**Simplex Algorithm for solving linear programming problems.**

**Example 1:**

**Consider the following LP problem:**

**Maximise Z = 10x + 12y subject to the constraints:**

**x + y ≤ 40 (i)**

**x + 2y ≤ 75 (ii)**

**x ≥ 0, y ≥ 0.**

**Step 1: Introduce *slack variables* to convert the non-trivial inequalities into equalities:**

**Equation (i): x + y + s = 40 s ≥ 0**

**Equation (ii): x + 2y + t = 75 t ≥ 0**

**s, t are slack variables.**

**Step 2: Rewrite the objective function so that the RHS is a number:**

**Z = 10x + 12y → Z – 10x – 12y = 0.**

**Step 3: Write the objective function and the slack variables as initial basic variables and construct the *Initial* *tableau* in the following format:**



**The aim is to solve the equations by combining rows together. The solution is reached when all entries in the first row (except possibly the value in the last column) are non-negative.**

**We begin by identifying the most negative entry in the objective function row, here -12 in the y column.**

**We highlight the pivotal column. We then divide every entry in the *ɭ* column by the corresponding value in the highlighted column. Pick the least positive of these. This is the pivotal row.**



**Divide the pivotal row by the pivot value (to get Eqn.6).**

**The aim is to now get 0 entries elsewhere in the pivotal column.**

**We now repeat the process, first selecting the new pivotal column, i.e. the one with the most negative value in the objective function row.**



**Having identified the pivotal row and the pivot value, we now divide every entry in the pivotal row by the pivot value.**



**The process is now finished as every entry on the objective function row is non-negative.**

**The values of x, y and Z can be read from the Table:**

**x = 5, y = 35, Z = 470.**

**This is the optimal solution.**

**Example 2:**

**Consider the following linear programming problem:**

**MAX Z = 10x + 30y**

***s.t.* 4x + 6y ≤ 12**

**8x + 4y ≤ 16**

**x ≥ 0, y ≥ 0.**

**Create the initial tableau and perform 1 pivot.**

**Answer: Standard form of LP is**

**Max. Z = 10x + 30y → Z – 10x – 30y = 0.**

**s.t. 4x +6 y + s = 12**

**8 x + 4y + t = 16**

**s ≥ 0, t ≥ 0**

**s, t are slack variables.**

**Initial tableau:**



**Pivot 1 or First Iteration:**



**Example 3**:

The owner of a shop producing automobile trailers wishes to determine the best mix for his three products: Flat-bed trailers, Economy trailers, and Luxury trailers. His shop is limited to working 24 days per month on metalworking and 60 days per month on woodworking for these products. The following table indicates the production data for the trailers.

**Flat-bed Economy Luxury Resource**

**Available**

**Metalworking (days) 0.5 2 1 24**

**Woodworking (days) 1 2 4 60**

**Unit Profit ($) 6 14 13**

Let the decision variables of the problem be:

x1 = Number of at-bed trailers produced per month

x2 = Number of economy trailers produced per month

x3 = Number of luxury trailers produced per month

The model is

max Z=6x1 + 14x2 + 13x3

s.t.

0.5x1 + 2x2 + x3≤ 24

x1 + 2x2 + 4x3 ≤ 60

x1,x2 ≥ 0

Let x4 and x5 be slack variables corresponding to unused hours of metalworking and woodworking capacity. Then the problem above is equivalent to the following standard

form problem.

max Z-6x1 - 14x2 - 13x3+ 0x4+ 0x5 =0

s.t.

0.5x1 + 2x2 + x3 + x4 = 24

x1 + 2x2 + 4x3 + x5 = 60

x1,x2 ≥ 0

The pivot row and column are indicated in green; the pivot element is red. We pick the variable with the most negative coefficient to enter the basis.

**Tableau I**

**BASIS Z x1 x2 x3 x4 x5 RHS Ratio Eqn.**

**Z 1 -6 -14 -13 0 0 0 0 (1)**

**x4 0 0.5 2 1 1 0 24 12 (2)**

**x5 0 1 2 4 0 1 60 30 (3)**

**Tableau II**

**BASIS Z x1 x2 x3 x4 x5 RHS Ratio Eqn.**

**Z 1 -2.5 0 -6 7 0 168 -28 (4)=(1)+14(5)**

**x2 0 0.25 1 0.5 0.5 0 12 24 (5)=(2)/2**

**x5 0 0.5 0 3 -1 1 36 12 (6)=(3)-2(5)**

**Tableau III**

**BASIS Z x1 x2 x3 x4 x5 RHS Ratio Eqn.**

**Z 1 -1.5 0 0 5 2 240 -160 (7)=(4)+6(9)**

**x2 0 1/6 1 0 2/3 -1/6 6 36.0 (8)=(5)-(9)/2**

**x3 0 1/6 0 1 -1/3 1/3 12 72 (9)=(6)/3**

**Tableau IV**

**BASIS Z x1 x2 x3 x4 x5 RHS Eqn.**

**Z 1 0 9 0 11 0.5 294 (10)=(7)+1.5(11)**

**x1 0 1 6 0 4 -1 36 (11)=6(8)**

**x3 0 0 -1 1 -1 0.5 6 (12)=(9)-(11)/6**

**Thus, the optimal value of to the z = 294. The optimal solution is x = (36; 0; 6; 0; 0).**