Some Suggested pbs for math 106
Final Exam (I Sem. 1438/1439)

Q(1) Evaluate each of the following Integral

a) $\int \frac{e^{\tan x}}{\cos^2 x}\,dx$

b) $\int_0^{1/\sqrt{2}} \sin^{-1} x\,dx$

c) $\int \frac{2x^3+4x^2+10x+13}{x^4+9x^2+20}\,dx$

d) $\int \frac{1}{\sqrt{e^{2x}-36}}\,dx$

Ans: a) $\int \frac{e^{\tan x}}{\cos^2 x}\,dx$

$= \int e^{\tan x}\sec^2 x\,dx$

$= e^{\tan x} + C$

b) $I = \int_0^{1/\sqrt{2}} \sin^{-1} x\,dx$

$u = \sin^{-1} x \qquad dv = dx$

$du = \frac{dx}{\sqrt{1-x^2}}, \quad v = x$

$I = \left[x \sin^{-1} x\right]_0^{1/\sqrt{2}} - \int_0^{1/\sqrt{2}} \frac{x\,dx}{\sqrt{1-x^2}}$

$I = \frac{1}{\sqrt{2}} \sin^{-1}\frac{1}{\sqrt{2}} + \left[\sqrt{1-x^2}\right]_0^{1/\sqrt{2}}$

$I = \frac{1}{\sqrt{2}} \cdot \frac{\pi}{4} + \frac{1}{\sqrt{2}} - 1$

$I \simeq 0.26$

c) $I = \int \frac{2x^3+4x^2+10x+13}{(x^2+5)(x^2+4)}\,dx$

$\Rightarrow I = \int \left[\frac{Ax+B}{x^2+5} + \frac{Cx+D}{x^2+4}\right] dx$

$(Ax+B)(x^2+4) + (Cx+D)(x^2+5)$

$= 2x^3+4x^2+10x+13$

$\Rightarrow A+C = 2$

$, \; 4A+5C = 10$

$, \; B+D = 4$

$, \; 4B+5D = 13$

$\Rightarrow A = 0, \; B = 7, \; C = 2, \; D = -3$

$\Rightarrow I = \int \left[\frac{7}{x^2+5} + \frac{2x}{x^2+4} - \frac{3}{x^2+4}\right] dx$

$I = 7/\sqrt{5} \tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + \ln(x^2+4) - \frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$

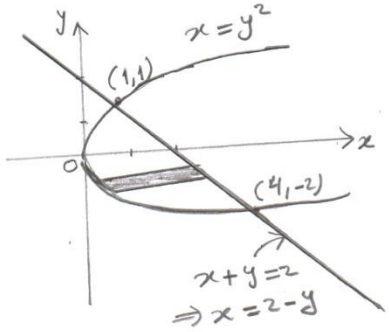
d) $I = \int \frac{1}{\sqrt{e^{2x}-36}}\,dx$

$I = \int \frac{du}{u\sqrt{u^2-36}}, \quad u = e^x$

$I = \frac{1}{6}\sec^{-1}(e^x/6) + C$ #

P.2

Q(2) Find the area of the region bounded by the graphs of the eqns: $x = y^2$ and $x + y = 2$



Ans: $x = y^2$ ①

$x = 2 - y$ ②

①, ② $\Rightarrow y^2 = 2 - y$

$\Rightarrow y^2 + y - 2 = 0$

$(y+2)(y-1) = 0$

$\Rightarrow y = -2, y = 1$

$\Rightarrow$ intersection points are $(4,-2)$ and $(1,1)$

$$\text{Area} = \int_{-2}^{1} [(2-y) - y^2]\, dy$$

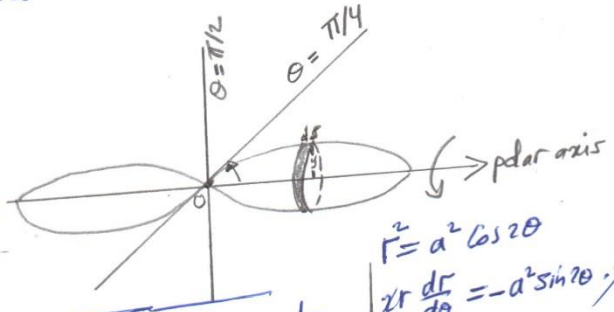
$$\therefore \text{Area} = \left[2y - \frac{y^2}{2} - \frac{y^3}{3}\right]_{-2}^{1} = \frac{9}{2}$$

Q(3) Find the area of the surface generated by revolving the lemniscate $r^2 = a^2 \cos 2\theta$ about the polar axis

Ans:

$$S.A = 2\pi \int_{\theta_1}^{\theta_2} y\, ds$$

$$\Rightarrow S.A = 2\pi \int_{\theta_1}^{\theta_2} r\sin\theta \cdot \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$



$$S.A = 2\pi \cdot 2 \int_0^{\pi/4} r\sin\theta \cdot a\sqrt{\cos 2\theta + \sin 2\theta \tan 2\theta}\, d\theta$$

$$S.A = 4\pi a^2 \int_0^{\pi/4} \sin\theta \sqrt{\cos^2 2\theta + \sin^2 2\theta}\, d\theta$$

$$S.A = 4\pi a^2 \int_0^{\pi/4} \sin\theta\, d\theta = 4\pi a^2 \left[-\cos\theta\right]_0^{\pi/4}$$

$r^2 = a^2 \cos 2\theta$

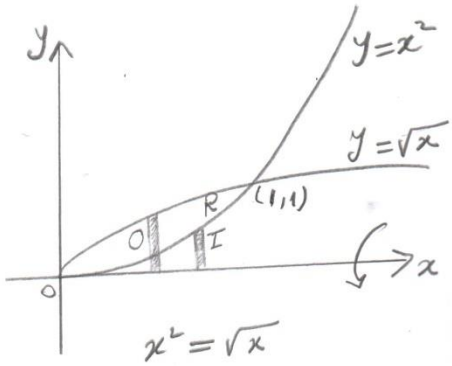
$2r \frac{dr}{d\theta} = -a^2 \sin 2\theta \cdot 2$

$\Rightarrow \left(\frac{dr}{d\theta}\right)^2 = a^2 \sin 2\theta \tan 2\theta$

P.3

Q(4) Sketch the region R bounded by the graphs of the eqns $y = x^2$, $y = \sqrt{x}$ and find the volume of solid by revolving R about x-axis

Ans:



V. of solid is

$$V = \pi \int_0^1 \left[(\sqrt{x})^2 - (x^2)^2\right] dx$$

$$V = \pi \int_0^1 (x - x^4)\, dx$$

$$V = \pi \left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1$$

$$V = \pi \left[\frac{1}{2} - \frac{1}{5}\right] = \frac{3\pi}{10}$$

$x^2 = \sqrt{x}$

$\Rightarrow x^4 = x$

$\Rightarrow x(x^3 - 1) = 0$

$\Rightarrow x = 0, x = 1$

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Q(5) Find $f'(x)$ for each of the following

a) $f(x) = \cosh \sqrt{4x^2+3}$

Ans:

a) $f'(x) = \sinh \sqrt{4x^2+3} \cdot \frac{1}{2\sqrt{4x^2+3}} \cdot 8x$

$\therefore f'(x) = \frac{4x \sinh \sqrt{4x^2+3}}{\sqrt{4x^2+3}}$

b) $f(x) = \log_2(\cos x)$

b) $f'(x) = \frac{1}{\cos x}(-\sin x) \cdot \frac{1}{\ln 2}$

$\therefore f'(x) = \frac{-\tan x}{\ln 2}$

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