J. King Saud Univ., Vol. 14, *Eng. Sci.* (1), pp. 137-145, Riyadh (1422/2002)

Simple Correlation to Evaluate Mohr-Coulomb Failure Criterion Using Uniaxial Compressive Strength

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(Received 01 February, 2000; accepted for publication 26 November, 2000)

Abstract. The evaluation of Mohr-Coulomb failure criterion as well as other mechanical properties for reservoir rocks is essential for well planning, development and characterization of oil and gas reservoirs. This is because the understanding of the rock-stress relationship can solve many reservoir problems and avoid cost of remedial work. For example, a Mohr-Coulomb failure criterion may be used for borehole instability analysis, water injection design, production optimization techniques, compaction and sand production prediction, etc.

A Mohr-Coulomb failure criterion is a function of the apparent cohesion (τ_0) and the angle of internal friction (ϕ). The evaluation of these two parameters requires testing of many rock samples using an expensive and time-consuming triaxial testing set-up. In this study, a correlation between the apparent cohesion and the uniaxial compressive strength was developed. It is based on laboratory data of more than 200 rock samples of different types obtained from the literature. The correlation coefficient of the developed correlation equals to 0.82. Verification of the developed correlation using data from other references has shown an average error of estimation equal to 10%. Therefore, the Mohr-Coulomb failure criterion's parameters as well as Poisson's ratio can be estimated using the developed correlation based on fast and cheap measurements of the uniaxial compressive strength.

Introduction

Several problems occur repeatedly during oil and gas well drilling such as sloughing shale, stuck drillstring or casing, etc. Normally, the first attempt to solve such problems is based on experience. If all methods fail to solve such problems, a rock mechanics study is considered to be the last hope. At this stage, rock mechanics analysis will be difficult due to the lack of data and rock samples, therefore, back analysis may be the only solution if new offset wells are not drilled. Thus, rock mechanics must be considered from the beginning and a rock mechanical dartabase must be establish to in solving new problem [1]. For example, borehole instability during drilling costs the industry around $400–500 millions per year [2]. When producing reservoir fluids over a long period, several problems related to rock mechanics may be encountered such as sand production, perforation instability, subsidence, permeability mechanical damage, etc. In order to solve any of the previously mentioned reservoir problems that may be encountered, a rock failure criterion and rock mechanical properties must be implemented as shown in Table 1. In order to evaluate the required mechanical properties, a huge number of rock specimens are needed in addition to an expensive triaxial testing set-up. Therefore, simple correlations are needed to estimate rock mechanical properties using a limited number of rock samples and inexpensive testing equipment. This paper introduces a correlation that can be used to evaluate the failure criterion and Poisson's ratio of reservoir rock using only the uniaxial compressive strength values.

Table 1. Wellbore and reservoir problems related to rock mechanics

Stage	Problems	Data possibly required to solve these problems
Exploration	Petroleum migration, Traps, Reserve estimation, etc.	- Failure criteria. - Rock-fluid compatibility.
Drilling	Borehole instability, Loss of circulation, Casing collapse, Bit balling, etc.	- Rock elastic properties. - In-situ stress state.
Production	Sand production, Perforation stability, Hydraulic fracturing, Propant crushing, etc.	- Well orientation and inclination. - Drilling fluid properties. - Swelling tendency. - Mud cake efficiency.
Reservoir	Permeability-stress sensitivity, Rock-fluid interaction, Subsidence, Fracture identification, etc.	- P & S velocities. - etc.

Theoretical Background

Mohr-Coulomb failure criterion was introduced to rock mechanics by Jaeger [3] in the year 1959 by combining the work of Mohr and Coulomb. This criterion stated that shear failure across a plane is restricted by the cohesion of the material. This criterion can be expressed mathematically as follows:

$$|\tau| = \tau_o + \sigma \tan\phi \quad (1)$$

where, τ and σ are the shear and normal stresses respectively, τ_o is the apparent or inherent cohesion and ϕ is the angle of internal friction. The evaluation of the Mohr-Coulomb failure criteria needs to carry out many triaxial tests on rock samples at various confining pressures. From these data a series of Mohr's circles can be plotted as shown in Fig. 1. Then the locus of the tangent points of circles is drawn, developing the failure envelope for the tested rock which defines the boundary between stable and unstable stress states. Once the failure criteria is established, the failure state (instability) at any other conditions can be predicted.

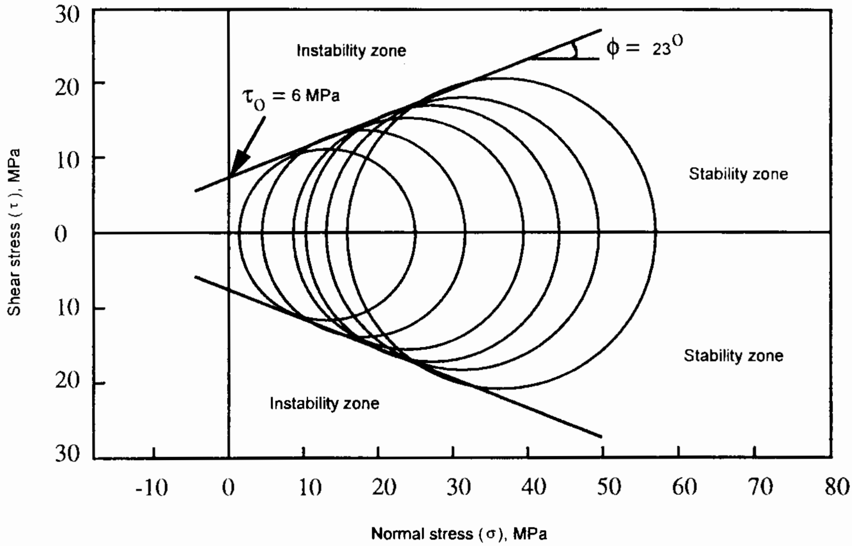

Fig. 1. Mohr-Coulomb failure criteria for natural intact shale.

Poisson's ratio is another important rock property which is defined as the ratio of the lateral strain to the axial strain in an axially stressed sample. If the axial and lateral strains are measured during triaxial testing to determine the Mohr-Coulomb failure criterion, Poisson's ratio (v) can be calculated as well. Alternatively, Poisson's ratio can also be estimated using the following relationship [4]:

$$\phi = \mathrm{Sin}^{-1}\left[\frac{1-v}{1+v}\right] \tag{2}$$

Equation 2 provides only estimated values of Poisson's ratio. However, for more accurate values of Poisson's ratio, data obtained using the triaxial tests must be used. The uniaxial (unconfined) compressive strength (σ_c), apparent cohesion and angle of internal friction are combined together in the following equation [2, 4]:

$$\frac{\sigma_c}{\tau_o} = \frac{2\,\mathrm{Cos}\phi}{1-\mathrm{Sin}\phi} \tag{3}$$

The objective of this study is to find a simple correlation between the uniaxial compressive strength and the apparent cohesion of rocks. Using this correlation, several important mechanical parameters can be estimated including, angle of internal friction, Poisson's ratio and Mohr-Coulomb failure criterion.

Results and Discussion

The failure criterion and mechanical properties of more than 200 rock samples of different types were obtained from the published literature [1-24]. The obtained rock properties are: uniaxial compressive strength, apparent cohesion, angle of internal friction and Poisson's ratio. Rock apparent cohesion is a measure of the degree of grain-to-grain bonding. Therefore, it might correlate with the uniaxial (unconfined) compressive strength which is also a measure of grain-to-grain bonding magnitude. For this reason, the uniaxial compressive strength was plotted versus the apparent cohesion of the literature-cited data. A well-defined trend was obtained from this plot as shown in Fig. 2. Thus, a relationship between the apparent cohesion and the uniaxial compressive strength for various types of rocks ranging from soft to very strong is represented by the following equation with a coefficient of correlation (r^2) equal to 0.82:

$$\tau_o = -0.41713 + 0.28907\left(\sigma_c\right) - 0.00051878\left(\sigma_c\right)^2 \quad (4)$$

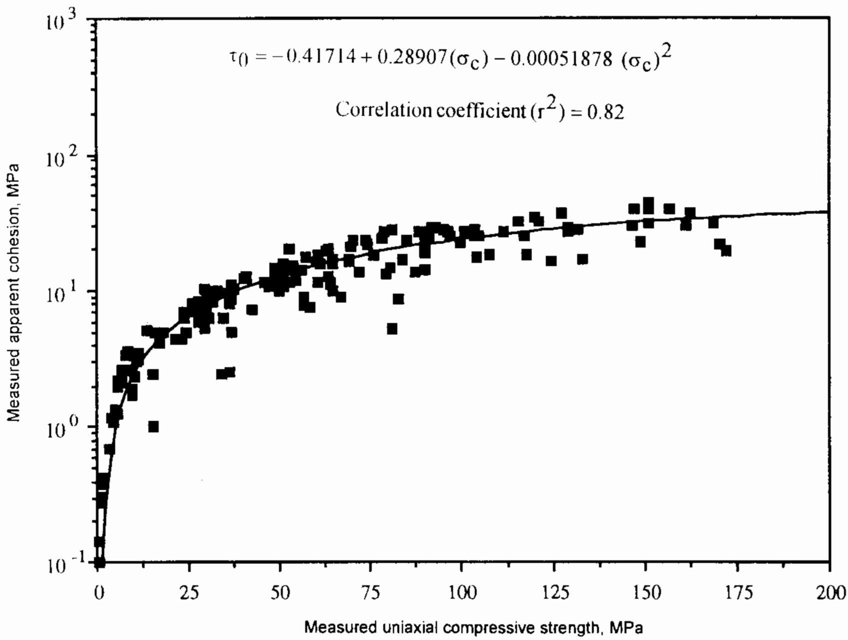

Fig. 2. Relationship between measured uniaxial compresive strength and measured apparent cohesion of various types of rocks obtained from literature.

Figure 3 represents the relationship between the literature obtained and correlation predicted apparent cohesion values. It can be seen that most of the data points are located on the 45-degree line. Similar result was obtained for the uniaxial compressive strength as shown in Fig. 4 indicating the existence of a relationship between rock uniaxial compressive strength and apparent cohesion. To verify the developed correlation, data from references [25-29] other than those used in the development of the correlation were used. The results showed an average error of estimation equal to 10% which is considered as a good confirmation of the validity of the developed correlation (see Table. 2). When more accurate rock mechanical properties are required, triaxial tests using sufficient number of rock samples should be performed. Therefore, this work provides a cheap and rapid estimation of rock failure criterion and some other mechanical properties using inexpensive laboratory measurements.

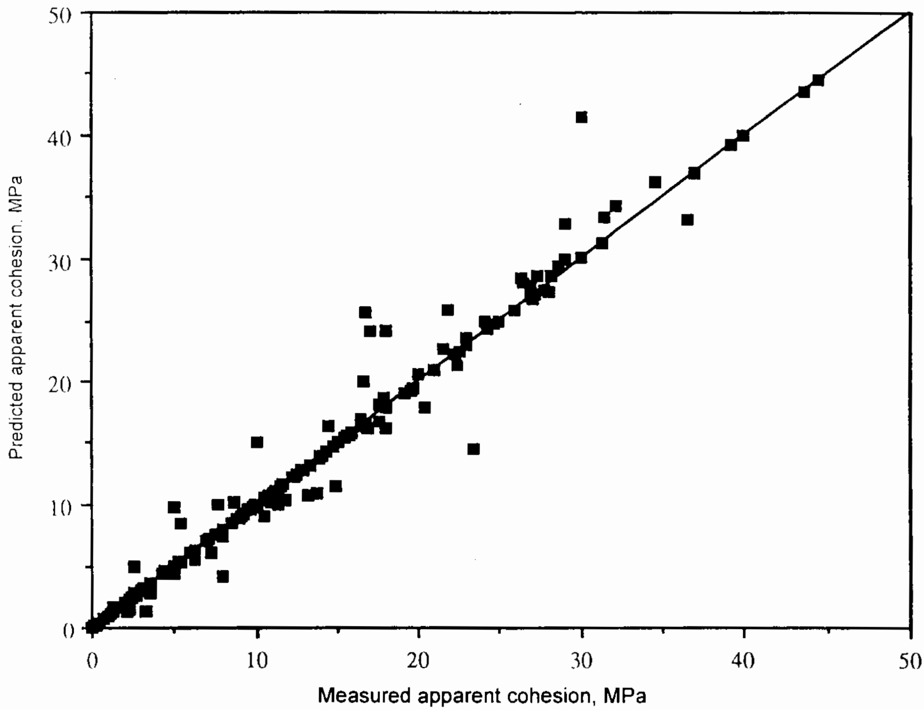

Fig. 3. Comparison between laboratory measured and predicted apparent cohesion of various types of rocks.

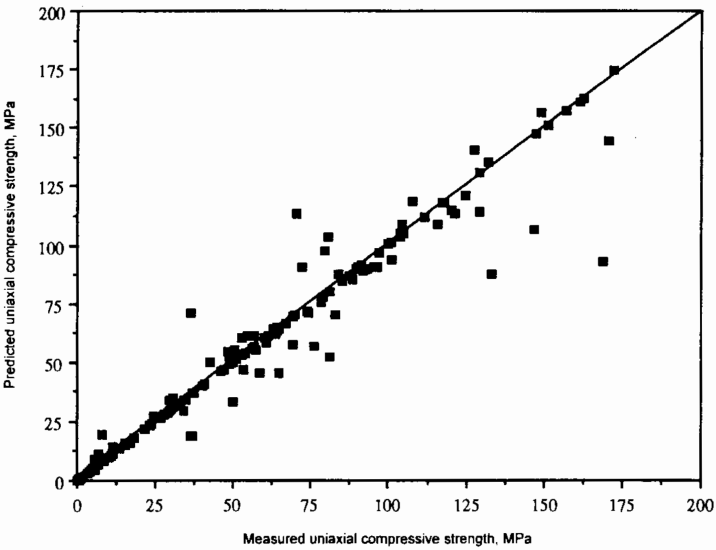

Fig. 4. Comparison between laboratory measured and predicted uniaxial compressive strength of various types of rocks.

Table 2. Verification of the developed correlation

Rock mechanical properties	Reference no. [25]		Reference no. [25]	
	Literature data	Predicted data	Literature data	Predicted data
Uniaxial compressive strength, MPa	105.8	111***	36.9	41***
Apparent cohesion, MPa	28*	24.4***	11.5*	9.5***
Apparent cohesion, MPa	28*	24.4***	11.5*	9.5***
Angle of internal friction, degree	34	37.8** 34*	26	39.7** 26*
Poisson's ratio	0.24	0.28**	0.22	0.35**
Rock mechanical properties	**Reference no. [26]**		**Reference no. [27]**	
	Literature data	**Predicted data**	**Literature data**	**Predicted data**
Uniaxial compressive strength, Mpa	60	67***	24.4	15***
Apparent cohesion, MPa	18*	15.1***	4.0	6.3***
Angle of internal friction, degree	26	26*	51.2	52.5** 51*
Poisson's ratio	----	0.36**	0.18	0.13**
Rock mechanical properties	**Reference no. [28]**		**Reference no. [29]**	
	Literature data	**Predicted data**	**Literature data**	**Predicted data**
Uniaxial compressive strength, MPa	50	44***	41.3	46***
Apparent cohesion, MPa	12.5	12.7***	12.9	10.6***
Angle of internal friction, degree	37	37*	25.6	25.6***
Poisson's ratio	----	0.245**	----	0.4**

* Calculated using Eq. 3 ** Calculated using Eq. 2 *** Calculated using Eq. 4

Conclusions

- A simple correlation between rock apparent cohesion and uniaxial compressive strength was developed.

- The developed correlation can be used to evaluate Mohr-Coulomb failure criterion's parameters as well as rock mechanical properties using simple and cheap deformation tests.

- The developed correlation provides only estimated values of rock mechanical properties (within an average error of estimation equal to 10%).

- For accurate mechanical properties, triaxial tests using satisfactory rock samples must be performed.

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علاقة مبسطة لتقويم متغيّرات خاصية موهر- كولومب لانهيار الصخر باستخدام اختبار مقاومة الانضغاط غير المحصور

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الرياض ١١٤٢١ المملكة العربية السعودية

(قُدّم للنشر في ٢٠٠٠/٢/١م، وقبل للنشر في ٢٠٠٠/٩/٢٦م)

ملخص البحث. من الضروري إيجاد علاقة موهر- كولومب لانهيار الصخر وكذلك قياس الخواص الميكانيكية لصخور المكمن البترولي وذلك لعمل خطط متقنة لحفر الآبار وتقويم مكامن النفط والغاز وتطويرها. هذا لأن فهم العلاقة بين صخور المكمن والإجهادات الموضعية ستمكننا من حل العديد من المشاكل وبالتالي تجنب المصاريف اللازمة لإصلاح الأعطال المترتبة عليها. وعلى سبيل المثال، فمعرفة علاقة موهر- كولومب لانهيار الصخر تمكننا من التخطيط لجوانب مهمة في تطوير المكمن البترولي مثل عدم ثباتية قاع الحفرة، تصميم عملية حقن المكمن بالماء، تصميم معدلات الإنتاج والتوقع بهبوط الطبقات وكميات الرمل المنتجة مع النفط.

إن علاقة موهر- كولومب لانهيار الصخر هي دالة في درجة التماسك الظاهري بين الحبيبات المكونة للصخر (τ_0) وزاوية الاحتكاك الداخلي بين تلك الحبيبات (ϕ) ويتطلب تقويم هاتين الخاصيتين اختبار العديد من عيّنات الصخر باستخدام أجهزة ذوات تكلفة عالية وزمن اختبار طويل.

تم في هذه الدراسة تطوير علاقة رياضية بين درجة التماسك الظاهري بين الحبيبات المكونة للصخر (τ_0) و مقاومة الانضغاط غير المحصور للصخر (σ_c) . واعتمد استنباط تلك العلاقة على نتائج اختبارات معملية لأكثر من ٢٠٠ نوع من الصخور أجريت بواسطة العديد من الباحثين والمنشورة في المقتنيات العلمية المختلفة. ووجد أن قيمة معامل العلاقة يساوي ٨٢ و٠ وتم التأكد من دقة تلك العلاقة باستخدام قياسات معملية لصخور مختلفة منشورة في المقتنيات العلمية و غير تلك القياسات التي تم استخدامها في تطوير العلاقة ووجد أن قيمة متوسط الانحراف المعياري تساوي ١٠٪. وعلى ذلك، أصبح بالإمكان حساب متغيرات خاصية موهر- كولومب لانهيار الصخر و كذلك نسبة بويزون باستخدام العلاقة المطوّرة في هذه الدراسة بعد معرفة قيمة مقاومة الانضغاط غير المحصور للصخر والتي يمكن قياسها معمليا بسهولة تامة وبتكلفة قليلة.