# Photogrammetry SE 321 <br> Parallax-Height Relation <br> Solved Problem 

A pair of overlapping aerial vertical photographs were taken from a flying height of $\mathbf{1 8 0 0} \mathbf{m}$ above mean sea level (MSL) with a $\mathbf{1 5 0 . 0 0} \mathbf{~ m m}$ focal length camera. The air base was $\mathbf{9 0 0 . 0 0 m}$ and x-photo coordinates of a point $\mathbf{q}$ on the left and right photos were recorded as $\mathbf{8 5 . 0 0} \mathbf{m m}$ and $\mathbf{- 0 5 . 0 0 \mathrm { mm }}$ respectively. Parallax bar readings of $\mathbf{1 0 . 5 0}, \mathbf{1 5 . 2 0}$ and $\mathbf{1 0 . 0 0 \mathrm { mm }}$ were taken on image point $\mathbf{q}, \mathbf{s}$ and $\mathbf{k}$, respectively, compute the levels of ground points $\mathbf{Q}, \mathbf{S}$ and $\mathbf{K}$.
Also the $\mathbf{x}$ and $\mathbf{y}$ photo coordinates of points $q$ and s on the left photograph were $x_{q}=85.00 \mathrm{~m}, y_{q}=50.80 \mathrm{~mm}, x_{s}=90.00 \mathrm{~mm}$ and $y_{s}=-60.00 \mathrm{~mm}$, Calculate the horizontal ground distance between $Q$ and $S$ to the nearest $\mathbf{c m}$.

## Solution

x-parallax of reference point $q=x^{\prime}-x^{\prime \prime}=85.00-(-05.00)=\mathbf{9 0 . 0 0 m m}$
Parallax Bar Constant, $\mathrm{C}=\mathrm{x}$-parallax of $\mathrm{q}-$ parallax bar reading at $\mathrm{q}=$

$$
\mathrm{C}=90.00-10.50=\mathbf{7 9 . 5 0} \mathrm{mm}
$$

$x$-parallax of image $\mathrm{s}=\mathrm{C}+$ parallax bar reading at $\mathrm{s}=79.50+15.20=\mathbf{9 4 . 7 0} \mathbf{m m}$ x-parallax of image $\mathrm{k}=\mathrm{C}+$ parallax bar reading at $\mathrm{k}=79.50+10.00=\mathbf{8 9 . 5 0} \mathbf{m m}$

Use parallax-height formula to calculate height of point Q :
$\mathbf{h}_{\mathbf{Q}}=\mathbf{H}-\mathbf{B} \mathbf{f} / \mathbf{p}_{\mathbf{q}}$
$\mathrm{h}_{\mathrm{Q}}=1800.00-[900.00 \times 150.00 / 90.00]=300.00 \mathrm{~m}$ above MSL
$\Delta \mathrm{h}=\mathrm{h}_{\mathrm{I}}-\mathrm{h}_{\mathrm{Q}}=\left(\mathrm{H}-\mathrm{Bf} / \mathrm{p}_{\mathrm{i}}\right)-\left(\mathrm{H}-\mathrm{Bf} / \mathrm{p}_{\mathrm{q}}\right)=\mathrm{H}-\mathrm{Bf} / \mathrm{p}_{\mathrm{i}}-\mathrm{H}+\mathrm{Bf} / \mathrm{p}_{\mathrm{q}}$
$\mathrm{H}_{\mathrm{I}}-\mathrm{h}_{\mathrm{Q}}=\mathrm{Bf} / \mathrm{p}_{\mathrm{q}}-\mathrm{Bf} / \mathrm{p}_{\mathrm{i}}=\operatorname{Bf}\left(\mathrm{p}_{\mathrm{i}}-\mathrm{p}_{\mathrm{q}}\right) /\left(\mathrm{p}_{\mathrm{i}} \mathrm{p}_{\mathrm{q}}\right)=\mathrm{Bf} \Delta \mathrm{p} /\left[\mathrm{p}_{\mathrm{q}}\left(\mathrm{p}_{\mathrm{q}}+\Delta \mathrm{p}\right)\right]$
$\mathbf{h}_{\mathrm{I}}=\mathbf{h}_{\mathrm{Q}}+\mathbf{B} \mathbf{f} \Delta \mathbf{P} /\left[\mathbf{p}_{\mathrm{q}}\left(\mathbf{P}_{\mathbf{q}}+\Delta \mathbf{P}\right)\right] ;$ where $\Delta \mathbf{p}=\mathbf{p}_{\mathbf{i}}-\mathbf{p}_{\mathrm{q}}$
$\mathrm{h}_{\mathrm{S}}=300.00+[900 \times 150 \times(94.70-90.00) /(90.00 \times 94.70)=300+74.446$
$=\underline{\mathbf{3 7 4 . 4 4 6 m}}$
$\mathrm{h}_{\mathrm{K}}=300.00+[900 \times 150 \times(89.50-90.00) /(90.00 \times 89.50)=300.00-8.380$
$=291.620 \mathrm{~m}$

To calculate horizontal distance QS, determine ground coordinates of Q and S in a 2D coordinates system parallel to image coordinate system:

Ground coordinates of Q :

$$
\begin{aligned}
& \mathrm{X}_{\mathrm{Q}}=\mathrm{x}_{\mathrm{q}}\left(\mathrm{H}-\mathrm{h}_{\mathrm{Q}}\right) / \mathrm{f}=85.00(1800.00-300.00) / 150.00=850.00 \mathrm{~m} \\
& \mathrm{Y}_{\mathrm{Q}}=\mathrm{y}_{\mathrm{q}}\left(\mathrm{H}-\mathrm{h}_{\mathrm{Q}}\right) / \mathrm{f}=50.80(1800.00-300.00) / 150.00=508.00 \mathrm{~m} \\
& \mathrm{X}_{\mathrm{S}}=\mathrm{x}_{\mathrm{S}}\left(\mathrm{H}-\mathrm{h}_{\mathrm{S}}\right) / \mathrm{f}=90.00(1800.00-374.446) / 150.00=855.33 \mathrm{~m} \\
& \mathrm{Y}_{\mathrm{S}}=\mathrm{y}_{\mathrm{S}}\left(\mathrm{H}-\mathrm{h}_{\mathrm{S}}\right) / \mathrm{f}=60.00(1800.00-374.446) / 150.00=570.22 \mathrm{~m}
\end{aligned}
$$

Ground distance $\mathbf{Q S}=\left[\left(\mathbf{X}_{\mathbf{S}}-\mathbf{X}_{\mathbf{Q}}\right)^{2}+\left(\mathbf{Y}_{\mathbf{S}}-\mathbf{Y}_{\mathbf{Q}}\right)^{2}\right]^{1 / 2}$

$$
\begin{aligned}
& =\left[(855.33-850.00)^{2}+(570.22-508.00)^{2}\right]^{1 / 2} \\
& =\mathbf{6 2 . 4 5 m}
\end{aligned}
$$

## Exercise Problem

A pair of overlapping aerial vertical photographs were taken from a flying height of $\mathbf{1 8 0 0 m}$ above mean sea level (MSL) with a $\mathbf{1 5 0 . 0 0} \mathbf{~ m m}$ focal length camera. The air base was $\mathbf{7 2 0 . 0 0 m}$ and elevation of ground point $\mathbf{Q}$ is $\mathbf{6 0 0 . 0 0 m}$ above MSL. Parallax bar readings of $\mathbf{1 0 . 5 0}$ and $\mathbf{1 5 . 2 0} \mathbf{m m}$ were taken on image point $\mathbf{q}$ and $s$ respectively, compute the $\mathbf{x}$-parallax for image points $\mathbf{q}$ and $s$ and reduced level of ground point $\mathbf{S}$.

