = \mu_{\hat{p}}$, the mean \hat{p} .
- (2) Find $Var(\hat{p}) = \sigma_{\hat{p}}^2$, the variance of \hat{p} .
- (3) Find an approximate distribution of \hat{p} .
- (4) Find $P(\hat{p} > 0.25)$.

Solution:

$$1) \hat{P} = 0.2$$

$$2) Var(\hat{P}) = \frac{pq}{n} = \frac{0.2 \cdot 0.8}{5} = 0.032$$

Q2: Suppose that you take a random sample of size $n=100$ from a binomial population with parameter $p=0.25$ (proportion of successes). Let $\hat{p} = X/n$ be the sample proportion of successes, where X is the number of successes in the sample.

- (a) What is the approximated sampling distribution of \hat{p} ?
- (b) What is the mean of \hat{p} ?
- (c) What is the standard error (standard deviation) of \hat{p} ?
- (d) Find the probability that the sample proportion \hat{p} is less than 0.2.

Solution:

a) $\hat{P} \sim N(0.25, 0.0433)$

b) $\hat{P} = 0.25$

c) $\sqrt{\text{Var}(\hat{P})} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.25 \cdot 0.75}{100}} = 0.0433$

d) $P(\hat{P} < 0.2) = P\left(Z < \frac{0.2 - 0.25}{\sqrt{0.25 \cdot 0.75 / 100}}\right) = P(Z < -1.15) = 0.1251$

10.5 t-distribution:

Q1. Using t-table with degrees of freedom $df=14$, find $t_{0.02}$, $t_{0.985}$.

Q2. From the table of t-distribution with degrees of freedom $\nu = 15$, the value of $t_{0.025}$ equals to

- (A) 2.131 (B) 1.753 (C) 3.268 (D) 0.0

$$t_{14,0.02} = 2.264$$

$$t_{14,0.985} = -t_{14,0.015} = -2.415$$

$$t_{15,0.025} = 2.131$$

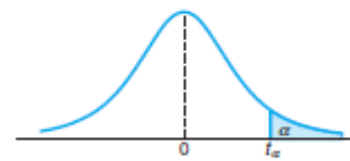


Table A.4 Critical Values of the *t*-Distribution

<i>v</i>	α						
	0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

Table A.4 (continued) Critical Values of the *t*-Distribution

<i>v</i>	α						
	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	15.894	21.205	31.821	42.433	63.656	127.321	636.578
2	4.849	5.643	6.965	8.073	9.925	14.089	31.600
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.689
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.660
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.290

ESTIMATION AND CONFIDENCE INTERVALS:

11.1. Single Mean:

Q1. An electrical firm manufacturing light bulbs that have a length of life that is normally distributed with a standard deviation of 30 hours. A sample of 50 bulbs were selected randomly and found to have an average of 750 hours. Let μ be the population mean of life lengths of all bulbs manufactured by this firm.

- (1) Find a point estimate for μ .
- (2) Construct a 94% confidence interval for μ .

Solution:

$$\hat{\mu} = 750$$

$$\bar{X} - Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

$$1 - \alpha = 0.94, \quad \alpha = 0.06, \quad \frac{\alpha}{2} = 0.03$$

$$Z_{1-\frac{\alpha}{2}} = Z_{0.97} = 1.88$$

$$750 - 1.88 \frac{30}{\sqrt{50}} < \mu < 750 + 1.88 \frac{30}{\sqrt{50}}$$

$$742 < \mu < 758$$

Q2. Suppose that we are interested in making some statistical inferences about the mean, μ , of a normal population with standard deviation $\sigma=2.0$. Suppose that a random sample of size $n=49$ from this population gave a sample mean $\bar{X}=4.5$.

- (1) The distribution of \bar{X} is
(A) $N(0,1)$ (B) $t(48)$ (C) $N(\mu, 0.2857)$ (D) $N(\mu, 2.0)$ (E) $N(\mu, 0.3333)$
- (2) A good point estimate of μ is
(A) 4.50 (B) 2.00 (C) 2.50 (D) 7.00 (E) 1.125
- (3) The standard error of \bar{X} is $\frac{2}{7} = 0.2857$
(A) 0.0816 (B) 2.0 (C) 0.0408 (D) 0.5714 (E) 0.2857
- (4) A 95% confidence interval for μ is
(A) (3.44,5.56) (B) (3.34,5.66) (C) (3.54,5.46) (D) (3.94,5.06) (E) (3.04,5.96)
- (5) If the upper confidence limit of a confidence interval is 5.2, then the lower confidence limit is
(A) 3.6 (B) 3.8 (C) 4.0 (D) 3.5 (E) 4.1
- (6) The confidence level of the confidence interval (3.88, 5.12) is
(A) 90.74% (B) 95.74% (C) 97.74% (D) 94.74% (E) 92.74%
- (7) If we use \bar{X} to estimate μ , then we are 95% confident that our estimation error will not exceed
(A) $e=0.50$ (B) $E=0.59$ (C) $e=0.58$ (D) $e=0.56$ (E) $e=0.51$
- (8) If we want to be 95% confident that the estimation error will not exceed $e=0.1$ when we use \bar{X} to estimate μ , then the sample size n must be equal to
(A) 1529 (B) 1531 (C) 1537 (D) 1534 (E) 1530

Solution:

$$4) \bar{X} - Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

$$1 - \alpha = 0.95, \quad \alpha = 0.05, \quad \frac{\alpha}{2} = 0.025$$

$$Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$$

$$4.5 - 1.96 \frac{2}{\sqrt{49}} < \mu < 4.5 + 1.96 \frac{2}{\sqrt{49}}$$

$$3.94 < \mu < 5.06$$

$$7) Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 1.96 \frac{2}{\sqrt{49}} = 0.56$$

$$8) n = \left(\frac{Z_{1-\frac{\alpha}{2}} \sigma}{e} \right)^2 = \left(\frac{1.96 * 2}{0.1} \right)^2 = 1536.64 = 1537$$

Q3. The following measurements were recorded for lifetime, in years, of certain type of machine: 3.4, 4.8, 3.6, 3.3, 5.6, 3.7, 4.4, 5.2, and 4.8. Assuming that the measurements represent a random sample from a normal population, then a 99% confidence interval for the mean life time of the machine is

- (A) $-5.37 \leq \mu \leq 3.25$ (B) $4.72 \leq \mu \leq 9.1$
 (C) $4.01 \leq \mu \leq 5.99$ (D) $3.37 \leq \mu \leq 5.25$

Solution:

$$\bar{X} = 4.31, S^2 = 0.7087$$

$$t_{n-1, \frac{\alpha}{2}} = t_{8, 0.005} = 3.355$$

$$\bar{X} - t_{n-1, \frac{\alpha}{2}} \frac{S}{\sqrt{n}} < \mu < \bar{X} + t_{n-1, \frac{\alpha}{2}} \frac{S}{\sqrt{n}}$$

$$4.31 - 3.355 \frac{\sqrt{0.7087}}{\sqrt{9}} < \mu < 4.5 + 1.96 \frac{\sqrt{0.7087}}{\sqrt{9}}$$

$$3.31 < \mu < 5.31$$

Q4.H.W A researcher wants to estimate the mean lifespan of a certain light bulbs. Suppose that the distribution is normal with standard deviation of 5 hours.

1. Determine the sample size needed on order that the researcher will be 90% confident that the error will not exceed 2 hours when he uses the sample mean as a point estimate for the true mean.
2. Suppose that the researcher selected a random sample of 49 bulbs and found that the sample mean is 390 hours.
 - (i) Find a good point estimate for the true mean μ .
 - (ii) Find a 95% confidence interval for the true mean μ .

Q5.H.W The amount of time that customers using ATM (Automatic Teller Machine) is a random variable with a standard deviation of 1.4 minutes. If we wish to estimate the population mean μ by the sample mean \bar{X} , and if we want to be 96% confident that the sample mean will be within 0.3 minutes of the population mean, then the sample size needed is

- (A) 98 (B) 100 (C) 92 (D) 85

Q6 H.W A random sample of size $n=36$ from a normal quantitative population produced a mean $\bar{X}=15.2$ and a variance $S^2=9$.

- (a) Give a point estimate for the population mean μ .
- (b) Find a 95% confidence interval for the population mean μ .

Q7.H.W A group of 10 college students were asked to report the number of hours that they spent doing their homework during the previous weekend and the following results were obtained:

7.25, 8.5, 5.0, 6.75, 8.0, 5.25, 10.5, 8.5, 6.75, 9.25

$$\sum X = 75.75, \sum X^2 = 600.563 \}$$

It is assumed that this sample is a random sample from a normal distribution with unknown variance σ^2 . Let μ be the mean of the number of hours that the college student spend doing his/her homework during the weekend.

- Find the sample mean and the sample variance.
- Find a point estimate for μ .
- Construct a 80% confidence interval for μ .

Q8.H.W An electronics company wanted to estimate its monthly operating expenses in thousands riyals (μ). It is assumed that the monthly operating expenses (in thousands riyals) are distributed according to a normal distribution with variance $\sigma^2=0.584$.

(I) Suppose that a random sample of 49 months produced a sample mean $\bar{X} = 5.47$.

- Find a point estimate for μ .
- Find the standard error of \bar{X} .
- Find a 90% confidence interval for μ .

(II) Suppose that they want to estimate μ by \bar{X} . Find the sample size (n) required if they want

their estimate to be within 0.15 of the actual mean with probability equals to 0.95.

Q9. The tensile strength of a certain type of thread is approximately normally distributed with standard deviation of 6.8 kilograms. A sample of 20 pieces of the thread has an average tensile strength of 72.8 kilograms. Then,

(a) A point estimate of the population mean of the tensile strength (μ) is:

- (A) 72.8 (B) 20 (C) 6.8 (D) 46.24 (E) None of these

(b) Suppose that we want to estimate the population mean (μ) by the sample mean (\bar{X}). To be 95% confident that the error of our estimate of the mean of tensile strength will be less than 3.4 kilograms, the minimum sample size should be at least:

- (A) 4 (B) 16 (C) 20 (D) 18 (E) None of these

(c) For a 98% confidence interval for the mean of tensile strength, we have the lower bound equal to:

- (A) 68.45 (B) 69.26 (C) 71.44 (D) 69.68 (E) None of these

(d) For a 98% confidence interval for the mean of tensile strength, we have the upper bound equal to:

- (A) 74.16 (B) 77.15 (C) 75.92 (D) 76.34 (E) None of these

Solution:

$$\bar{X} = 72.8, \quad \sigma = 6.8, \quad n = 20$$

a) $\hat{\mu} = \bar{X} = 72.8$

b) $Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$

$$n = \left(\frac{Z_{1-\frac{\alpha}{2}} \sigma}{e} \right)^2 = \left(\frac{1.96 * 6.8}{3.4} \right)^2 = 15.3664 \approx 16$$

c&d) $\bar{X} - Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$

$$1 - \alpha = 0.98, \quad \alpha = 0.02, \quad \frac{\alpha}{2} = 0.01$$

$$Z_{1-\frac{\alpha}{2}} = Z_{0.99} = 2.055$$

$$72.8 - 2.055 \frac{6.8}{\sqrt{20}} < \mu < 72.8 + 2.055 \frac{6.8}{\sqrt{20}}$$

$$69.675 < \mu < 75.924$$

9.2 An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed with a standard deviation of 40 hours. If a sample of 30 bulbs has an average life of 780 hours, find a 96% confidence interval for the population mean of all bulbs produced by this firm.

Solution:

$$\bar{X} = 780, \quad \sigma = 40, \quad n = 30$$

$$\bar{X} - Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

$$1 - \alpha = 0.96, \quad \alpha = 0.04, \quad \frac{\alpha}{2} = 0.02$$

$$Z_{1-\frac{\alpha}{2}} = Z_{0.98} = 1.755$$

$$780 - 1.755 \frac{40}{\sqrt{30}} < \mu < 780 + 1.755 \frac{40}{\sqrt{30}}$$

$$767.12 < \mu < 792.87$$

9.6 How large a sample is needed in Exercise 9.2 if we wish to be 96% confident that our sample mean will be within 10 hours of the true mean?

$$1 - \alpha = 0.96, \quad \alpha = 0.04, \quad \frac{\alpha}{2} = 0.02$$

$$Z_{1-\frac{\alpha}{2}} = Z_{0.98} = 1.755$$

$$n = \left(\frac{Z_{1-\frac{\alpha}{2}} \sigma}{e} \right)^2$$

9.4 The heights of a random sample of 50 college students showed a mean of 174.5 centimeters and a standard deviation of 6.9 centimeters.

(a) Construct a 95% confidence interval for the mean height of all college students.

(b) What can we assert with 95% confidence about the possible size of our error if we estimate the mean height of all college students to be 174.5 centimeters?

Solution:

$$\bar{X} = 174.5, \quad S = 6.9, \quad n = 50$$

$$\text{a) } \bar{X} - Z_{1-\frac{\alpha}{2}} \frac{S}{\sqrt{n}} < \mu < \bar{X} + Z_{1-\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$$

$$1 - \alpha = 0.95, \quad \alpha = 0.05, \quad \frac{\alpha}{2} = 0.025$$

$$Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$$

$$174.5 - 1.96 \frac{6.9}{\sqrt{50}} < \mu < 174.5 + 1.96 \frac{6.9}{\sqrt{50}}$$

$$172.54 < \mu < 176.46$$

b) $e = 1.96 \frac{6.9}{\sqrt{50}} = 1.91$

H.W: 9.5 , 9.8

11.2. Two Means:

Q1.(I) The tensile strength of type I thread is approximately normally distributed with standard deviation of 6.8 kilograms. A sample of 20 pieces of the thread has an average tensile strength of 72.8 kilograms. Then,

- 1) To be 95% confident that the error of estimating the mean of tensile strength by the sample mean will be less than 3.4 kilograms, the minimum sample size should be:
 (A) 4 (B) 16 (C) 20 (D) 18 (E) None of these
- 2) The lower limit of a 98% confidence interval for the mean of tensile strength is
 (A) 68.45 (B) 69.26 (C) 71.44 (D) 69.68 (E) None of these
- 3) The upper limit of a 98% confidence interval for the mean of tensile strength is
 (A) 74.16 (B) 77.15 (C) 75.92 (D) 76.34 (E) None of these

Q1.(II). The tensile strength of type II thread is approximately normally distributed with standard deviation of 6.8 kilograms. A sample of 25 pieces of the thread has an average tensile strength of 64.4 kilograms. Then for the 98% confidence interval of the difference in tensile strength means between type I and type II , we have:

- 1) the lower bound equals to:
 (A) 2.90 (B) 4.21 (C) 3.65 (D) 6.58 (E) None of these
- 2) the upper bound equals to:
 (A) 13.90 (B) 13.15 (C) 12.59 (D) 10.22 (E) None of these

Solution:

I) $\bar{X}_1 = 70, \sigma_1 = 6.8, n_1 = 20$

II) $\bar{X}_2 = 64.4, \sigma_2 = 6.8, n_2 = 25$

$$(\bar{X}_1 - \bar{X}_2) - Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$1 - \alpha = 0.98, \alpha = 0.02, \quad \frac{\alpha}{2} = 0.01$$

$$Z_{1-\frac{\alpha}{2}} = Z_{0.99} = 2.055$$

$$(70 - 64.4) - 2.055 \sqrt{\frac{6.8^2}{20} + \frac{6.8^2}{25}} < \mu_1 - \mu_2 < (70 - 64.4) + 2.055 \sqrt{\frac{6.8^2}{20} + \frac{6.8^2}{25}}$$

$$4.208 < \mu_1 - \mu_2 < 12.5922$$

Q2.H.W Two random samples were independently selected from two normal populations with equal variances. The results are summarized as follows.

	First Sample	Second Sample
sample size (n)	12	14
sample mean (\bar{X})	10.5	10.0

sample variance (S^2)	4	5
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Let μ_1 and μ_2 be the true means of the first and second populations, respectively.

- Find a point estimate for $\mu_1 - \mu_2$.
- Find 95% confidence interval for $\mu_1 - \mu_2$.

Q3.H.W A researcher was interested in comparing the mean score of female students, μ_f , with the mean score of male students, μ_m , in a certain test. Two independent samples gave the following results:

Sample	Observations							mean	Variance
Scores of Females	89.2	81.6	79.6	80.0	82.8			82.63	15.05
Scores of Males	83.2	83.2	84.8	81.4	78.6	71.5	77.6	80.04	20.79

Assume the populations are normal with equal variances.

- The pooled estimate of the variance s_p^2 is
 (A) 17.994 (B) 17.794 (C) 18.094 (D) 18.294 (E) 18.494
- A point estimate of $\mu_f - \mu_m$ is
 (A) 2.63 (B) -2.59 (C) 2.59 (D) 0.00 (E) 0.59
- The lower limit of a 90% confidence interval for $\mu_f - \mu_m$ is
 (A) -1.97 (B) -1.67 (C) 1.97 (D) 1.67 (E) -1.57
- The upper limit of a 90% confidence interval for $\mu_f - \mu_m$ is
 (A) 6.95 (B) 7.45 (C) -7.55 (D) 7.15 (E) 7.55

Q4.H.W A study was conducted to compare to brands of tires A and B. 10 tires of brand A and 12 tires of brand B were selected randomly. The tires were run until they wear out. The results are:

$$\begin{aligned} \text{Brand A: } \quad \bar{X}_A &= 37000 \text{ kilometers} & S_A &= 5100 \\ \text{Brand B: } \quad \bar{X}_B &= 38000 \text{ kilometers} & S_B &= 6000 \end{aligned}$$

Assuming the populations are normally distributed with equal variances,

- Find a point estimate for $\mu_A - \mu_B$.
- Construct a 90% confidence interval for $\mu_A - \mu_B$.

Q5.H.W The following data show the number of defects of code of particular type of software program made in two different countries (assuming normal populations with unknown equal variances)

Country	observations							mean	standard dev.
A	48	39	42	52	40	48	54	46.143	5.900
B	50	40	43	45	50	38	36	43.143	5.551

- A point estimate of $\mu_A - \mu_B$ is
 (A) 3.0 (B) -3.0 (C) 2.0 (D) -2.0 (E) None of these
- A 90% confidence interval for the difference between the two population means $\mu_A - \mu_B$ is
 (A) $-2.46 \leq \mu_A - \mu_B \leq 8.46$ (B) $1.42 \leq \mu_A - \mu_B \leq 6.42$
 (C) $-1.42 \leq \mu_A - \mu_B \leq -0.42$ (D) $2.42 \leq \mu_A - \mu_B \leq 10.42$

Q6.DELET A study was made by a taxi company to decide whether the use of new tires (A) instead of the present tires (B) improves fuel economy. Six cars were equipped with tires (A) and driven over a prescribed test course. Without changing drivers and cares, test course was

made with tires (B). The gasoline consumption, in kilometers per liter (km/L), was recorded as follows: (assume the populations are normal with equal unknown variances)

Car	1	2	3	4	5	6
Type (A)	4.5	4.8	6.6	7.0	6.7	4.6
Type (B)	3.9	4.9	6.2	6.5	6.8	4.1

A 95% confidence interval for the true mean gasoline consumption for brand A is:

- (a)
- (A) $4.462 \leq \mu_A \leq 6.938$ (B) $2.642 \leq \mu_A \leq 4.930$
 (C) $5.2 \leq \mu_A \leq 9.7$ (D) $6.154 \leq \mu_A \leq 6.938$

A 99% confidence interval for the difference between the true means consumption of type (A) and type (B) ($\mu_A - \mu_B$) is:

- (b)
- (A) $-1.939 \leq \mu_A - \mu_B \leq 2.539$ (B) $-2.939 \leq \mu_A - \mu_B \leq 1.539$
 (C) $0.939 \leq \mu_A - \mu_B \leq 1.539$ (D) $-1.939 \leq \mu_A - \mu_B \leq 0.539$

Q7.H.W A geologist collected 20 different ore samples, all of the same weight, and randomly divided them into two groups. The titanium contents of the samples, found using two different methods, are listed in the table:

Method (A)					Method (B)				
1.1	1.3	1.3	1.5	1.4	1.1	1.6	1.3	1.2	1.5
1.3	1.0	1.3	1.1	1.2	1.2	1.7	1.3	1.4	1.5
$\bar{X}_1 = 1.25, S_1 = 0.1509$					$\bar{X}_2 = 1.38, S_2 = 0.1932$				

- (a) Find a point estimate of $\mu_A - \mu_B$ is
 (b) Find a 90% confidence interval for the difference between the two population means $\mu_A - \mu_B$. (Assume two normal populations with equal variances).

From book:

9.35 A random sample of size $n_1 = 25$, taken from a normal population with a standard deviation $\sigma_1 = 5$, has a mean $\bar{X}_1 = 80$. A second random sample of size $n_2 = 36$, taken from a different normal population with a standard deviation $\sigma_2 = 3$, has a mean $\bar{X}_2 = 75$. Find a 95% confidence interval for $\mu_1 - \mu_2$.

Solution:

I) $\bar{X}_1 = 80, \sigma_1 = 5, n_1 = 25$

II) $\bar{X}_2 = 75, \sigma_2 = 3, n_2 = 36$

$$(\bar{X}_1 - \bar{X}_2) - Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$1 - \alpha = 0.95, \alpha = 0.05, \frac{\alpha}{2} = 0.025$$

$$Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$$

$$(80 - 75) - 1.96 \sqrt{\frac{5^2}{25} + \frac{3^2}{36}} < \mu_1 - \mu_2 < (80 - 75) + 1.96 \sqrt{\frac{5^2}{25} + \frac{3^2}{36}}$$

$$2.808 < \mu_1 - \mu_2 < 7.192$$

9.41 The following data represent the length of time, in days, to recovery for patients randomly treated with one of two medications to clear up severe bladder infections:

Medication1	Medication2
.n₁= 14	.n₂=16

Find a 99% confidence interval for the difference $\mu_2 - \mu_1$ in the mean recovery times for the two medications, assuming normal populations **with equal variances**.

Solution:

I) $\bar{X}_1 = 80, \sigma_1 = 5, n_1 = 25$

II) $\bar{X}_2 = 75, \sigma_2 = 3, n_2 = 36$

$$(\bar{X}_1 - \bar{X}_2) - t_{n_1+n_2-2, \frac{\alpha}{2}} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + t_{n_1+n_2-2, \frac{\alpha}{2}} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$1 - \alpha = 0.99, \alpha = 0.01, \frac{\alpha}{2} = 0.005$$

$$t_{n_1+n_2-2, \frac{\alpha}{2}} = t_{28, 0.005} = 2.763$$

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} = \frac{(13)1.5 + (15)1.8}{28} = 1.6607$$

$$S_p = 1.289$$

$$(17 - 19) - 2.763 * 1.289 \sqrt{\frac{1}{14} + \frac{1}{16}} < \mu_1 - \mu_2 < (17 - 19) + 2.763 * 1.289 \sqrt{\frac{1}{14} + \frac{1}{16}}$$

$$-3.303 < \mu_1 - \mu_2 < -0.303$$

H.W: 9.36 , 9.43