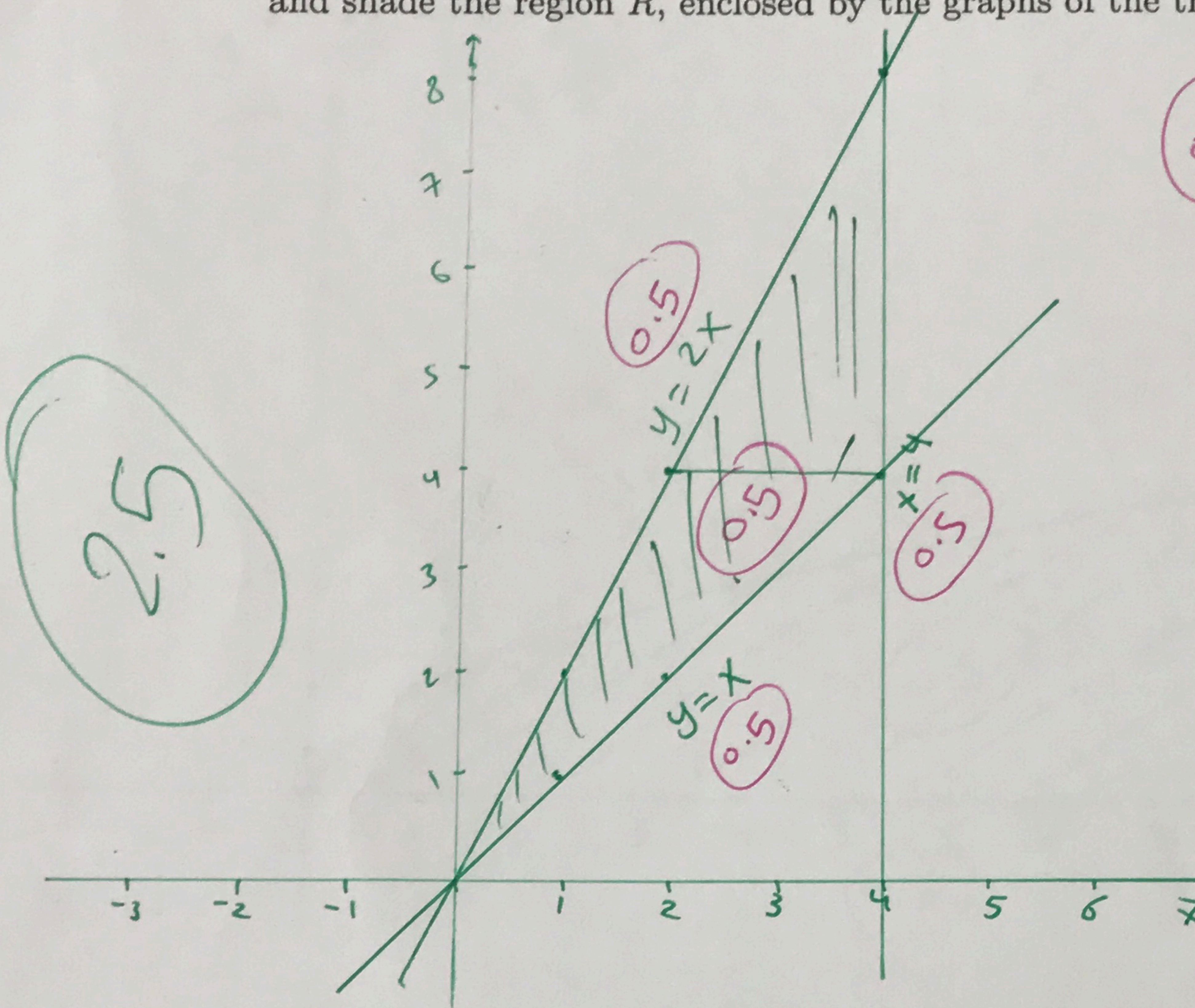
41548

$\frac{96-64}{3}$ $\frac{32}{3}$

I) [5 *marks*]

A) Sketch the graphs of functions $y = 2x$, $y = x$ and $x = 4$ on the same system of coordinates and shade the region R, enclosed by the graphs of the three functions.

1 2 3



2.5

0.5

$2x = 2x \rightarrow x=0$ (0,0)	تقاطع ① مع ②
(4,4)	تقاطع ② مع ③
(4,8)	تقاطع ① مع ③

B) Find the area of the region R.

$$A = \int_0^4 (2x - x)\,dx = \int_0^4 x\,dx$$

0.5 0.5 0.5

$$= \frac{x^2}{2}\Big|_0^4 = \frac{16}{2} - 0 = 8$$

0.5 0.5

2.5

x

$$A = \int_0^4 \left(y - \frac{y}{2}\right) dy + \int_4^8 \left(4 - \frac{y}{2}\right) dy$$

0.5 التقسيم

1 A_1 تكامل

1 A_2 تكامل

$$= \frac{y^2}{4}\Big|_0^4 + 4y - \frac{y^2}{4}\Big|_4^8$$

$$= 4 + \cancel{32} - \cancel{16} - \cancel{16} + 4$$

$$= 8$$

y

$16 - 32 = -16$

$-24 + 12$

$16 - 32 - 4 + 16$

$\left(\frac{8\cdot 8}{4} - 4\cdot 8\right) - \left(\frac{4\cdot 4}{4} - 4\cdot 4\right)$

$16 - 32 + 12$

$16 - 20 = -4$

$4 - 8$

$(f(x))^2$ ما طلوب تربيع 1.25

$-\frac{4}{2} + \frac{1}{8}$

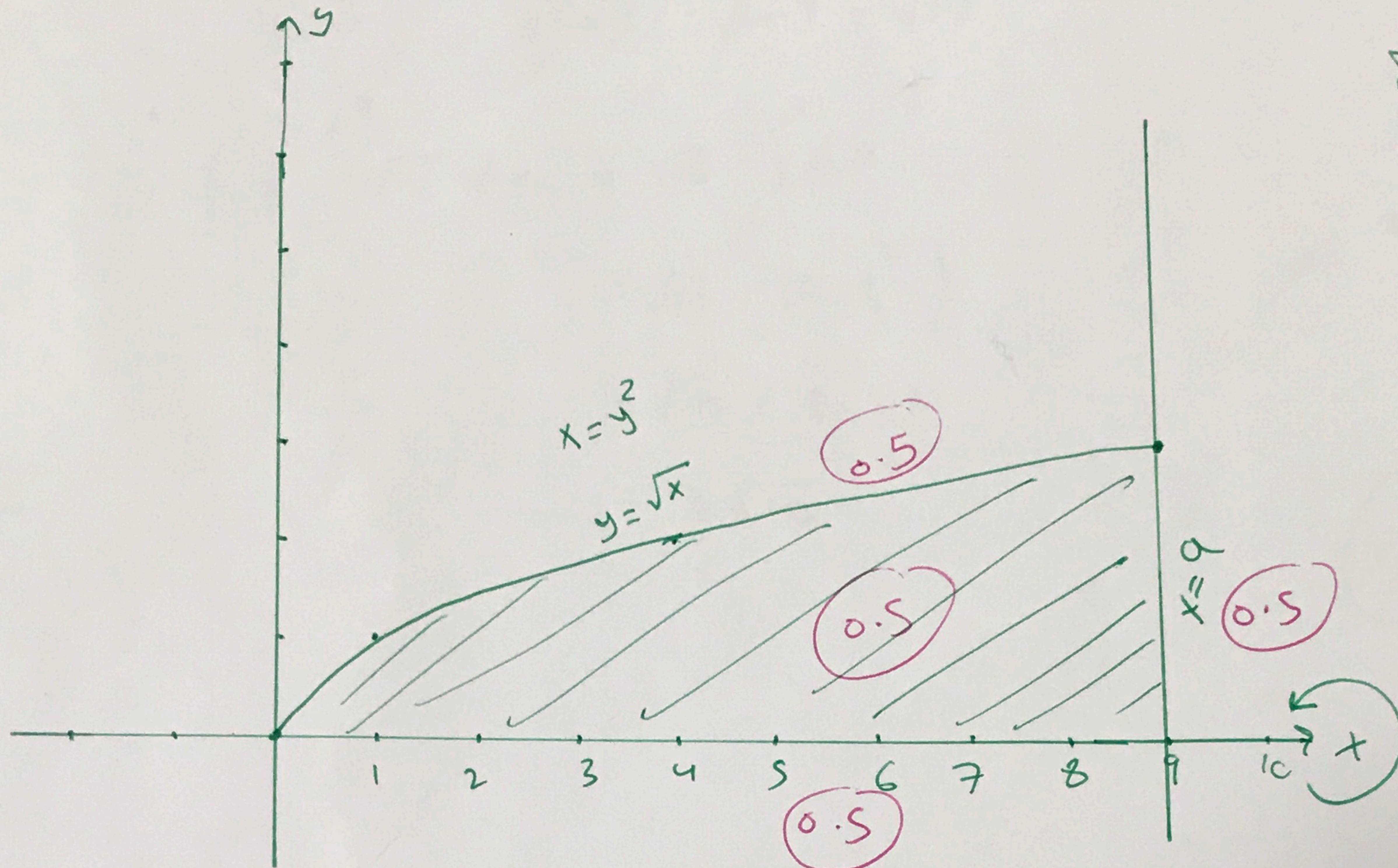
II) [5 *marks*]

A) Sketch the graph of the region R determined, in the first quadrant, by the graphs of $y = \sqrt{x}$ and $x = 9$.

نقطة التقاطع

(9,3) (9,-3)

0.5



B) Find the volume of the solid obtained by revolving the region R about the x-axis.

كتابة القانون 1 (π 0.5 / التربيع 0.5)

الحدود 0.5

التكامل 0.5

التعويض بالحدود 0.5

disk

$$V = \int_0^9 \pi (\sqrt{x})^2 \, dx = \pi \int_0^9 x \, dx$$

$$= \pi \left. \frac{x^2}{2} \right|_0^9 = \pi \left(\frac{81}{2} \right) = \frac{81}{2} \pi$$

cylindical shells

$$V = \int_0^3 2\pi y (9 - y^2) \, dy$$

$$V = 2\pi \left(\left. \frac{9y^2}{2} - \frac{y^4}{4} \right|_0^3 \right) = 2\pi \left(\frac{81}{2} - \frac{81}{4} \right) = 2\pi \left(\frac{2 \cdot 81 - 81}{4} \right)$$

$$= \cancel{2\pi} (\cancel{81} - \cancel{9}) = \cancel{2\pi}(18) = 36\pi$$

$$= \cancel{2}\pi \left(\frac{81}{\cancel{4}_2} \right)$$

$$= \frac{81}{2} \pi$$

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III) [5 *marks*]

A) Compute the arc length of the graph of $y = 3x + 2$, from $(1, 5)$ to $(4, 14)$.

$y' = 3$, $\sqrt{1+(y')^2} = \sqrt{1+9} = \sqrt{10}$

$L_1^4 = \int_1^4 \sqrt{10}\, dx = \sqrt{10} \int_1^4 dx$

$= \sqrt{10} \left(x \Big|_1^4 \right)$

$= \sqrt{10}\,(4-1)$

$= 3\sqrt{10}$

2.5

B) The graph of the curve $4x = y^2$, from $(0, 0)$ to $(1, 2)$, is revolved about the x-axis. Find the area of the resulting surface.

$S = 2\pi \int f(x) \sqrt{1+(f'(x))^2}\, dx$

2.5

$y' = \frac{4}{2\sqrt{4x}} = \frac{4}{2 \cdot 2\sqrt{x}} = \frac{1}{\sqrt{x}}$

$\sqrt{1+(y')^2} = \sqrt{1+\frac{1}{x}} = \frac{\sqrt{x+1}}{\sqrt{x}}$

$S = 2\pi \int_0^1 2\cancel{\sqrt{x}} \cdot \frac{\sqrt{x+1}}{\cancel{\sqrt{x}}}\, dx$

$= 4\pi \int_0^1 (x+1)^{1/2}\, dx$

$= \frac{2 \cdot 4\pi}{3} (x+1)^{3/2} \Big|_0^1$

$= \frac{8}{3}\pi \left[2\sqrt{2} - 1\right]$

IV) [5 *marks*]

A) Use logarithmic differentiation to compute $\frac{dy}{dx}$, if $y = \frac{x^7 \cos x}{(x+1)^2}$.

$\ln y = \ln(x^7 \cos x) - 2\ln(x+1)$

$\rightarrow \ln y = 7\ln x + \ln \cos x - 2\ln(x+1)$

$\rightarrow \frac{1}{y} y' = \frac{7}{x} - \frac{\sin x}{\cos x} - \frac{2}{x+1}$

$\rightarrow y' = \left(\frac{x^7 \cos x}{(x+1)^2}\right)\left[\frac{7}{x} - \tan x - \frac{2}{x+1}\right].$

2.5

B) Use implicit differentiation to compute y', if $x \ln y - y \ln x = 1$.

$\left(1 \cdot \ln y + x \cdot \frac{y'}{y}\right) - \left(y' \ln x + y \cdot \frac{1}{x}\right) = 0$

$\rightarrow \ln y + \frac{xy'}{y} - \ln x \cdot y' - \frac{y}{x} = 0$

$\rightarrow \left(\frac{x}{y} - \ln x\right) y' = \frac{y}{x} - \ln x$

2.5

$\rightarrow y' = \dfrac{\frac{y}{x} - \ln x}{\frac{x}{y} - \ln x}$