NAME:

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244
Second Midterm, December 2013

I) Let A be the following matrix:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & t \\ 1 & 4 & t^2 \end{bmatrix}$$

a) Compute the determinant of A (your answer will be in terms of t).

b) For what values of t is A invertible?

II) Solve the linear system of equations

$$\begin{cases} 3x + 2y - z &= -3 \\ x - y + z &= 1 \\ -2x + 3y - 2z &= 0 \end{cases}$$

using **Cramer's rule**.

III) Find all scalars c_1, c_2 and c_3, such that

$$c_1(1,-1,0) + c_2(3,2,1) + c_3(0,1,4) = (-1,1,19).$$

IV) Let $u = (-3, 2, 1, 0)$, $v = (4, 7, -3, 2)$, $w = (5, -2, 8, 1)$.

a) Find the vector x that satisfies $5x - 2v = 2(w - 5x)$.

b) Find $\|u\| - 2\|v\| - 3\|w\|$ and $\|u\| + \| - 2v\| + \| - 3w\|$.

c) If $y = (1, -1, -3, k)$, find k such that vectors v and y are orthogonal.

V) Let $V = \{(x, 0) | x \in \mathbb{R}\}$. Prove that V is not a vector space, when endowed with the operations

$$(x, 0) + (y, 0) = (x + y, 0) \quad \text{and} \quad k(x, 0) = (k^2 x, 0), \quad \forall\ (x, 0), (y, 0) \in V, \quad \forall\ k \in \mathbb{R}.$$

VI) Prove that the vectors

$$v_1 = (2, -1, 3), \quad v_2 = (4, 1, 2), \quad v_3 = (8, -1, 8)$$

do not span $\mathbb{R}^3$.