CH#14
Spur and Helical Gears

ME-305 Machine Design II

14-1 The Lewis Bending Equation

- Wilfred Lewis introduced an equation for estimating the bending stress in gear teeth in 1892
- In figure;

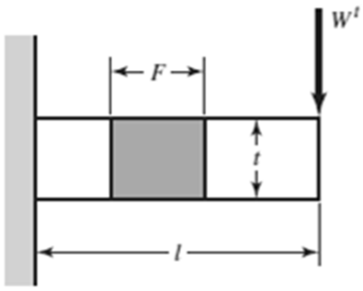
$$I = \frac{1}{12}bh^3 = \frac{1}{12}Ft^3$$

$$c = \frac{t}{2}$$

- Which gives

$$\sigma = \frac{M}{I/c} = \frac{6W^t l}{Ft^2} \rightarrow \text{(a)}$$

- Where *F* is the face width and *t* is the tooth thickness.



(a)

ME-30

14-1 The Lewis Bending Equation…

- Assume stress is maximum at *a*.
  By similar triangle

$$\frac{t/2}{x} = \frac{l}{t/2} \qquad \text{or} \qquad x = \frac{t^2}{4l}$$

- Rearrange (a)

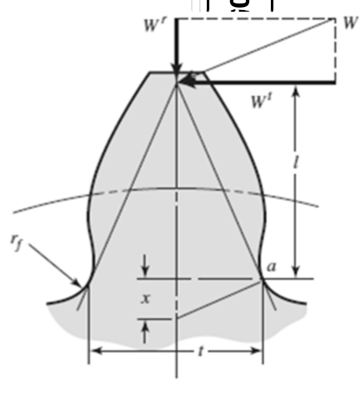
$$\sigma = \frac{6W^t l}{Ft^2} = \frac{W^t}{F}\frac{1}{t^2/6l} = \frac{W^t}{F}\frac{1}{t^2/4l}\frac{1}{\frac{4}{6}}$$

- Put for *x* in (a) and divide & multiply by *p* (circular pitch)

$$\sigma = \frac{W^t p}{F\left(\frac{2}{3}\right)xp}$$

- Let $y = 2x/3p$

$$\sigma = \frac{W^t}{Fpy}$$



(*b*)

ME-30

14-1 The Lewis Bending Equation…

- The factor *y* is called the *Lewis form factor*
- It is obtained by a graphical layout of the gear tooth or by digital computation
- Put $P=\pi/p$ and $Y=\pi y$, we get

$$\sigma = \frac{W^t P}{FY}$$

- Where

$$Y = \frac{2xP}{3}$$

- Only bending is considered
- Compression due to the radial force component is neglected

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14-1 The Lewis Bending Equation…

Table 14–2

Values of the Lewis Form Factor Y (These Values Are for a Normal Pressure Angle of 20°, Full-Depth Teeth, and a Diametral Pitch of Unity in the Plane of Rotation)

Number of Teeth	Y	Number of Teeth	Y
12	0.245	28	0.353
13	0.261	30	0.359
14	0.277	34	0.371
15	0.290	38	0.384
16	0.296	43	0.397
17	0.303	50	0.409
18	0.309	60	0.422
19	0.314	75	0.435
20	0.322	100	0.447
21	0.328	150	0.460
22	0.331	300	0.472
24	0.337	400	0.480
26	0.346	Rack	0.485

14-1 The Lewis Bending Equation…

- Examination of run-in teeth will show that the heaviest loads occur near the middle of the tooth.
- The maximum stress probably occurs while a single pair of teeth is carrying the full load, at a point where another pair of teeth is just on the verge of coming into contact.

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Dynamic effect “K_v”

- At moderate and high speed, noise occurs which is a sign of the dynamic effect.
- Dynamic effect can be incorporate by increasing the gear load by a factor.
- The factor is K_v and is calculate by the empirical relations as shown on the next slide.
- Equation of K_v is determined experimentally.
- If a pair of gear fails at 2 kN tangential load at 0 velocity and the identical pair of gear fails at 1 kN at V_1 velocity, then K_v = 2

Dynamic effect “K_v”…

- In SI units

$$K_v = \frac{3.05 + V}{3.05} \qquad \text{(cast iron, cast profile)}$$

$$K_v = \frac{6.1 + V}{6.1} \qquad \text{(cut or milled profile)}$$

$$K_v = \frac{3.56 + \sqrt{V}}{3.56} \qquad \text{(hobbed or shaped profile)}$$

$$K_v = \sqrt{\frac{5.56 + \sqrt{V}}{5.56}} \qquad \text{(shaved or ground profile)}$$

- Where V is in m/s
- Introducing K_v into the Lewis equation

$$\sigma = \frac{K_v W^t P}{FY} \quad \text{or} \quad \sigma = \frac{K_v W^t}{FmY}$$

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Example 14-1

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- A stock spur gear is available having a module of 3 mm, a 38mm face, 16 teeth, and a pressure angle of 20◦ with full-depth teeth. The material is AISI 1020 steel in as-rolled condition. Use a design factor of n_d = 3 to rate the power output of the gear corresponding to a speed of 1200 rev/m and moderate applications.

Solution

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$$H = \frac{\pi W^t d n}{60000} = \mathbf{4.75kW}$$

  - *H* is in kW
  - W^t is in kN
  - *d* is in mm
  - *n* is in rpm
- and

$$W^t = \frac{mFY\sigma_{all}}{K_v}$$

  - *m* is module in mm
  - *F* is face width in mm
  - *Y* is form factor determined from Table 14-2
  - σ_{all} is 210 MPa from Table A-20. With n = 3, $\sigma_{all} = \frac{210}{3} = 70\ \text{Mpa}$
  - Use equation for K_v for milled.

14-3 AGMA Stress equation/Analysis

- Only spur gear will be analysed
- Gear bending stress equation "σ"

$$W^t = \frac{1000\ H}{V}$$

$$\sigma = W^t K_O K_V K_S \frac{1}{bm} \frac{K_H K_B}{Y_J}$$

1 [or Eq. (a), Sec. 14–10]
Eq. (14–30)
Eq. (14–40)
Fig. 14–6
module
Eq. (14–27)
Table below

SI units

$$W^t = \frac{33\,000\ H}{V}$$

$$\sigma = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}$$

1 [or Eq. (a), Sec. 14–10]; p. 759
Eq. (14–30); p. 759
Eq. (14–40); p. 764
Fig. 14–6; p. 753
Eq. (14–27); p. 756
Table below

US customary units

- Bending factor of safety "S_F"

$$S_F = \frac{\sigma_{FP} Y_N/(Y_\theta Y_Z)}{\sigma}$$

SI units

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma}$$

US customary units

14-3 AGMA Stress equation/Analysis…

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- The endurance strength "σ_{all}" equation

Tables 14–3, 14–4

$$\sigma_{\text{all}} = \frac{\sigma_{FP}}{S_F} \frac{Y_N}{Y_\theta Y_Z}$$

Fig. 14–14
Eq. (14–38)
1 if $T < 120°\text{C}$

SI units

$_{0.99}(S_t)_{10^7}$ Tables 14–3, 14–4; pp. 748, 749

$$\sigma_{\text{all}} = \frac{S_t}{S_F} \frac{Y_N}{K_T K_R}$$

Fig. 14–14; p. 763
Table 14–10, Eq. (14–38); pp. 763, 764
1 if $T < 250°\text{F}$

US customary units

14-3 AGMA Stress equation/Analysis…

- Gear Contact stress "σ_c" equation (Wear)

$$W^t = \frac{1000\,\mathrm{H}}{V}$$

$$\sigma_c = Z_E \left(W^t K_O K_v K_S \frac{K_H}{d_{w1} b} \frac{Z_R}{Z_I} \right)^{1/2}$$

$$Z_I = \frac{\cos\phi_t \sin\phi_t}{2} \frac{m_G}{m_G \pm 1}$$

1 [or Eq. (a), Sec. 14–10]
Eq. (14–30)
1
Table 14–8
Eq. (14–27)
Table below

SI units

$$W^t = \frac{33\,000\,\mathrm{H}}{V}$$

$$\sigma_c = C_p \left(W^t K_o K_v K_s \frac{K_m}{d_P F} \frac{C_f}{I} \right)^{1/2}$$

1 [or Eq. (a), Sec. 14–10]; p. 759
Eq. (14–30); p. 759
1
Eq. (14–23); p. 755
Table 14–8; pp. 744, 757
Eq. (14–27); p. 756
Table below

US customary units

- Wear factor of safety "S_H"

$$S_H = \frac{\sigma_{HP} Z_N Z_W / (Y_\theta Y_Z)}{\sigma_c}$$

SI units

$$S_H = \frac{S_c Z_N C_H / (K_T K_R)}{\sigma_c}$$

Gear only

US customary units

14-3 AGMA Stress equation/Analysis…

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- The gear contact endurance strength "$\sigma_{c,\mathrm{all}}$" equation is

$$\sigma_{c,\mathrm{all}} = \frac{\sigma_{HP} Z_N Z_W}{S_H Y_\theta Y_Z}$$

Tables 14–6, 14–7
Fig. 14–15
Section 14–12, gear only
Eqs. (14–38)
1 if $T < 120°\mathrm{C}$

SI units

$$\sigma_{c,\mathrm{all}} = \frac{S_c Z_N C_H}{S_H K_T K_R}$$

$0.99(S_c)_{10^7}$ Tables 14–6, 14–7; pp. 751, 752
Fig. 14–15; p. 763
Section 14–12, gear only; pp. 761, 762
Table 14–10, Eq. (14–38); pp. 763, 764
1 if $T < 250°\mathrm{F}$

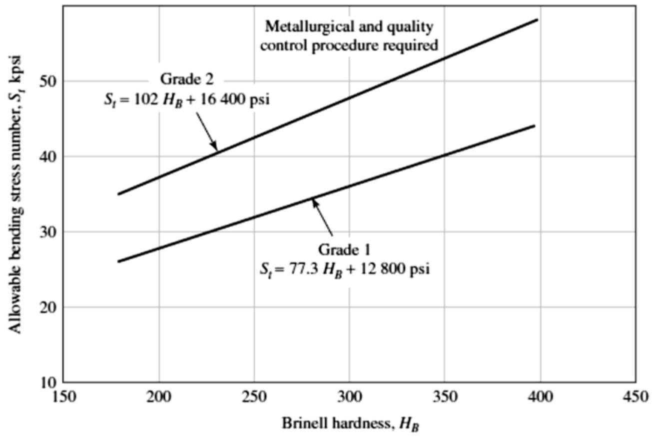
US customary units

Allowable bending stress σ_{FP} (S_t)

- For steel, from figures 14-2, 14-3, 14-4
- For other materials, refer to Table 14-3 & 14-4

Figure 14–2

Allowable bending stress number for through-hardened steels. The SI equations are $S_t = 0.533H_B + 88.3$ MPa, grade 1, and $S_t = 0.703H_B + 113$ MPa, grade 2. *(Source: ANSI/AGMA 2001-D04 and 2101-D04.)*

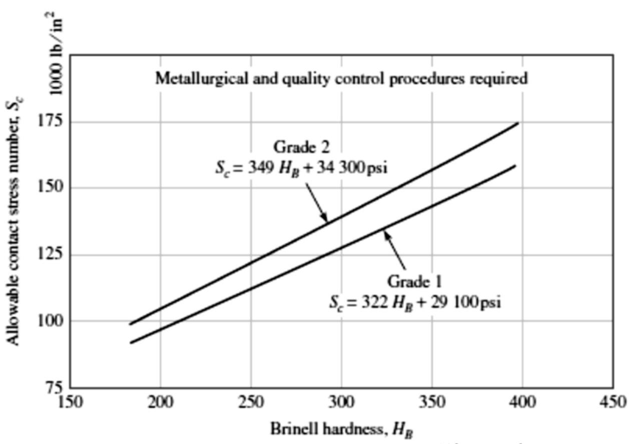


Allowable contact stress σ_{HP} (S_c)

- For through hardened steel use Fig. 14-5
- For all other, use Table 14-6 and 14-7

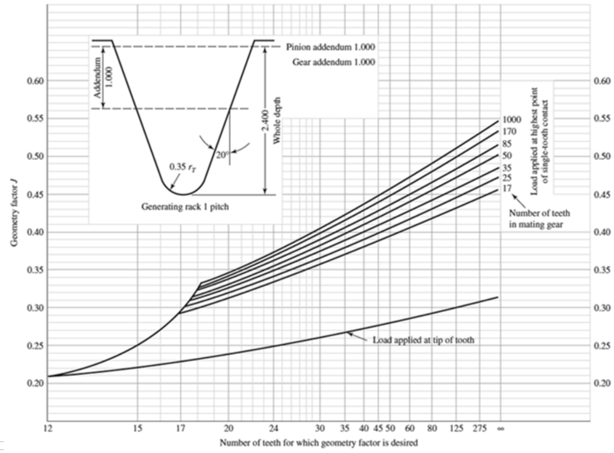
Figure 14–5

Contact-fatigue strength S_c at 10^7 cycles and 0.99 reliability for through-hardened steel gears. The SI equations are $S_c = 2.22H_B + 200$ MPa, grade 1, and $S_c = 2.41H_B + 237$ MPa, grade 2. *(Source: ANSI/AGMA 2001-D04 and 2101-D04.)*



14-5 Geometry Factors Y_J *(J)* and Z_I (*I*)

- Y_J (*I*) is the bending-strength geometry factor used in the bending-stress equation



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14-5 Geometry Factors Y_J *(J)* and Z_I (*I*)…

- Z_I (*I*) is the surface-strength geometry factor used in the contact-stress equation
- Can be determined as (for external gear)

$$Z_I = I = \left[\frac{cos\emptyset\ sin\emptyset}{2M_N} + \frac{m_G}{m_G + 1}\right]$$

- Where
  - M_N = 1 for spur gear
  - m_G is the gear ratio

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14-6 The Elastic Coefficient Z_E (C_p)

- The elastic coefficient is calculated from Table 14-8

Table 14-8

Elastic Coefficient C_p (Z_E), $\sqrt{\text{psi}}$ ($\sqrt{\text{MPa}}$) *Source: AGMA 218.01*

Pinion Material	Pinion Modulus of Elasticity E_p psi (MPa)*	Gear Material and Modulus of Elasticity E_G, lbf/in² (MPa)*: Steel 30×10^6 (2×10^5)	Malleable Iron 25×10^6 (1.7×10^5)	Nodular Iron 24×10^6 (1.7×10^5)	Cast Iron 22×10^6 (1.5×10^5)	Aluminum Bronze 17.5×10^6 (1.2×10^5)	Tin Bronze 16×10^6 (1.1×10^5)
Steel	30×10^6 (2×10^5)	2300 (191)	2180 (181)	2160 (179)	2100 (174)	1950 (162)	1900 (158)
Malleable iron	25×10^6 (1.7×10^5)	2180 (181)	2090 (174)	2070 (172)	2020 (168)	1900 (158)	1850 (154)
Nodular iron	24×10^6 (1.7×10^5)	2160 (179)	2070 (172)	2050 (170)	2000 (166)	1880 (156)	1830 (152)
Cast iron	22×10^6 (1.5×10^5)	2100 (174)	2020 (168)	2000 (166)	1960 (163)	1850 (154)	1800 (149)
Aluminum bronze	17.5×10^6 (1.2×10^5)	1950 (162)	1900 (158)	1880 (156)	1850 (154)	1750 (145)	1700 (141)
Tin bronze	16×10^6 (1.1×10^5)	1900 (158)	1850 (154)	1830 (152)	1800 (149)	1700 (141)	1650 (137)

Poisson's ratio = 0.30.

*When more exact values for modulus of elasticity are obtained from roller contact tests, they may be used.

14-7 Dynamic factor K_v

- used to account for inaccuracies in the manufacture and meshing of gear teeth in action.
- Inaccuracies produced in the generation of the tooth profile; these include errors in tooth spacing, profile lead, and runout
- Vibration of the tooth during meshing due to the tooth stiffness
- Magnitude of the pitch-line velocity
- Dynamic unbalance of the rotating members
- Wear and permanent deformation of contacting portions of the teeth
- Gear-shaft misalignment and the linear and angular deflection of the shaft
- Tooth friction

$$K_v = \left(\frac{A+\sqrt{200V}}{A}\right)^B, \ V \text{ in m/s}$$

$$B = 0.25(12 - Q_v)^{2/3}$$

$$A = 50 + 56(1 - B)$$

14-8 Overload factor K_0

- is intended to make allowance for all externally applied loads in excess of the nominal tangential load W_t in a particular application
- Examples include variations in torque from the mean value due to firing of cylinders in an internal combustion engine or reaction to torque variations in a piston pump drive. There are other similar factors such as application factor or service factor.
- These factors are established after considerable field experience in a particular application

Table of Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

14-9 Surface Condition factor Z_R (C_f)

- It depends on
  - Surface finish affected by cutting, shaving, lapping, grinding, shot peening.
  - Residual stresses
  - Work hardening
  - The value should be greater than unity if desired.
  - For normal condition use $Z_R = C_f = 1$

14-10 Size factor K_s

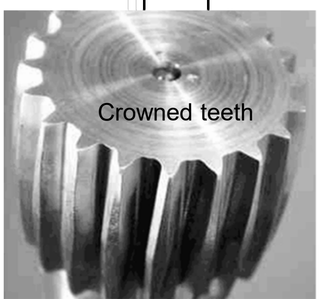
- The size factor reflects non-uniformity of material properties due to size. It depends upon
  - Tooth size
  - Diameter of part
  - Ratio of tooth size to diameter of part
  - Face width
  - Area of stress pattern
  - Ratio of case depth to tooth size
  - Hardenability and heat treatment

$$K_s = \frac{1}{k_b} = 0.904\left(bm\sqrt{Y}\right)^{0.0535} \text{ (SI units)}$$

$$K_s = \frac{1}{k_b} = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535} \text{ (US customary units)}$$

- If K_s is less than 1 use equal to 1.

14-11 Load distribution factor K_H (K_m)



- The load-distribution factor modifies the stress equations to reflect non-uniform distribution
- $K_H = K_m$ depends on different constants like

$$\boxed{K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)}$$

- where

$$\boxed{C_{mc} = \begin{cases} 1 & \text{for uncrowned teeth} \\ 0.8 & \text{for crowned teeth} \end{cases}}$$

- and C_{pf} is (in SI Units)

$$C_{pf} = \begin{cases} \dfrac{b}{10d} - 0.025 & b \le 25mm \\ \dfrac{b}{10d} - 0.0375 + (4.92 \times 10^{-4})b & 25 < b \le 425mm \\ \dfrac{b}{10d} - 0.1109 + (8.15 \times 10^{-4})b - (3.53 \times 10^{-7})b^2 & 425 < b \le 1000mm \end{cases}$$

- Note that $\frac{b}{10d} = 0.05$ if $\frac{b}{10d} < 0.05$

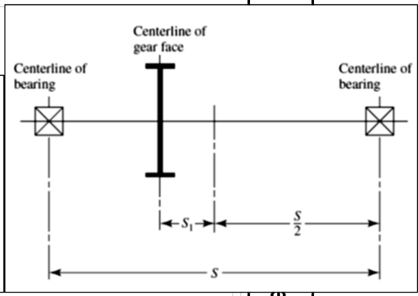
14-11 Load distribution factor K_H (K_m)…

– where

$$C_{pm} = \begin{cases} 1 & \text{for straddle-mounted pinion with } S_1/S < 0.175 \\ 1.1 & \text{for straddle-mounted pinion with } S_1/S \ge 0.175 \end{cases}$$

$$C_{ma} = A + BF + CF^2 \qquad \text{(see Table 14–9 for values of } A, B, \text{ and } C)$$

$$C_e = \begin{cases} 0.8 & \text{for gearing adjusted at assembly, or compatibility is improved by lapping, or both} \\ 1 & \text{for all other conditions} \end{cases}$$



Condition	A	B	C
Open gearing	0.247	0.0167	$-0.765(10^{-4})$
Commercial, enclosed units	0.127	0.0158	$-0.930(10^{-4})$
Precision, enclosed units	0.0675	0.0128	$-0.926(10^{-4})$
Extraprecision enclosed gear units	0.00360	0.0102	$-0.822(10^{-4})$

*See ANSI/AGMA 2101-D04, pp. 20–22, for SI formulation.

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14-12 Hardness Ratio factor Z_w (C_H)

- $Z_w = C_H = 1$ for pinion
- It is used only for the gear
- It is used if the hardness of the mating gears is not the same as pinion.
- The value is obtained as;

$$Z_w = C_H = 1.0 + A'(m_G - 1.0)$$

- Where
  - $A' = 8.98 \times 10^{-3}\left(\frac{BH_P}{BH_G}\right) - 8.29 \times 10^{-3}$ if $1.2 \le \frac{BH_P}{BH_G} \le 1.7$
  - $A' = 0$ if $\frac{BH_P}{BH_G} < 1.2$
  - $A' = 0.00689$ if $\frac{BH_P}{BH_G} > 1.7$
- When Pinion of 48 Rockwell hardness is run with 180-400 BH gear, then use equation 14-37.

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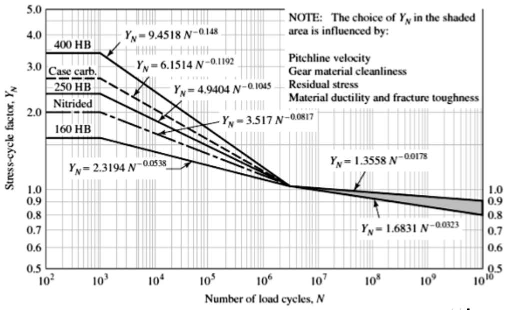
14-13 Stress cycle factors

- Y_N = Bending stress cycle factor
  - The AGMA strengths as given in Figs. 14–2 through 14–4, in Tables 14–3 and 14–4 for bending fatigue, and in Fig. 14–5 and Tables 14–5 and 14–6 for contact-stress fatigue are based on 10^7 load cycles applied.
  - The purpose of the load cycle factor Y_N is to modify the gear strength for lives other than 10^7 cycles.
  - Y_N =1 for 10^7

Figure 14–14
Repeatedly applied bending strength stress-cycle factor Y_N. *(ANSI/AGMA 2001-D04.)*

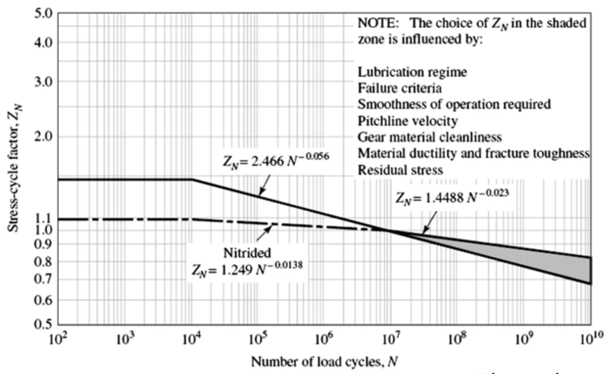


ME-305 Machine

14-13 Stress cycle factors...

- Z_N = Stress cycle factor for pitting resistance

Figure 14–15
Pitting resistance stress-cycle factor Z_N. *(ANSI/AGMA 2001-D04.)*



ME-

14-14 Reliability factor Y_Z (K_R)

- It can be determined as

$$K_R = \begin{cases} 0.658 - 0.0759 \ln(1 - R) & 0.5 < R < 0.99 \\ 0.50 - 0.109 \ln(1 - R) & 0.99 \leq R \leq 0.9999 \end{cases}$$

- Or from table

Table 14–10
Reliability Factors $K_R (Y_Z)$
Source: ANSI/AGMA 2001-D04.

Reliability	K_R (Y_Z)
0.9999	1.50
0.999	1.25
0.99	1.00
0.90	0.85
0.50	0.70

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14-15 Temperature factor Y_θ (K_t)

- Use $Y_\theta = K_t = 1$ for temperature up to 120°C
- Use larger than 1 if temperatrue is large.

ME-305 Machine Design II

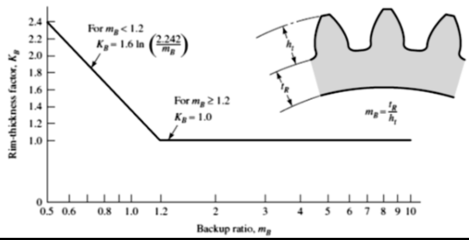
14-16 Rim thickness factor K_B

- When the rim thickness is not sufficient to provide full support for the tooth root, the location of bending fatigue failure may be through the gear rim rather than at the tooth fillet. In such cases, the use of a stress-modifying factor K_B or (t_R) is recommended.

$$K_B = \begin{cases} 1.6 \ln \dfrac{2.242}{m_B} & m_B < 1.2 \\ 1 & m_B \geq 1.2 \end{cases} \qquad m_B = \frac{t_R}{h_t}$$

- It is a function of the backup ratio m_B,

Figure 14–16
Rim-thickness factor K_B.
(ANSI/AGMA 2001-D04.)



Example 14-4 (in SI units)

A 17-tooth 20° pressure angle spur pinion rotates at 1800 rev/min and transmits 3 kW to a 52-tooth disk gear. The module is 2.5 mm, the face width is 38 mm, and the quality standard is No. 6. The gears are straddle-mounted with bearings immediately adjacent. The pinion is a grade 1 steel with a hardness of 240 Brinell tooth surface and through-hardened core. The gear is steel, through-hardened also, grade 1 material, with a Brinell hardness of 200, tooth surface and core. The geometry factor J_P = 0.30 and J_G = 0.40. The loading is smooth because of motor and load. Assume a pinion life of 10^8 cycles and a reliability of 0.90, and use $Y_N = 1.6831(N)^{-0.0323}$. The tooth profile is uncrowned. This is a commercial enclosed gear unit.

a) Find the factor of safety of the gears in bending.
b) Find the factor of safety of the gears in wear.
c) By examining the factors of safety, identify the threat to each gear and to the mesh.

Solution (in SI units)

(a) Based on bending Stress *(Steps)*

- FOS is given by; $(S_F)_i = \left(\frac{\sigma_{FP} Y_N / (Y_\theta Y_Z)}{\sigma}\right)_i \rightarrow (1)$ (*i = P or G*)
- and $\sigma_i = \frac{K_o K_v K_s K_H K_B}{Y_J} \times \frac{W^t}{m \times b} \rightarrow (2)$

Where;

- $W^t = \frac{60000 \times H}{\pi d n} = \frac{60000 \times H}{\pi \times m \times N \times n} = \frac{60000 \times 3}{\pi \times 2.5 \times 17 \times 1800} = \mathbf{0.75\ kN}$
- Loading factor; $\boldsymbol{K_o)_P} = \boldsymbol{K_o)_G} = \mathbf{1}$ (as the loading is smooth)
- Dynamic factor; $K_v)_i = \left(\frac{A + \sqrt{200V}}{A}\right)^B \rightarrow (3)$
- $B = \frac{(12 - Q_v)^{2/3}}{4} = \frac{(12-6)^{2/3}}{4} = 0.826$
- $A = 50 + 56(1 - B) = 59.74$
- (3) becomes; $\boldsymbol{K_v)_p} = \boldsymbol{K_v)_G}$ **= 1.37**

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Solution…

Steps…

- Size Factor; $\boldsymbol{K_s)_P} = 0.904\left(mb\sqrt{Y_P}\right)^{0.0535} = 0.904\left(2.5 \times 38\sqrt{0.303}\right)^{0.0535} = \mathbf{1.117}$

  $\boldsymbol{K_s)_G} = 0.904\left(mb\sqrt{Y_G}\right)^{0.0535} = 0.904\left(2.5 \times 38\sqrt{0.412}\right)^{0.0535} = \mathbf{1.126}$
- Load distribution factor; $K_H = 1 + C_{mc}\left(C_{Pf} C_{pm} + C_{ma} C_e\right) \rightarrow (4)$

Where

- $C_{mc} = 1$ (as the tooth is uncrowned)
- $C_{pf} = \frac{b}{10d} - 0.0375 + (4.92 \times 10^{-4})b = 0.1456$ (for b = 38 mm, and, use same value of $\frac{b}{10d}$ as $\frac{b}{10d} > 0.05$ i.e $\frac{b}{10d} = \frac{38}{10 \times 42.5} = 0.089 > 0.05$)
- $C_{Pm} = 1$ (as the bearings are immediately installed, $\frac{S_t}{S} < 0.175$)
- $C_{ma} = A + BF + CF^2 \rightarrow (5)$
- *A* = 0.127, *B* = 0.0158, *C* = -0.93E-4 (Commercial unit, Table 14-9)

Solution…

Steps…

- (5) becomes; C_{ma} = 0.1276
- C_e = 1 (as no information about assembly/condition/compatibility)
- (4) becomes; $\boldsymbol{K_H)_p} = \boldsymbol{K_H)_G}$ **= 1.273**
- Geometry factor; $\boldsymbol{Y_J)_P}$ **= 0.30** and $\boldsymbol{Y_J)_G}$ **= 0.04** (Given)
- Allowable bending Strength; $\sigma_{FP})_i = 0.533H_B + 88.3 = 216 \text{ Mpa}$

$$\left.\begin{aligned}\sigma_{FP})_P &= 0.533 \times 240 + 88.3 = 216 \text{ Mpa}\\ \sigma_{FP})_G &= 0.533 \times 200 + 88.3 = 195 \text{ Mpa}\end{aligned}\right\}$$

For Grade 1 steel
Table 14-2

- Stress Cycle factor; $Y_N)_P = 1.6831N^{-0.0323} = 1.6831(10^8)^{-0.0323}$

$$Y_N)_P = 0.9283$$

$$Y_N)_G = 1.6831\left(\frac{N}{m_G}\right)^{-0.0323} \rightarrow (6)$$

Solution…

Steps…

- Where m_G is the gear ratio and is given by; $m_G = \frac{N_G}{N_P} = \frac{52}{17} = 3.06$
- (6) becomes; $\boldsymbol{Y_N)_G} = 1.6831\left(\frac{10^8}{3.06}\right)^{-0.0323} = \mathbf{0.9625}$
- Temperature factor; $\boldsymbol{Y_\theta}$ ***= 1*** (Room temperature)
- Reliability factor; $\boldsymbol{Y_Z)_P} = \boldsymbol{Y_Z)_G} = \mathbf{0.85}$ (From Table 14-10)
- Put values in (2) and (1), to get;

$$\sigma_P = \frac{0.75 \times 10^3}{2.5 \times 38} \times \frac{1 \times 1.377 \times 1.117 \times 1 \times 1.273}{0.3} = 51.5 \text{ Mpa}$$

$$S_F)_P = \frac{(216 \times 0.9283) \big/ (1 \times 0.85)}{51.5} = \mathbf{4.6}$$

Solution…

Steps…

- For Gear

$$\sigma_G = \frac{0.75 \times 10^3}{2.5 \times 38} \times \frac{1 \times 1.377 \times 1.126 \times 1 \times 1.273}{0.4} = 38.6 \text{ Mpa}$$

$$S_F)_G = \frac{(195 \times 0.9625)\big/(1 \times 0.85)}{38.6} = \mathbf{5.72}$$

- Since Factors of Safety are greater than 1, the gear set is safe based on ***Bending Stress.***

Solution…

(b) Based on Wear *(Steps)*

- FOS is given by; $(S_H)_i = \left(\frac{\sigma_{HP} Z_N Z_W / (Y_\theta Y_Z)}{\sigma_c}\right)_i \rightarrow (1)$

$$\sigma_c)_i = C_p \left(\frac{K_o K_v K_s K_H Z_R}{Z_I} \times \frac{W^t}{d_p \times b}\right)_i^{1/2} \rightarrow (2)$$

- Contact Fatigue Strength;

$$\sigma_{HP})_p = 2.22BH + 200 \text{ MPa} = 732.8 \text{ Mpa}$$
$$\sigma_{HP})_G = 2.22BH + 200 \text{ MPa} = 644 \text{ Mpa}$$

- Pitting resistance stress-cycle factor

$$Z_N)_p = 1.4488(10^8)^{-0.023} = 0.948$$

$$Z_N)_G = 1.4488\left(\frac{N}{m_G}\right)^{-0.023} = 1.4488\left(\frac{10^8}{3.06}\right)^{-0.023} = 0.973$$

Solution…

(b) Based on Wear *(Steps)*

- Hardness ratio factor

  $Z_w)_p = 1$ (For pinion, always equal to 1)

  $Z_w)_G = 1.005$ (after using eqn. 14-36)

- Surface condition factor

  $Z_R = 1$ (for normal condition use always 1)

- Surface strength geometry factor

  $$Z_I = \frac{cos20^o \times sin20^0}{2 \times 1}\left(\frac{3.06}{3.06 + 1}\right) = 0.121$$

- Elastic coefficient factor

  $C_p = 191\sqrt{MPa}$ (from Table 14-8)

Solution…

ME-305 Machine Design II

- (2) becomes

  $\boldsymbol{\sigma_c)_p = 522}$ **Mpa**

- (1) becomes

  $\boldsymbol{S_H)_p = 1.61}$

- Adopt similar procedure to calculate $\boldsymbol{S_H)_G = 1.5}$
- Since Factors of Safety are greater than 1, the gear set is safe based on ***Wear.***

(c) Threat to the gear set

- Compare S_F to $(S_H)^2$ for the pinion, i.e.

  $4.6 > (1.61)^2 = 4.6 > 2.6$

- So threat to the gear set is due to ***Pinion Wear.***

Example 14-5 (Helical gear set)

- Several parameters in this example are the same as in Example 14-4, with the exception that we are using helical gears.
- For the same loading conditions, the fos are larger for Helical gears as compared to spur gears. Why?

(Home task)

Problems

- 14-2, 14-4, 14-5, 14-9, 14-10
- 14-19, 14-22, 14-24, 14-27, 14-28

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