PHYSICS 301
Solutions 1st HOMEWORK
Dr. V. Lempesis

Hand in: Tuesday 2nd of March 2014

1. For two complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ ($z_2 \neq 0$) find the number z_1 / z_2.

Solution:

$$z_1 / z_2 = (x_1 + iy_1)/(x_2 + iy_2) = \frac{(x_1 + iy_1)(x_2 - iy_2)}{(x_2 + iy_2)(x_2 - iy_2)} =$$
$$\frac{x_1 x_2 - ix_1 y_2 + iy_1 x_2 + iy_1(-iy_2)}{x_2^2 + y_2^2} = \frac{(x_1 x_2 + y_1 y_2) + i(y_1 x_2 - x_1 y_2)}{(x_2^2 + y_2^2)} =$$
$$\frac{(x_1 x_2 + y_1 y_2)}{(x_2^2 + y_2^2)} + i\frac{(y_1 x_2 - x_1 y_2)}{(x_2^2 + y_2^2)}$$

2. What geometrical object is represented by the equation $|z - 1 + 3i| = 2$?

Solution:

Let $z = x + iy$ then we have:

$$|z - 1 + 3i| = 2 \Rightarrow |z - 1 + 3i|^2 = 4 \Rightarrow (z - 1 + 3i)\cdot(\bar{z} - 1 - 3i) = 4$$
$$\Rightarrow (x + iy - 1 + 3i)\cdot(x - iy - 1 - 3i) = 4 \Rightarrow$$
$$[(x-1) + i(y+3)]\cdot[(x-1) - i(y+3)] = 4 \Rightarrow$$
$$(x-1)^2 + (y+3)^2 = 4$$

So the object is a circle with radius equal to 2 and centered at the point $(x_0, y_0) = (1, -3)$.

3. Express the following complex numbers in exponential form:
$2 + 2\sqrt{3}i$, $\quad -5 + 5i$.

Solution:

a) $z = 2 + 2\sqrt{3}i \Rightarrow |z| = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{16} = 4$. Also

$$Arg(z) = \arctan(2\sqrt{3}/2) \Rightarrow Arg(z) = \arctan(\sqrt{3})$$
$$\Rightarrow Arg(z) = \pi / 3$$

Thus $z = 4e^{i\pi/3}$.

b) $z = -5 + 5i \Rightarrow |z| = \sqrt{(-5)^2 + (5)^2} = 5\sqrt{2}$

$$\begin{aligned} &Arg(z) = \arctan(5/-5) \Rightarrow Arg(z) = \arctan(-1) \\ &\Rightarrow Arg(z) = 3\pi/4 \end{aligned}$$

Thus $z = 5\sqrt{2}e^{i3\pi/4}$.

4. Represent graphically on the x-y plane the following complex numbers: $6\left(\cos 240^0 + i\sin 240^0\right)$, $2e^{-i\pi/4}$.

Solution:

5. Find the square root of *i*.

Solution:

$z = i \Rightarrow |z| = z = 1e^{i\pi/2}$. So if w is the square root we have

$$\begin{aligned} &w^2 = z \Rightarrow w^2 = 1e^{i\pi/2} \Rightarrow w^2 = 1e^{i(\pi/2+2k\pi)} \Rightarrow \\ &w = \sqrt{1}e^{i(\pi/2+2k\pi)/2}, \quad (k = 0,\ 1) \end{aligned}$$

$$w = \begin{cases} e^{i\pi/4} & k = 0 \\ e^{i3\pi/4} & k = 1 \end{cases}$$