Case 2: Simple Hypothesis against One-Sided Hypothesis

1. Consider a normal distribution of the form $N(\theta, 4)$. Test the simple hypothesis $H_0: \theta = 0$ against the alternative composite hypothesis $H_1: \theta > 0$ and find the best rejection region.
2. Let $X_1, X_2, \dots, X_n$ be a random sample from a distribution with pdf $f(x;\theta) = \theta x^{\theta-1}, 0 < x < 1$, zero elsewhere, where $\theta > 0$. Calculate the best critical region to reject $H_0: \theta = 6$ versus $H_1: \theta < 6$.
3. Let X have the pdf $f(x;\theta) = \theta^x(1-\theta)^{1-x}, x = 0, 1$, zero elsewhere. We test $H_0: \theta = \frac{1}{2}$ against $H_1: \theta < \frac{1}{2}$ by taking a random sample $X_1, X_2, \dots, X_5$ of size $n = 5$. Find the best critical region of size α.
4. Suppose that $X_1, X_2, \dots, X_n$ is a random sample of size *n* from exponential population with parameter $\frac{1}{\theta}$. Determine the best rejection region of size α for $H_0: \theta = \theta_0$ against $H_1: \theta > \theta_0$.
5. If $X \sim Gamma\left(2, \frac{1}{\theta}\right)$. Find the best critical region for $H_0: \theta = 1$ against $H_1: \theta < 1$.
6. Let X be a random sample whose probability mass function is binomial distribution with parameters $n = 10$ and *p*. Test the hypotheses $H_0: p = 0.25$ against $H_1: p > 0.25$ and find the best critical region of size α of this test.