

## CH#10 Springs

Springs are mechanical elements that exert forces or torques and absorb energy. The absorbed energy is usually stored and later released. Springs are made of metal. For light loads the metal can be replaced by plastics. Some applications that require minimum spring mass use structural composite materials. Blocks of rubber can be used as springs, in bumpers and vibration isolation mountings of electric or combustion motors.



ME 305

## Types of Springs

- Springs may be classified as wire springs, flat springs, or special shaped springs and there are variations within these divisions
- Wire springs include helical springs of round or square wire, made to resist and deflect under tensile, compressive, and torsional loads
- Flat springs include cantilever or elliptical types, wound motor- or clock-type power springs and flat spring washers usually called Belleville springs

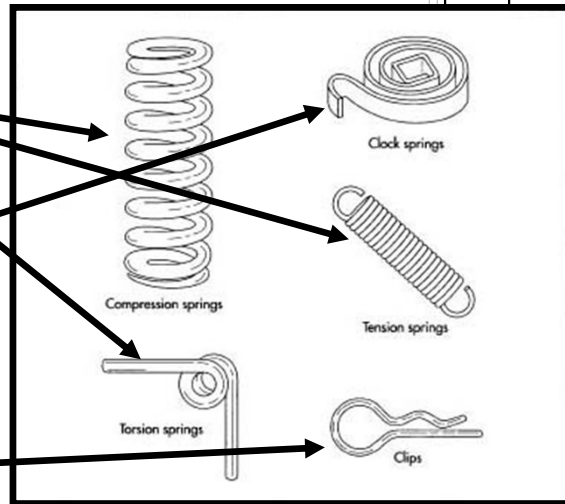
ME 305 (Machine Design II)

## Types of Springs...

Wire Springs

Flat Springs

Special Shaped Springs

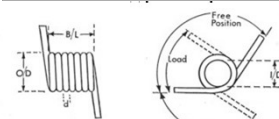


## Types of Springs...

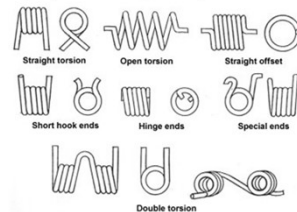
Helical Springs



Torsional Spring



Basic types of torsion springs which can be wound LH or RH. Legs may be variable in length and shape.



Helical springs used to apply torque or store rotational energy are commonly referred to as torsion or double torsion

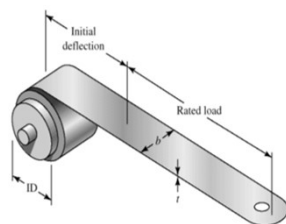
## Types of Springs...

	 <p>A <b>Belleville spring</b> is a piece of curved steel with an extremely high tensile strength.</p>	<p>ME 302</p>  <p>PARALLEL</p>  <p>SERIE</p>  <p>PARALLEL UND SERIE</p>
--	--	--

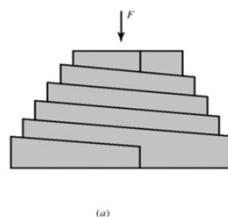
## Types of Springs...

### Miscellaneous Spring

Constant-  
force  
spring

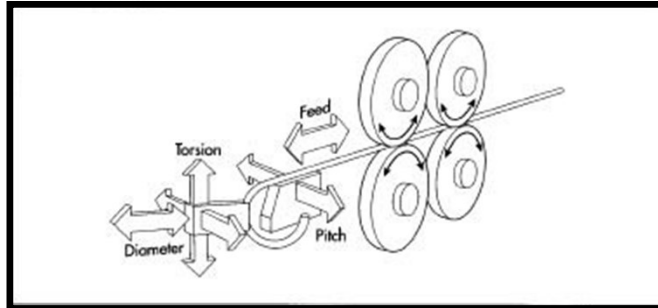


Volute  
spring



ME 305 (Machine Design)

## Manufacturing of Springs



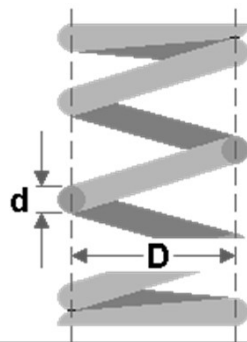
A diagram depicting spring coiling done by a CNC machine.

Show video

ME 305 (Machine Design II)

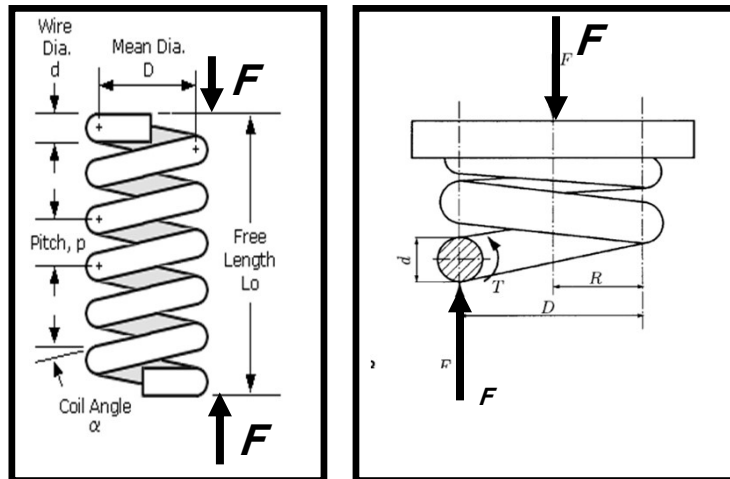
## Helical Compression Spring

A helical compression spring is an open-pitch spring which is used to resist applied compression forces or to store energy. It can be made in a variety of configurations and from different shapes of wire, depending on the application. Round, high-carbon-steel wire is the most common spring material,



Machine Design II)

## Helical Compression Spring...



ME 305 (Machine Design II)

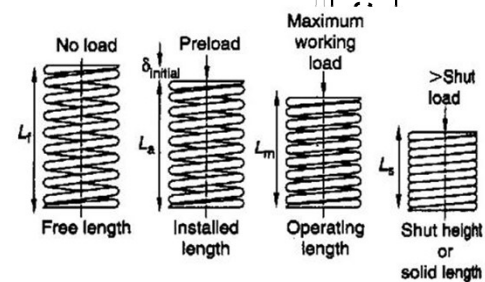
## Design Parameters

**Free Length ( $L_o$ )** - The length of the unloaded spring.

**Wire Diameter ( $d$ )** - The diameter of the wire that is wound into a helix.

**Coil Diameter ( $D$ )** - The mean diameter of the helix, i.e.,  $(D_{\text{outer}} + D_{\text{inner}})/2$ .

**Total Coils ( $N_t$ )** - The number of coils or turns in the spring.



ME :

1)

## 10-1 Stresses in Helical Springs

- $D$  is the mean dia. of the spring and  $d$  is the dia. of the wire.
- The spring wire is subjected to
  - direct shear stress ( $\tau_{shear} = \frac{F}{A}$ ) and
  - torsional shear stress ( $\tau_{torsion} = \frac{Tr}{J}$ )
- The maximum stress is the addition of the two

$$\tau_{max} = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}, \quad C = \frac{D}{d} \quad (4 \leq C \leq 12)$$

and

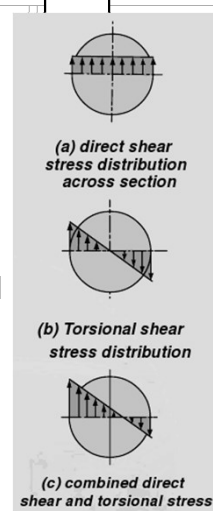
$$\tau_{max} = K_s \frac{8FD}{\pi d^3}, \quad K_s = \frac{2C + 1}{2C}$$

ME 305 (Machine Design II)

## 10-2 The Curvature Effect

- The curvature of the wire increases the stresses on the inside and decreases only slightly on the outside
- To account for this,  $K_s$  is modified as
 
$$K_B = \frac{4C + 2}{4C - 3}$$
- $K_B$  is determined by Bergstrasser, and is called *Bergstrasser factor*
- The equation for the largest shear stress becomes

$$\tau_{max} = K_B \frac{8FD}{\pi d^3}$$



### 10-3 Deflection of Helical Springs

- For deflection, use Castigliano's theorem

$$y = \frac{\partial U}{\partial F} \rightarrow (1)$$

- $U$  for the spring is given by

$$U = \frac{T^2 l}{2JG} + \frac{F^2 l}{2AG}$$

- Put for  $U$  in (1) to get

$$y = \frac{8D^3 N}{d^4 G} F \left(1 + \frac{1}{2C^2}\right) \approx \frac{8D^3 N}{d^4 G} F$$

- and

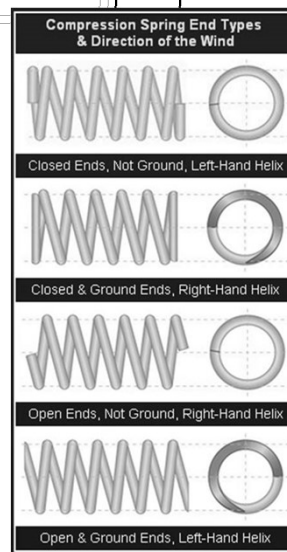
$$k = \frac{d^4 G}{8D^3 N}$$

ME 305 (Machine Design II)

### 10-4 Compression Springs

- The design of compression spring normally consists of four different types of ends;

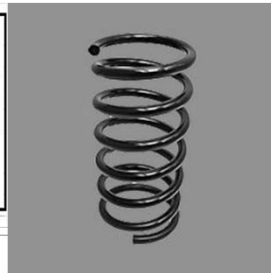
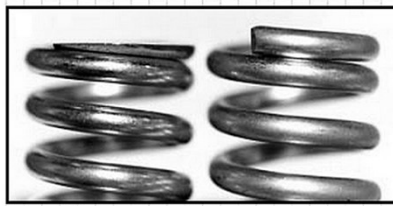
1. Closed ends, not ground *OR* Squared, not ground
2. Closed ends, ground *OR* Squared, ground
3. Open ends, not ground *OR* Plain end, not ground
4. Open ends, ground *OR* Plain ends, ground



## Formulae for compression spring dimensions (Table 10-1)

- Where  $N_a$  is the number of active coils

Term	Type of Spring Ends			
	Plain	Plain and Ground	Squared or Closed	Squared and Ground
End coils, $N_e$	0	1	2	2
Total coils, $N_t$	$N_a$	$N_a + 1$	$N_a + 2$	$N_a + 2$
Free length, $L_0$	$pN_a + d$	$p(N_a + 1)$	$pN_a + 3d$	$pN_a + 2d$
Solid length, $L_s$	$d(N_t + 1)$	$dN_t$	$d(N_t + 1)$	$dN_t$
Pitch, $p$	$(L_0 - d)/N_a$	$L_0/(N_a + 1)$	$(L_0 - 3d)/N_a$	$(L_0 - 2d)/N_a$



ME 305 (Machine Design II)

## Set Removal or Presetting

- A process used in the manufacture of compression spring to induce useful residual stresses
- It is done by making the spring longer than needed and then compressing it to its solid height
- Springs to be pre-set should be designed so that 10 to 30% of the initial free length is removed during the operation
- If the stress at the solid height is greater than 1.3 times the torsional yield strength, distortion may occur

ME 305 (Machine Design II)



## Set Removal or Presetting...

- If this stress is much less than 1.1 times, it is difficult to control the resulting free length
- Set removal increases the strength of the spring and so is specially useful when the spring is used for energy-storage purposes
- However, set removal should not be used when springs are subjected to fatigue

ME 305 (Machine Design II)

## 10-6 Spring materials

- Springs are manufactured either by hot- or cold-working processes, depending upon the size of the material, the spring index, and the properties desired
- In general, prehardened wire should not be used if  $D/d < 4$  or if  $d > 1/4$  in.
- Winding of the springs induces residual stresses through bending, however they are relieved after winding, by a mild thermal treatment

Name of Material	Similar Specifications	Description
Music wire, 0.80-0.95C	UNS G10850 AISI 1085 ASTMA228-51	This is the best, toughest, and most widely used of all spring materials for small springs. It has the highest tensile strength and can withstand higher stresses under repeated loading than any other spring material. Available in diameters 0.12 to 3 mm (0.005 to 0.125 in). Do not use above 120°C (250°F) or at subzero temperatures
Oil-tempered wire, 0.60-0.70C	UNS G10650 AISI 1065 ASTM 229-41	This general-purpose spring steel is used for many types of coil springs where the cost of music wire is prohibitive and in sizes larger than available in music wire. Not for shock or impact loading. Available in diameters 3 to 12 mm (0.125 to 0.500 in), but larger and smaller sizes may be obtained. Not for use above 180°C (350°F) or at subzero temperatures
Hard-drawn wire, 0.60-0.70C	UNS G10660 AISI 1066 ASTMA227-47	This is the cheapest general-purpose spring steel and should be used only where life, accuracy, and deflection are not too important. Available in diameters 0.8 to 12 mm (0.031 to 0.500 in). Not for use above 120°C (250°F) or at subzero temperatures
Chrome vanadium	UNS G6150C AISI 6150 ASTM 231-41	This is the most popular alloy spring steel for conditions involving higher stresses than can be used with the high-carbon steels and for use where fatigue resistance and long endurance are needed. Also good for shock and impact loads. Widely used for aircraft-engine valve springs and for temperatures to 220°C (425°F). Available in annealed or pretempered sizes 0.8 to 12 mm (0.031 to 0.500 in) in diameter

## 10-6 Spring materials...

- Graph b/w tensile strength and wire diameter is a straight line for some material. Then

$$S_{ut} = A/d^m$$

- Use Table 10-4 to find  $m$  and  $A$ .

Material	ASTM No.	Exponent $m$	Diameter, in	$A$ , kpsi·in <sup><math>m</math></sup>	Diameter, mm	$A$ , MPa·mm <sup><math>m</math></sup>	Relative Cost of wire
Music wire*	A228	0.145	0.004–0.256	201	0.10–6.5	2211	2.6
OQ&T wire†	A229	0.187	0.020–0.500	147	0.5–12.7	1855	1.3
Hard-drawn wire‡	A227	0.190	0.028–0.500	140	0.7–12.7	1783	1.0
Chrome-vanadium wire§	A232	0.168	0.032–0.437	169	0.8–11.1	2005	3.1
Chrome-silicon wire	A401	0.108	0.063–0.375	202	1.6–9.5	1974	4.0
302 Stainless wire#	A313	0.146	0.013–0.10	169	0.3–2.5	1867	7.6–11
		0.263	0.10–0.20	128	2.5–5	2065	
		0.478	0.20–0.40	90	5–10	2911	
Phosphor-bronze wire**	B159	0	0.004–0.022	145	0.1–0.6	1000	8.0
		0.028	0.022–0.075	121	0.6–2	913	
		0.064	0.075–0.30	110	2–7.5	932	

## Selection of Spring Material.....

The torsional yield strength can be obtained by assuming that the tensile yield strength is between 60% and 90% of the tensile strength. According to the distortion-energy theory, the torsional yield strength for steels is

$$0.35 S_{ut} \leq S_{sy} \leq 0.52 S_{ut}$$

## Example 10.1

A helical compression spring is made of no. 16 music wire. The outside diameter of the spring 11mm. The ends are squared and there are 12.5 total turns.

- Estimate the torsional yield strength ( $S_{sy}$ ) of the wire.
- Estimate the static load ( $F$ ) corresponding to the torsional yield strength of part a.
- Estimate the scale ( $k$ ) of the spring.
- Estimate the deflection ( $y$ ) that would be caused by the load in part (b).
- Estimate the solid length ( $L_s$ ) of the spring.
- What length should the spring be to ensure that when it is compressed solid and then released, there will be no permanent change in the free length ( $L_0$ )?
- Buckling (Not included)
- What is the pitch ( $p$ ) of the body coil?

ME 305 (Machine Design II)

## Solution

$$a) S_{sy} = 0.45S_{ut}$$

$$b) F = \frac{\pi d^3 S_{sy}}{8K_B D}, K_B = \frac{4C+2}{4C-3}$$

$$c) k = \frac{d^4 G}{8D^3 N_a}$$

$$d) y = \frac{F}{k}$$

$$e) L_s = (N_t + 1)d$$

$$f) L_0 = y + L_s$$

$$g) \text{ Not included}$$

$$h) p = \frac{(L_0 - 3d)}{N_a}$$

Table 10-5

Mechanical Properties of Some Spring Wires

Material	Elastic Limit, Percent of $S_{ut}$		Diameter $d$ , in	$E$ Mpsi	$E$ GPa	$G$ Mpsi	$G$ GPa
Music wire A228	65-75	45-60	<0.032	29.5	203.4	12.0	82.7
			0.033-0.063	29.0	200	11.85	81.7
			0.064-0.125	28.5	196.5	11.75	81.0
			>0.125	28.0	193	11.6	80.0
				28.8	198.6	11.7	80.7
HD spring A227	60-70	45-55	<0.032	28.8	198.6	11.7	80.7
			0.033-0.063	28.7	197.9	11.6	80.0
			0.064-0.125	28.6	197.2	11.5	79.3
			>0.125	28.5	196.5	11.4	78.6
Oil tempered A239	85-90	45-50		28.5	196.5	11.2	77.2

ME 305 (Machine Design II)

<b>10-7 Helical Spring Design for Static Service</b>		
<ul style="list-style-type: none"> <li>Static service means light service (frequency of loading less than 1000 cycles)</li> <li>The preferred range of spring index is <math>4 \leq C \leq 12</math></li> <li>The recommended range of active turns is <math>3 \leq N_a \leq 15</math></li> <li>To maintain linearity when a spring is about to close, it is necessary to avoid the gradual touching of coils (due to non perfect pitch)</li> <li>A helical coil spring force-deflection characteristic is ideally linear, although practically not (<i>Due to non-uniform "Ls"</i>)</li> </ul>	ME 305 (Machine Design II)	

<b>10-7 Helical Spring Design for Static Service...</b>		
<ul style="list-style-type: none"> <li>Non-linear behaviour at closure.</li> <li>The designer confines the spring's operating point to central 75% of the curve between no load, <math>F = 0</math> and closure, <math>F = F_s</math></li> <li>Thus the maximum operating force should be limited to <math>F_{max} \leq 7/8 F_s</math></li> <li>Defining the <i>Fractional overrun to closure</i> as <math>\xi</math>, where <math>F_s = (1 + \xi)F_{max}</math></li> <li><math>\xi \geq 0.15</math></li> <li><math>F_s</math> is 15% larger than <math>F_{max}</math></li> <li>The FOS of the solid height (<math>n_s</math>) should be <math>\geq 1.2</math></li> </ul>	ME 305 (Machine Design II)	

## 10-7 Helical Spring Design for Static Service...

- To design a spring, the designer should check
 
$$4 \leq C \leq 12$$

$$3 \leq N_a \leq 15$$

$$n_s \geq 1.2$$
- Along with other conditions such as the spring lengths and diameter
- Normally  $\xi \geq 0.15$
- When considering designing a spring for high volume production, the figure of merit (fom) can be the cost of the wire from which the spring is wound
- The fom would be proportional to the relative material cost, weight density, and volume

$$fom = -(relative\_material\_cost) \frac{\gamma \pi^2 d^2 N_t D}{4}$$

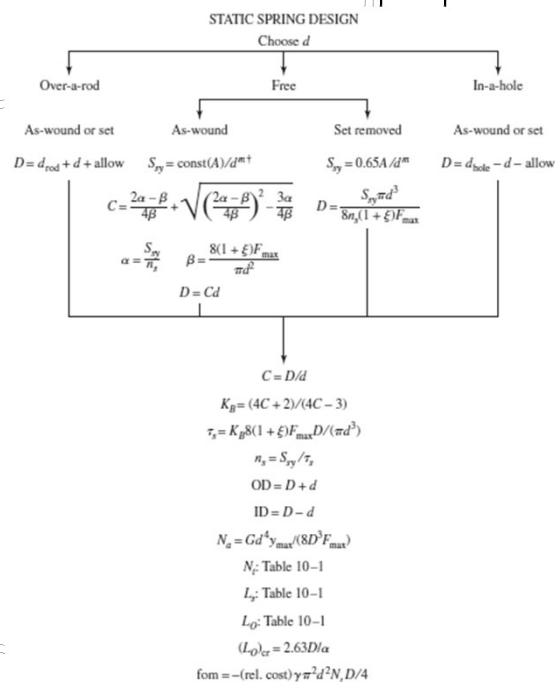
For comparison between steels, the specific weight  $\gamma$  can be omitted

ME 305 (Machine Design II)

## Design flow chart for static loading

Use MatLab or Excel for calculations...

Print or display:  $d, D, C, OD, ID, N_s, N_t, L_s, L_t, (L_0)_{cr}, n_s, fom$   
 Build a table, conduct an adequacy assessment by inspection  
 Eliminate infeasible designs by showing active constraints  
 Choose among satisfactory designs using the figure of merit



<b>Problem 10.21</b>		
<p>A static service music wire helical compression spring is needed to support a 90N load after being compressed 50 mm. The solid height of the spring cannot exceed 38mm. The free length must not exceed 100mm. The static factor of safety must equal or exceed 1.2. For robust linearity use a fractional overrun to closure <math>\xi</math> of 0.15. There are two springs to be designed. Start with a wire diameter of 1.9mm.</p> <p>(a) The spring must operate over a 20mm rod. A 0.1mm diametral clearance allowance should be adequate to avoid interference between the rod and the spring due to out-of-round coils. Design the spring.</p> <p>(b) The spring must operate in a 25mm diameter hole. A 0.1mm diametral clearance allowance should be adequate to avoid interference between the spring and the hole due to swelling of the spring diameter as the spring is compressed and out-of-round coils. Design the spring.</p>	ME 305 (Machine Design II)	

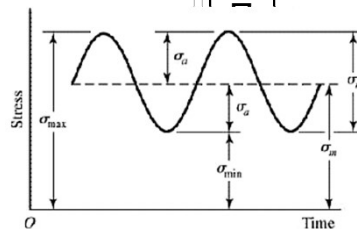
<b>10-9 Fatigue loading of Helical Compression Springs</b>		
<ul style="list-style-type: none"> <li>• Springs are always subjected to Fatigue loading</li> <li>• Life of springs vary from thousands to millions cycles</li> <li>• Toggle switch spring life is finite and that of engine valve spring is infinite</li> <li>• <i>Zimmerli</i> discovered that size, material, and tensile strength have no effect on the endurance limit (infinite life only) of spring steels under size 10mm</li> </ul>	ME 305 (Machine Design II)	

## 10-9 Fatigue loading of Helical Compression Springs...

- Helical springs are never used as both compression and extension springs. This is because they are usually assembled with a preload so that the working load is additional. Thus, the spring application fall under the condition of *fluctuating loads*. Thus,

$$F_a = \frac{F_{\max} - F_{\min}}{2}$$

$$F_m = \frac{F_{\max} + F_{\min}}{2} = F_{\min} + F_a$$



ME 305 (Machine I)

## 10-9 Fatigue loading of Helical Compression Springs...

- The amplitude and midrange stresses are;

$$\tau_a = K_B \frac{8F_a D}{\pi d^3}$$

$$\tau_m = K_B \frac{8F_m D}{\pi d^3}$$

- Endurance limits for infinite life are (determined experimentally by Zimmerli)

Un peened:  $S_{sa} = 241 \text{ MPa}$

$S_{sm} = 379 \text{ MPa}$

Peened:  $S_{sa} = 398 \text{ MPa}$

$S_{sm} = 534 \text{ MPa}$

ME 305 (Machine Design II)

## 10-9 Fatigue loading of Helical Compression Springs...

- For example, using Goodman failure criteria, the ordinate intercept  $S_{se}$  for the Zimmerli data is;

$$S_{se} = \frac{S_{sa(\text{Zimmerli Data})}}{1 - (S_{sm(\text{Zimmerli Data})} / S_{su})}$$

- Where the shearing ultimate strength  $S_{su} = 0.67S_{ut}$
- The fatigue factor of safety  $n_f$  is calculated as;  

$$n_f = \frac{S_{sa}}{\tau_a}$$
- Where  $S_{sa}$  is the alternating component of strength of the spring and is calculated using Table 6-6 to 6-8 for different failure criteria (refer to section 6-12)

ME 305 (Machine Design II)

## 10-9 Fatigue loading of Helical Compression Springs...

- $\tau_a$  is the alternate stress component

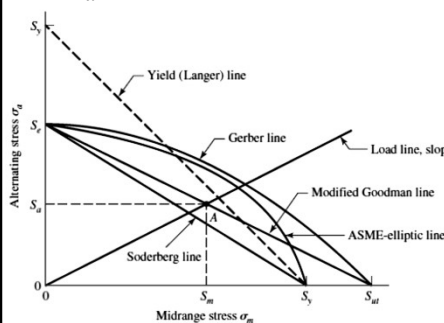


Table 6-7. Gerber failure criterion

Intersecting Equations	Intersection Coordinates
$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$	$S_a = \frac{r^2 S_{ut}^2}{2S_e} \left[ -1 + \sqrt{1 + \left(\frac{2S_e}{r S_{ut}}\right)^2} \right]$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_a}{r}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = \frac{r S_y}{1 + r}$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_y}{1 + r}$
$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$	$S_m = \frac{S_{ut}^2}{2S_e} \left[ 1 - \sqrt{1 + \left(\frac{2S_e}{S_{ut}}\right)^2 \left(1 - \frac{S_y}{S_e}\right)} \right]$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = S_y - S_m \cdot r_{crit} = S_a / S_m$

Fatigue factor of safety

$$n_f = \frac{1}{2} \left( \frac{S_{ut}}{\sigma_m} \right)^2 \frac{\sigma_a}{S_e} \left[ -1 + \sqrt{1 + \left( \frac{2\sigma_m S_e}{S_{ut} \sigma_a} \right)^2} \right] \quad \sigma_m > 0$$

ME 305 (



<b>Example 10-4</b>		
<p>An as-wound helical compression spring, made of music wire, has a wire size of 2.3mm, an outside coil diameter of 14mm, a free length of 98mm, 21 active coils, and both ends squared and ground. The spring is un-peened. This spring is to be assembled with a preload of 22 N and will operate with a maximum load of 156 N during use.</p> <ol style="list-style-type: none"> <li>Estimate the factor of safety guarding against fatigue failure using a torsional Gerber fatigue failure criterion with Zimmerli data.</li> <li>Repeat part (a) using the Sines torsional fatigue criterion (steady stress component has no effect), with Zimmerli data.</li> <li>Repeat using a torsional Goodman failure criterion with Zimmerli data.</li> <li>Not included</li> </ol>	ME 305 (Machine Design II)	

<b>Example 10-5</b>		
<ul style="list-style-type: none"> <li>A music wire helical compression spring with infinite life is needed to resist a dynamic load that varies from 12 to 50 N at 5 Hz while the end deflection varies from 12 to 50 mm. Because of assembly considerations, the solid height cannot exceed 25 mm and the free length cannot be more than 100 mm. The spring maker has the following wire sizes in stock: 1.7, 1.8, 2.0, 2.15, 2.3, 2.4, 2.6 and 2.8 mm.</li> </ul>	ME 305 (Machine Design II)	

## Solution...

### Priority Decisions are:

- Surface treatment: un-peened
- End treatment: squared and ground
- Robust linearity:  $\xi = 0.15$
- Set: use in as-wound condition
- Fatigue-safe:  $n_f = 1.5$  using the Sines-Zimmerli fatigue-failure criterion
- Spring operates free (no rod or hole)
- Decision variable: wire size  $d$

None of the “ $d$ ” satisfy the condition. The 2.6 mm wire is closest to satisfy most of the conditions with  $C = 13.4$  and  $L_s = 24.3$ . The selection is still debatable.

$d$ :	1.7	1.8	2.0	2.15	2.3	2.4	2.6	2.8
$D$	9.5	10.6	15.4	18.7	22.5	26.4	34.8	42.8
ID	6.3	7.2	11.9	15.0	18.6	22.6	32.1	40
OD	9.9	10.9	16.1	19.4	23.2	27.5	37.4	45.6
$C$	5.6	5.9	7.7	8.7	9.8	11.0	13.4	15.3
$N_a$	105.2	84.7	37.0	25.21	17.61	12.73	7.13	4.96
$L_s$	194.3	161.5	81.5	59.9	45.8	36.1	24.3	19.5
$L_0$	270.3	234.1	145.6	121.7	106.0	95.3	82.3	77
$(L_0)_{cr}$	42.6	47.6	80.8	90.7	110.1	131.8	182.8	225
$n_f$	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
$n_s$	1.82	1.81	1.78	1.77	1.75	1.74	1.71	1.7
$f_u$	86.7	88.9	96.0	98.8	101.0	102.8	105.6	107
fom	-1.17	-1.12	-0.98	-0.95	-0.93	-0.93	-0.96	-1.01

ME 305 (Machine D

## 10-11 Extension springs

An Extension Spring resists a ‘pulling force’ exerted against it through energy stored. Normally made from round wire, the coils are ‘close-wound’, with some ‘initial tension’ included



ME 305 (Machine Design II)

## 10-11 Extension springs...

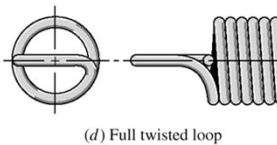
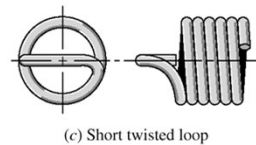
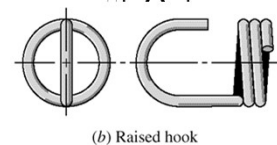
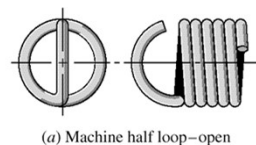
- Extension springs differ from compression springs in that they carry tensile loading, they require some means of transferring the load from the support to the body of the spring, and the spring body is wound with an initial tension
- The load transfer can be done with a threaded plug or a swivel hook; both of these add to the cost of the finished product
- Stresses in the body of the extension spring are handled the same as compression springs
- In designing a spring with a hook end, bending and torsion in the hook must be included in the analysis

ME 305 (Machine Design II)

## 10-11 Extension springs...

**Figure 10-2**

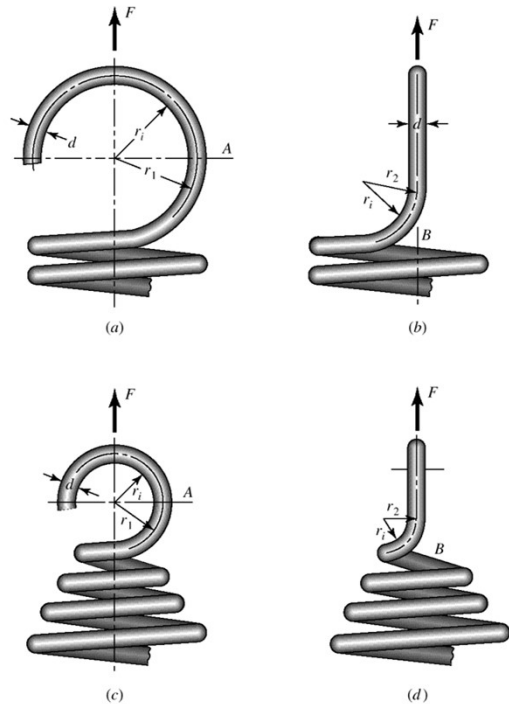
Types of ends used on extension springs. (Courtesy of Associated Spring Corporation.)



ME 305

**Figure 10-3**

Ends for extension springs. (a) Usual design; stress at A is due to combined axial force and bending moment. (b) Side view of part a; stress is mostly torsion at B. (c) Improved design; stress at A is due to combined axial force and bending moment. (d) Side view of part c; stress at B is mostly torsion.



Note: Radius  $r_1$  is in the plane of the end coil for curved beam bending stress. Radius  $r_2$  is at a right angle to the end coil for torsional shear stress.

## 10-11 Extension Springs...

- The maximum tensile stress at A, due to bending and axial loading, is given by

$$\sigma_A = F \left[ (K)_A \frac{16D}{\pi d^3} + \frac{4}{\pi d^2} \right]$$

- Where  $(K)_A$  is a bending stress correction factor for curvature, given by

$$(K)_A = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)} \quad C_1 = \frac{2r_1}{d}$$

- The maximum torsional stress at point B is given by

$$\tau_B = (K)_B \frac{8FD}{\pi d^3}$$

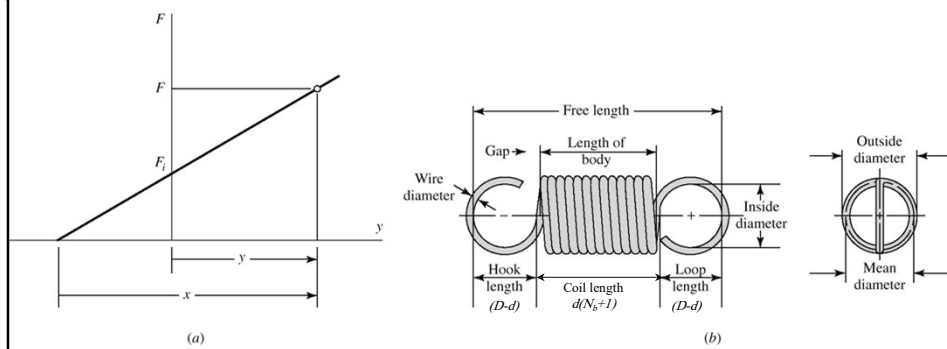
- Where the stress correction factor for curvature,  $(K)_B$  is

$$(K)_B = \frac{4C_2 - 1}{4C_2 - 4} \quad C_2 = \frac{2r_2}{d}$$

ME 305 (Machine Design II)

## 10-11 Extension Springs...

- When extension springs are made with their coils in contact with one another, they are said to be close-wound
- Some manufacturers prefer some initial tension in close-wound springs in order to hold the free length more accurately
- The corresponding load-deflection curve is shown, where  $y$  is the extension beyond the free length  $L_o$  and  $F_i$  is the initial tension in the spring that must be exceeded before the spring deflects



## 10-11 Extension Springs...

- The load-deflection relation is then
 
$$F = F_i + ky$$
- Where  $k$  is the spring rate. The free length  $L_o$  of a spring measured inside the end loops or hook can be expressed as

$$L_o = (2C - 1 + N_b)d$$

- Where  $D$  is the mean coil diameter,  $N_b$  is the number of body coils, and  $C$  is the spring index
- With ordinary twisted end loops, to account for the deflection of the loops in determining the spring rate  $k$ , the equivalent number of active helical turns  $N_a$  is

$$N_a = N_b + \frac{G}{E} \text{ (How? See Problem \#10-38)}$$

- Where  $G$  and  $E$  are the shear and tensile moduli of elasticity, respectively

10-11 Extension Springs...		
<ul style="list-style-type: none"> <li>The initial tension in an extension spring is created in the winding process by twisting the wire as it is wound onto the mandrel</li> <li>When the spring is completed and removed from the mandrel, the initial tension is locked in because the spring cannot get any shorter</li> <li>The preferred range of the initial tension can be expressed as (in terms of torsional stress).</li> </ul> $(\tau_i)_{pref} = \frac{231}{\exp(0.105C)} \pm 6.9 \left( 4 - \frac{C-3}{6.5} \right) MPa$		
		ME 305 (Machine Design II)

10-11 Extension Springs...		
<ul style="list-style-type: none"> <li>Guidelines for the maximum allowable corrected stresses for static applications of extension springs are given</li> </ul>		
		ME 305 (A)

Table 10-21	
Maximum Allowable Stresses ( $K_W$ or $K_B$ corrected) for Helical Extension Springs in Static Applications	
Source: Associated Spring-Barnes Group, Design Handbook, Bristol, Conn., 1987, p. 52.	

Materials	Percent of Tensile Strength		
	In Torsion		In Bending
	Body	End	End
Patented, cold-drawn or hardened and tempered carbon and low-alloy steels	45-50	40	75
Austenitic stainless steel and nonferrous alloys	35	30	55

This information is based on the following conditions: set not removed and low temperature heat treatment applied.  
For springs that require high initial tension, use the same percent of tensile strength as for end.

### Example 10.6

A hard-drawn steel wire extension spring has a wire diameter of  $0.9\text{mm}$ , an outside coil diameter of  $6.3\text{mm}$ , hook radii of  $r_1 = 2.7\text{mm}$  and  $r_2 = 2.3\text{mm}$  and an initial tension of  $5\text{N}$ . The number of body turns is  $12.17$ . From the given information:

- Determine the physical parameters of the spring i.e.  $C$ ,  $K_B$ ,  $N_a$ ,  $k$ ,  $L_0$ ,  $L$
- Check the initial preload stress conditions i.e.  $\tau_i$  should be in the preferred range.
- Find the factors of safety under a static load of  $23\text{N}$  for the
  - Spring
  - Hook at A
  - Hook at B

ME 305 (Machine Design II)

### Solution

(a)

- $C = \frac{D}{d}$
- $K_B = \frac{4C+2}{4C-3}$
- $N_a = N_b + \frac{G}{E}$
- $k = \frac{d^4 G}{8D^3 N_a}$
- $L_0 = (2C - 1 + N_b)d$
- $L = L_0 + y$
- $y = \frac{F - F_i}{k}$

(b)

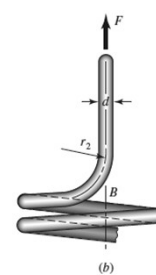
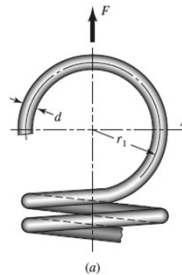
- $\tau_i = \frac{8F_i D}{\pi d^3}$
  - $(\tau_i)_{pref} = \frac{231}{\exp(0.105C)} \pm 6.9(4 - \frac{C-3}{6.5})$
- (C) Hook "at A"
- $n_A = \frac{S_y}{\sigma_A}$
  - $S_y = 0.75S_{ut}$
  - $\sigma_A = F_{max}(K)_A(\frac{16D}{\pi d^3} + \frac{4}{\pi d^2})$
  - $(K)_A = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)}$

ME 305 (Machine Design II)

## Extension Spring Design for Fatigue loading - **Example 10.7**

The helical coil extension spring of Ex. 10–6 is subjected to a dynamic loading from 6.5 to 20 N. Estimate the factors of safety using the Gerber failure criterion for

- Coil fatigue using ,
- Coil yielding,
- End-hook bending fatigue at point A of Fig. 10–6a , and
- End-hook torsional fatigue at point B of Fig. 10–6b .



sign II)

**FED1/FED1+  
Software for Compression Springs**

**FED2/FED2+  
Software for Extension Springs**

(C) Copyright 1990-2007 by HEXAGON Software, Berlin

[http://www.hexagon.de/fed\\_e.htm](http://www.hexagon.de/fed_e.htm)

ME 305 (Machine Design II)





Problems		
<p>Problems</p> <ul style="list-style-type: none"> <li>• 3 (part <i>d</i> omit), 5, 6, 7 (part <i>e</i> omit), 8, 9, 11, 16, 20, 26, 30, 31, 33, 37.</li> </ul> <p>From</p> <p><i>Shigley's Mechanical Engineering Design, 9<sup>th</sup> SI Ed.</i></p>	ME 305 (Machine Design II)	