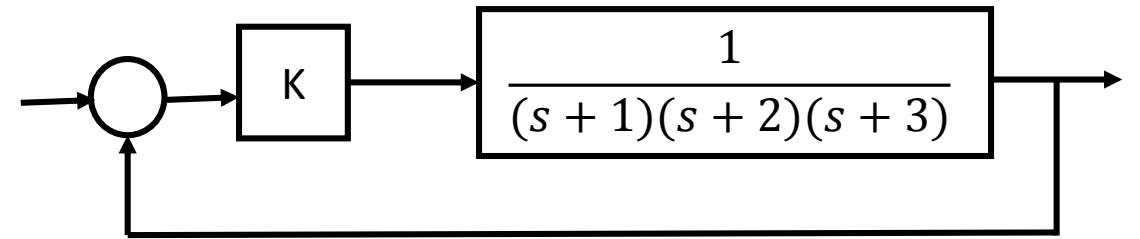


## Exercise 1 (Root Locus)

Sketch the root locus for the system shown in Figure

1. Locate the open-loop poles and zeros on the complex plane
2. Find the asymptotes.
3. Determine the intersection with the  $j\omega$  axis,
4. Determine the breakaway and break-in points



## Solution 1<sub>1</sub> (Root Locus)

1. Locate the open-loop poles and zeros on the complex plane. Root loci exist on the negative real axis between -1 and between -2 and -3.
2. The number of open-loop poles is 3 and the number of finite zeros is 0. This means that we have three branches ending at infinity and there are three asymptotes in the complex region of the s plane (three infinite zeros).

The angles of asymptotes:  $\varphi_A = \pm \frac{(2q + 1)\pi}{nb(poles) - nb(zeros)} \text{ for } q = 0, 1, 2 \rightarrow \varphi_A = 60^\circ, 180^\circ, 300^\circ$

Point of joint of asymptotes:  $\sigma_A = \frac{(poles) - (zeros)}{nb(poles) - nb(zeros)} = \frac{(-3 - 2 - 1) - (0)}{(3) - (0)} = -2$

3. Determine the breakaway and break-in points. The characteristic equation for the system is:

$$1 + \frac{K}{(s + 1)(s + 2)(s + 3)} = 0 \rightarrow K = -(s + 1)(s + 2)(s + 3) = -[s^3 + 6s^2 + 11s + 6] \rightarrow \frac{dK}{ds} = -[3s^2 + 12s + 11] = 0$$

we have two roots for this equation: **-1.4226** and -2.5773 the second root is not valid (not on the root locus branch)

## Solution 1<sub>2</sub> (Root Locus)

4. Determine the intersection with the  $j\omega$  axis,  
we use Routh-Hurwitz stability criterion, we have:

$$1 + G(s)H(s) = 0 \rightarrow s^3 + 6s^2 + 11s + (6 + K) = 0$$

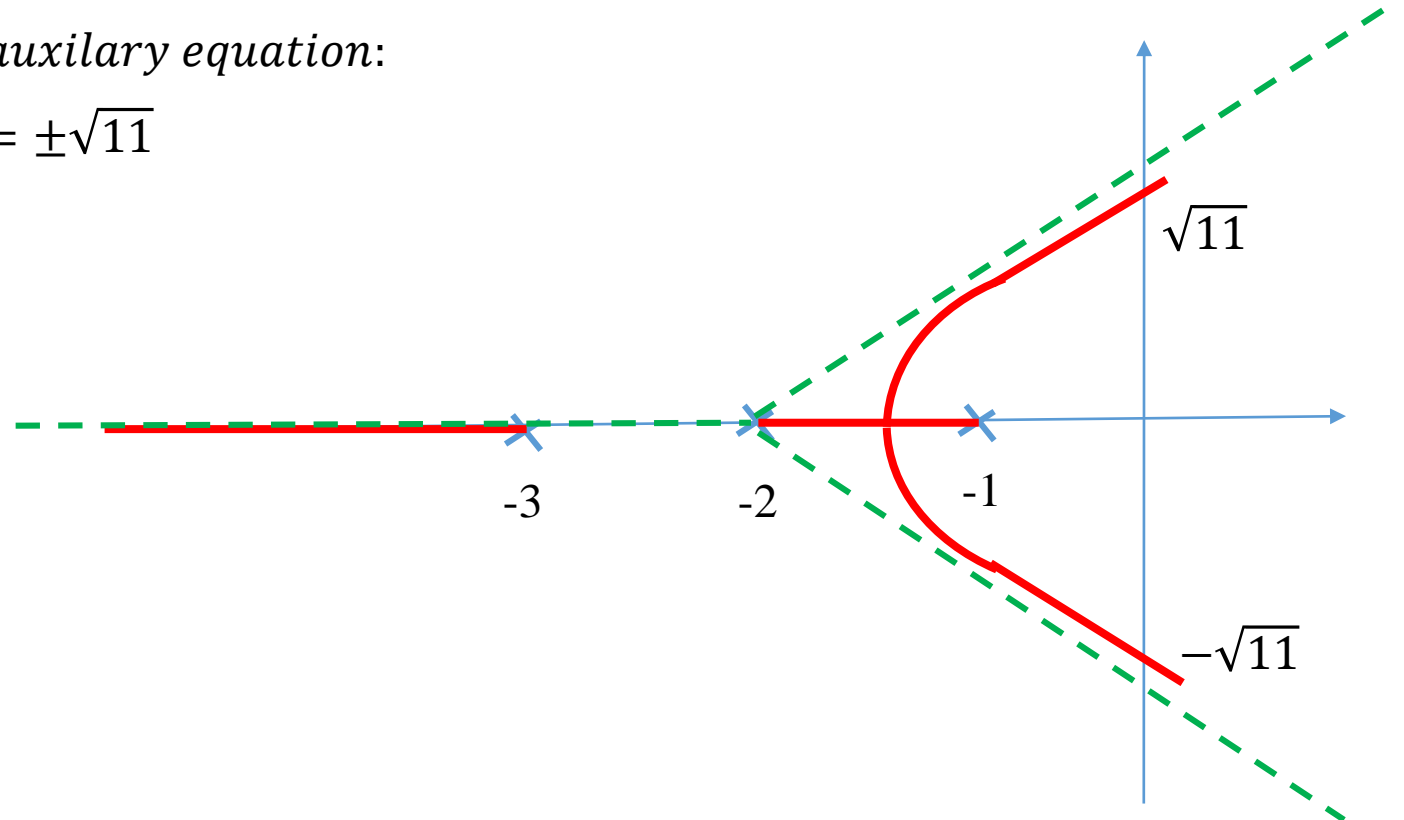
The line with odd power and find k to have complete zeros row

$$\frac{66 - 6 - K}{6} = 0 \rightarrow K = 60 \rightarrow \text{the above row is auxiliary equation:}$$

$$6s^2 + 66 = 0 \rightarrow s = \pm j\sqrt{11} \rightarrow \omega_{1,2} = \pm\sqrt{11}$$

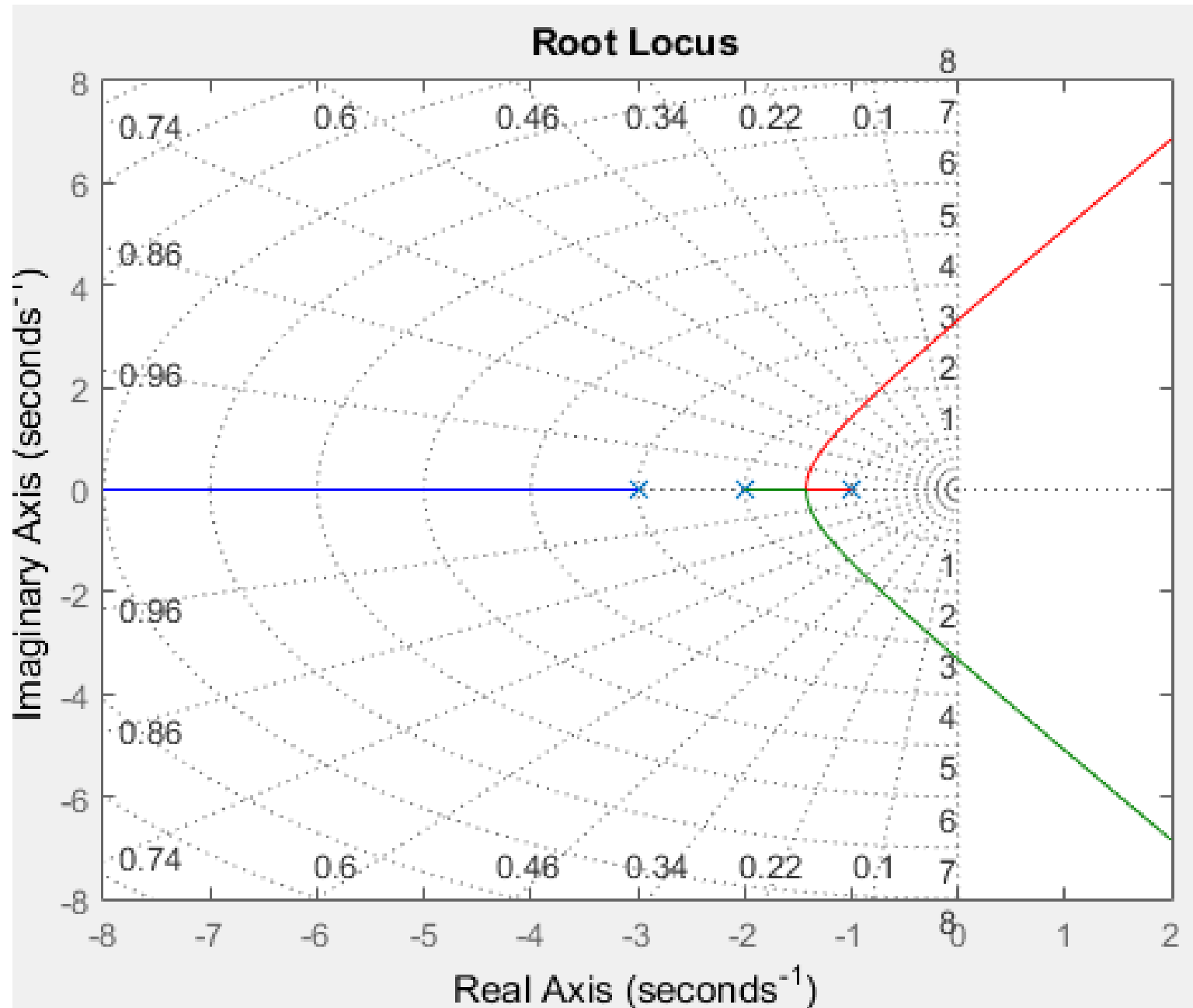
$s^3$	1	11	0
$s^2$	6	$6+K$	0
$s^1$	$\frac{66 - 6 - K}{6}$	0	0
$s^0$	$6+K$	0	0

- For  $K > 60$  the system will be unstable.



## Solution 1<sub>3</sub> (Root Locus Using MATLAB)

```
clear all;clc;  
s=tf('s');  
sys=1/((s+1)*(s+2)*(s+3)); sgrid;  
rlocus(sys);grid on;  
axis([-8 2 -8 8]);
```



## Exercise 2 (Lag Compensator)

A unity feedback system with the forward transfer function  $G(s)$  is operating with a closed-loop step response that has 15% overshoot. Do the following:

$$G(s) = \frac{K}{s(s + 7)}$$

- a. Evaluate the steady-state error for a unit ramp input.
- b. Design a lag compensator to improve the steady-state error by a factor of 20.
- c. Evaluate the steady-state error for a unit ramp input to your compensated system.
- d. Evaluate how much improvement in steady-state error was realized.

## Solution 2<sub>1</sub> (Lag Compensator)

- a. Uncompensated system analysis: The uncompensated system error. The root locus for the uncompensated system is shown in Figure. A damping ratio of 0.517 is represented by a radial line drawn on the s-plane at 121.1°.

$$15\% \text{overshoot} \rightarrow \xi = 0.517 \rightarrow \text{poles} = -3.5 \pm j5.82 \text{ with } K = 45.8$$

$$K_v = \lim_{s \rightarrow 0} sG(s) = \frac{K}{7} = 6.54 \rightarrow e(\infty) = \frac{1}{K_v} = 0.1527$$

- b. Lag compensator design

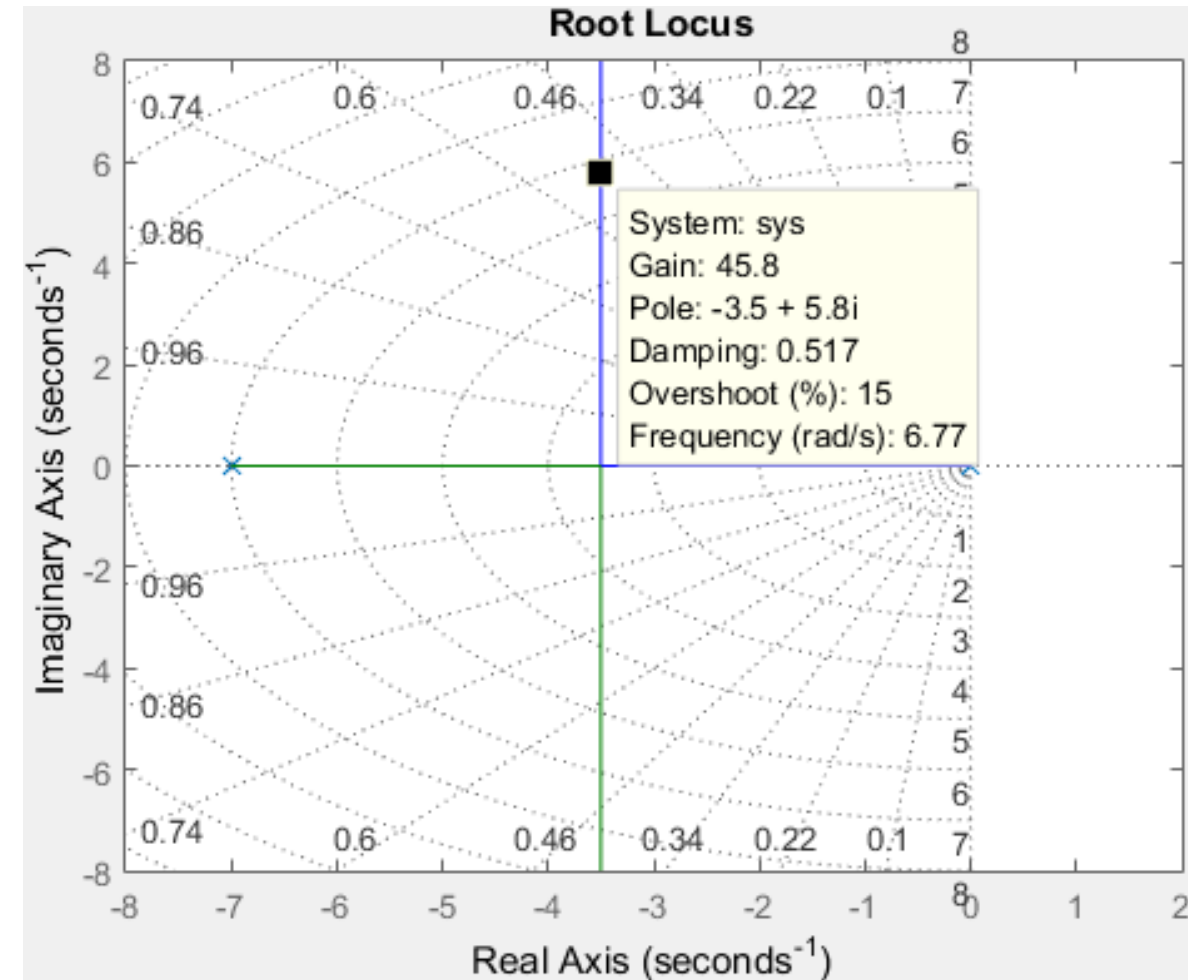
The uncompensated system error was 0.1527 with K=46.1, a factor improvement of 20:

$$e(\infty) = \frac{0.1527}{20} = 0.007635, \text{ since } e(\infty) = \frac{1}{K_v} \Rightarrow K_{vN} = 130.98$$

The improvement in  $K_v$  from the uncompensated system to the compensated system is the required ratio of the compensator zero to the compensator pole:

$$\frac{z_c}{p_c} = \frac{K_{vN}}{K_v} = \frac{130.98}{6.54} = 20.03 \quad \text{Arbitrarily selecting } p_c = 0.01$$

$$\rightarrow z_c = 20.03 p_c \approx 0.2 \rightarrow G_{Lag}(s) = \frac{s + 0.2}{s + 0.01}$$



## Solution 2<sub>2</sub> (Lag Compensator)

c. Error evaluation for the compensated system

$$G_N(s) = \frac{K(s + 0.2)}{s(s + 0.01)(s + 7)}$$

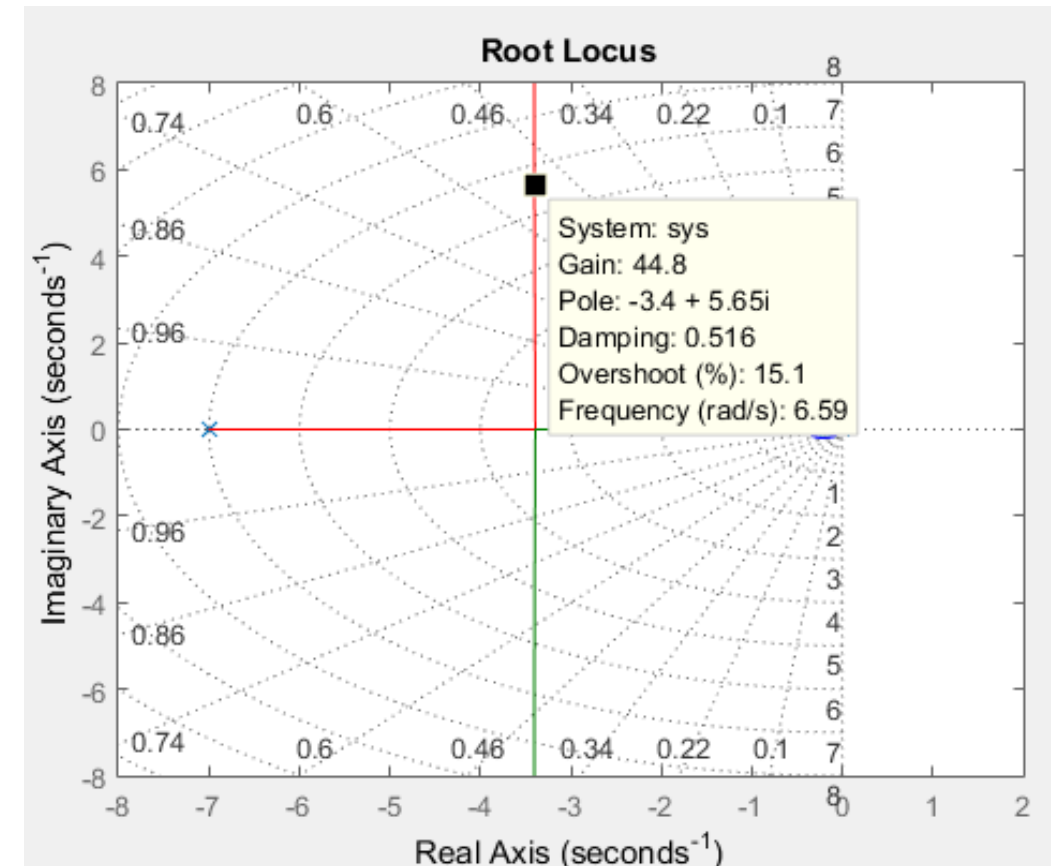
15% overshoot  $\rightarrow \xi = 0.517 \rightarrow \text{poles} = -3.4 \pm j5.65$  with  $K = 44.8$

$$K_v = \lim_{s \rightarrow 0} sG_N(s) = \frac{K \cdot 0.2}{(7)(0.01)} = 128 \rightarrow e_N(\infty) = \frac{1}{K_v} = 0.0078$$

$$e_N(\infty) = \frac{1}{K_v} = 0.0078$$

d. Realized improvement in steady-state error

$$\frac{e(\infty)}{e_N(\infty)} = \frac{0.1527}{0.0078} = 19.58$$



## Exercise 3 (Lead Compensator)

A unity feedback system with the forward transfer function

$$G(s) = \frac{K}{s(s + 7)}$$

is operating with a closed-loop step response that has 15% overshoot. Do the following:

- Evaluate the settling time.
- Design a lead compensator to decrease the settling time by three times.  
Choose the compensator's zero to be at -10.



# Solution 3<sub>1</sub> (Lead Compensator)

## • Characteristics of the uncompensated system

System operating at 30% overshoot

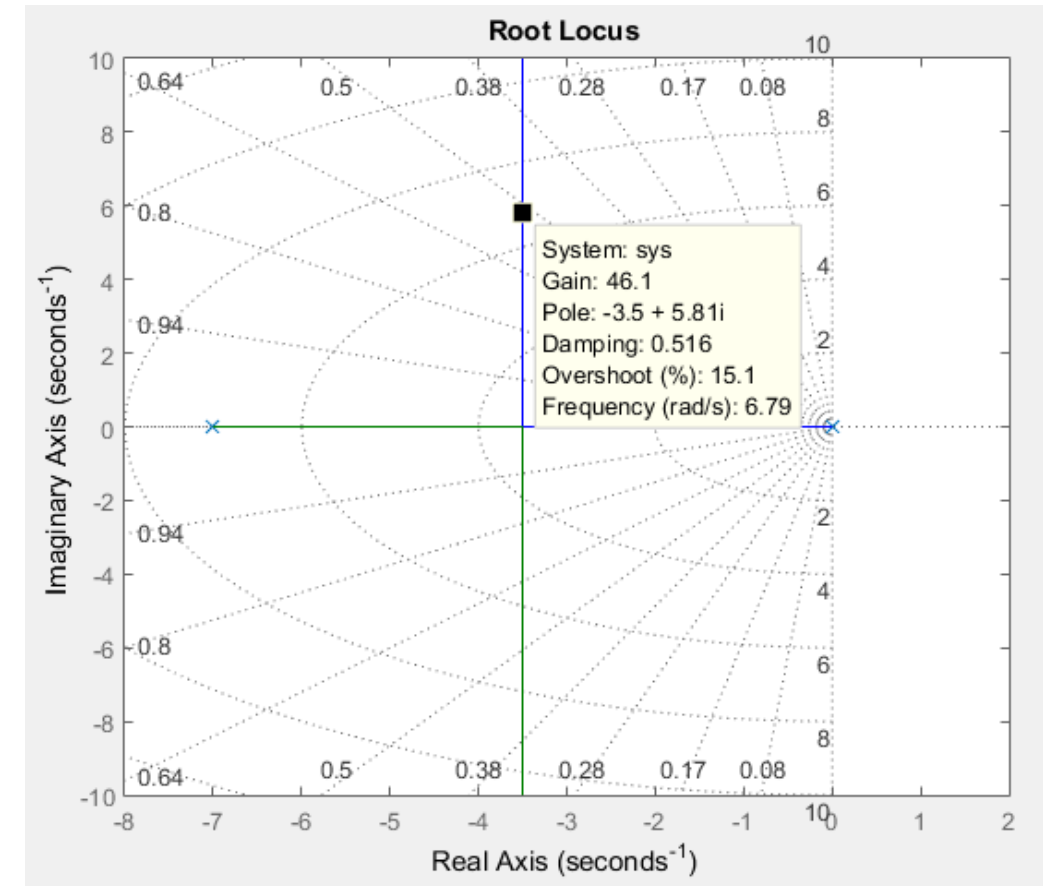
15% Overshoot  $\xrightarrow{\text{damping ratio}}$   $\zeta = 0.517$   
 $\alpha = 121.13^\circ$ ,  $\xrightarrow{\text{Dominant second-order pair of poles}}$   $-3.5 \pm j5.82$ .  
along damping ratio line

$\xrightarrow{\text{settling time}}$   $T_s = 4/3.5 = 1.143 \text{ sec}$   
From pole's real part

## • Design point

$\xrightarrow{\text{Threefold reduction in settling time}}$   $T_{sN} = 1.143/3 = 0.381 \text{ sec}$   $\xrightarrow{\text{real part of the desired pole location}}$   $-\zeta\omega_n = -4/T_{sN} = -10.499$

$\xrightarrow{\text{Imaginary part of the desired pole location}}$   $\omega_d = -10.499 \tan(121.13^\circ) = 17.38$



# Solution 3<sub>2</sub> (Lead Compensator)

- Lead compensator Design.

Place the zero on real axis at -10 (arbitrarily as possible solution).  
sum the angles (this zero and uncompensated system's poles and zeros),

resulting angle

→  $\theta = -222.52^\circ$

the angular contribution required  
from the compensator pole

→  $\theta_{pc} = -180^\circ + 222.52^\circ = 42.52^\circ$

location of the  
compensator pole

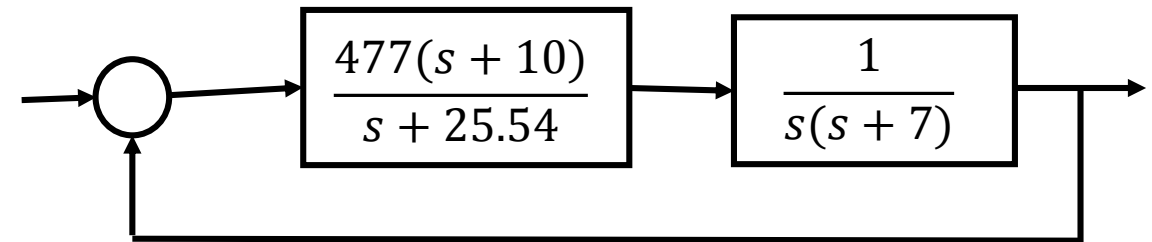
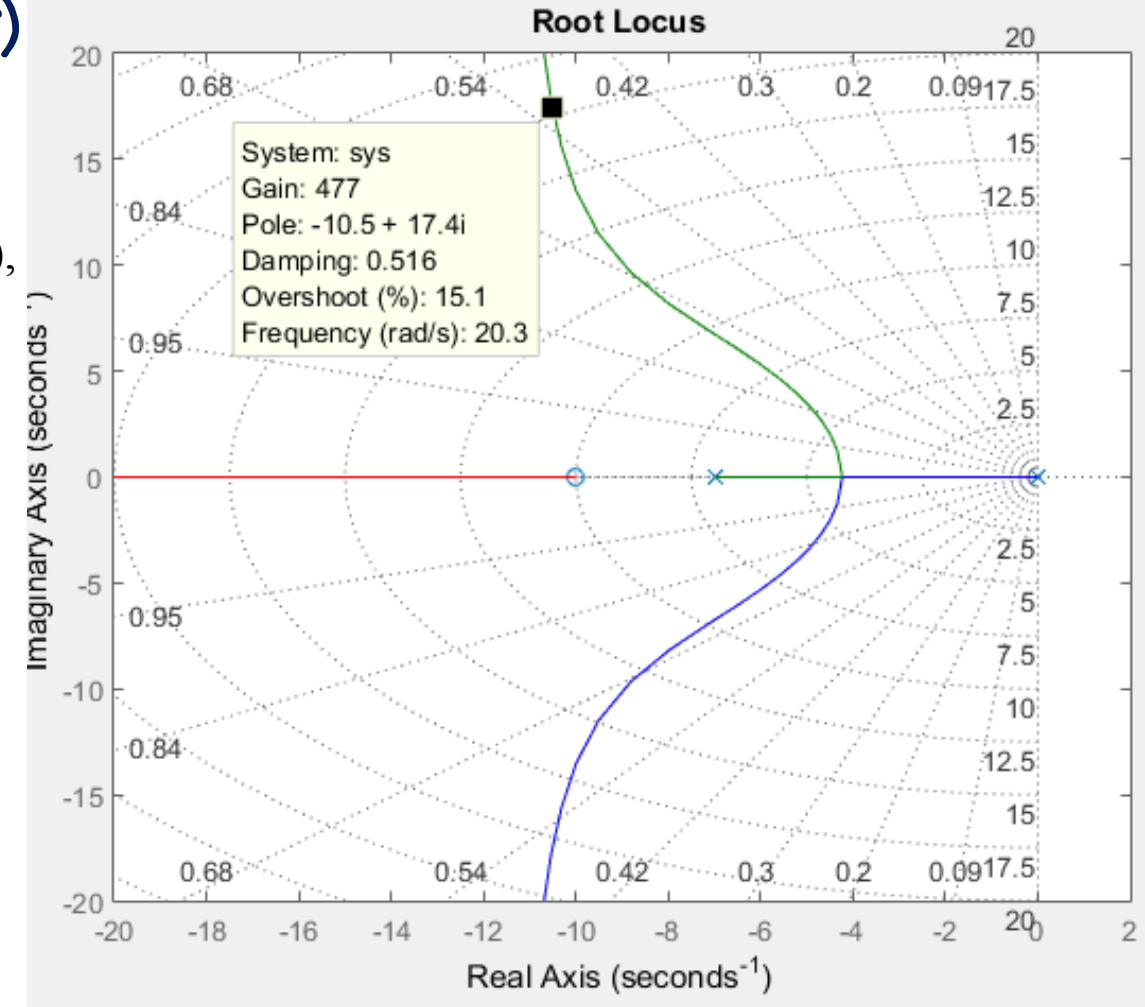
→  $\frac{17.38}{p_c - 10.499} = \tan(42.52^\circ)$   
From geometry  
in fig(a)

compensator pole

→  $p_c = -25.54$

$$G_{Led}(s) = \frac{s + 10}{s + 25.54}$$

With gain K = 477



## Exercise 4 (Implementation)

Implement the compensator  $G_c(s)$

$$G_c(s) = \frac{(s + 0.1)(s + 5)}{s}$$

Choose a passive realization if possible.

## Solution 4 (Implementation)

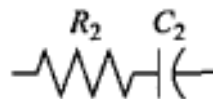
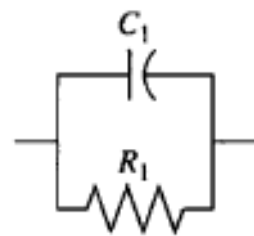
The transfer function of the controller  $G_c(s) = \frac{(s + 0.1)(s + 5)}{s}$

$$G_c(s) = \frac{s^2 + 5.1s + 0.5}{s} = 5.1 + s + \frac{0.5}{s}$$

$G_c(s)$  is a PID controller and thus requires active realization. From table

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

PID controller



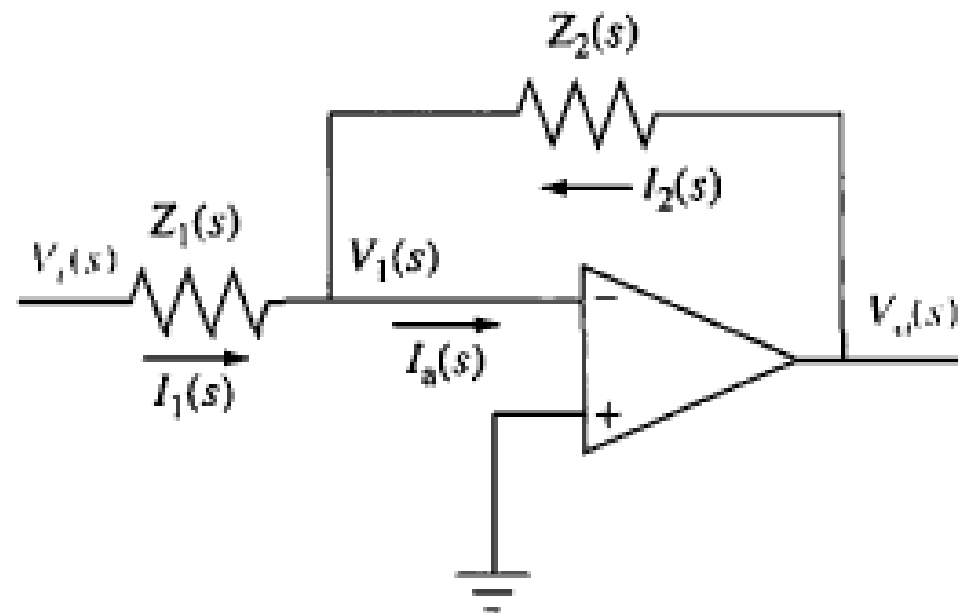
Matching the coefficients:

$$\frac{R_2}{R_1} + \frac{C_1}{C_2} = 5.1$$

$$R_2 C_1 = 1$$

$$\frac{1}{R_1 C_2} = 0.5$$

If we choose  $C_1 = 10\mu F$  and  $C_2 = 100\mu F \rightarrow R_2 = 100k\Omega$  and  $R_1 = 20k\Omega$



$$-\left[ \left( \frac{R_2}{R_1} + \frac{C_1}{C_2} \right) + R_2 C_1 s + \frac{1}{R_1 C_2 s} \right]$$