# (3) Some Statistical tests in Minitab

The are some statistical tests available in MINITAB, we will show some of such tests, namely;

- One-sample z-test
- One-sample t-test
- Two-sample t-test
- Paired-sample t-test
- One-sample proportion
- Two-sample-proportion
- One-sample variance test
- Two-sample variances test

# (3.1) 1-sample Z test

To perform this test, select

### **Stat > Basic Statistics > 1-Sample Z**

Use the 1-sample Z-test to estimate the mean of a population and compare it to a target or reference value when you know the standard deviation of the population. Using this test, you can:

- Determine whether the mean of a group differs from a specified value.
- Calculate a range of values that is likely to include the population mean.

For example, you take a sample of pencil stock and you want to know if the machine that cuts them to length has drifted from its intended settings.

This procedure is based on the normal distribution. So for small samples, this procedure works best if your data were drawn from a normal distribution or one

that is close to normal. Because of the central limit theorem, you can use this procedure if you have a large sample, substituting the sample standard deviation for  $\sigma$ .

For 1-Sample Z, the hypotheses are:

#### **Null hypothesis**

Η <sub>0</sub> : μ = μ <sub>0</sub>	The population mean ( $\mu$ ) equals the hypothesized mean ( $\mu_0$ ).
Πο. μ — μο	

#### **Alternative hypothesis**

#### Choose one:

H₁: μ ≠ μ₀	The population mean ( $\mu$ ) differs from the hypothesized mean ( $\mu$ <sub>0</sub> ).
H <sub>1</sub> : μ > μ <sub>0</sub>	The population mean $(\mu)$ is greater than the hypothesized mean $(\mu_0)$ .
H <sub>1</sub> : μ < μ <sub>0</sub>	The population mean $(\mu)$ is less than the hypothesized mean $(\mu_0)$ .

In general, the test can be done through 4 steps

### **Step-1**: Setup the hypotheses

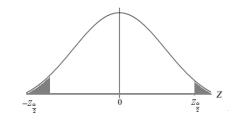
$$H_0$$
:  $\mu = \mu_0$  vs  $H_1$ :  $\mu \neq \mu_0$  ( $\mu > \mu_0$  or  $H_1$ :  $\mu < \mu_0$ )

### **Step-2**: Test Statistic

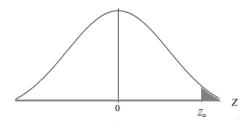
$$Z_0 = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

### **Step-3**: The critical region(s)

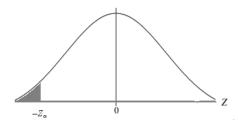
- when  $H_1$ :  $\mu \neq \mu_0$ 



- when  $H_1: \mu > \mu_0$ 



- when  $H_1$ :  $\mu \le \mu_0$ 



**Step-4**: Decision: When the calculated statistics Z0 belongs under the shaded areas, we reject H0, otherwise, we cannot reject H0.

Or one can use p-value approach (reject H0 in p-value  $\leq \alpha$ )

### Example:

Measurements were made on nine widgets. You know that the distribution of measurements has historically been close to normal with  $\sigma = 0.2$ . Because you know  $\sigma$ , and you wish to test if the population mean is 5 and obtain a 90% <u>confidence interval</u> for the mean, you use the Z-procedure.

- 1 Open the worksheet EXH STAT.MTW.
- 2 Choose Stat > Basic Statistics > 1-Sample Z.
- 3 In Samples in columns, enter Values.
- 4 In **Standard deviation**, enter 0.2.
- 5 Check Perform hypothesis test. In Hypothesized mean, enter 5.
- 6 Click Options. In Confidence level, enter 90. Click OK.
- 7 Click **Graphs**. Check **Individual value plot**. Click **OK** in each dialog box.

Session window output

One-Sample Z: Values

Test of mu = 5 vs not = 5

The assumed standard deviation = 0.2

Variable N Mean StDev SE Mean 90% CI Z P

Values 9 4.7889 0.2472 0.0667 (4.6792, 4.8985) -3.17 0.002

From the results, we can write down the test steps as:

**Step-1**: Setup the hypotheses

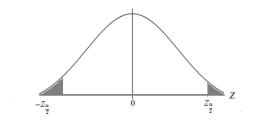
 $H_0\colon \mu=5 \quad vs \qquad \quad H_1\colon \mu\neq 5$ 

**Step-2**: Test Statistic

$$Z_0 = \frac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} = -3.17$$

### **Step-3**: The critical region(s)

- when  $H_1$ :  $\mu \neq \mu_0$ 



#### where

$$-Z_{\alpha/2}=-1.96$$
 and  $Z_{\alpha/2}=1.96$ 

Step-4: Decision: reject H0.

On the other hand since p-value =0.002  $\leq \alpha = 0.1$ ), the reject H0

# 3.2) 1-sample t test

To perform this test, select

## **Stat > Basic Statistics > 1-Sample t**

Use the 1-sample t-test to estimate the mean of a population and compare it to a target or reference value when you do not know the standard deviation of the population. Using this test, you can:

- Determine whether the mean of a group differs from a specified value.
- Calculate a range of values that is likely to include the population mean.

#### For 1-Sample t, the hypotheses are:

#### **Null hypothesis**

H <sub>0</sub> : μ = μ <sub>0</sub>	The population mean ( $\mu$ ) equals the hypothesized mean ( $\mu$ <sub>0</sub> ).
-------------------------------------	--

#### **Alternative hypothesis**

Choose one:

H₁: μ ≠ μ₀	The population mean ( $\mu$ ) differs from the hypothesized mean ( $\mu_0$ ).
H <sub>1</sub> : μ > μ <sub>0</sub>	The population mean $(\mu)$ is greater than the hypothesized mean $(\mu_0)$ .
H <sub>1</sub> : μ < μ <sub>0</sub>	The population mean $(\mu)$ is less than the hypothesized mean $(\mu_0)$ .

In general, the test can be done through 4 steps

**Step-1**: Setup the hypotheses

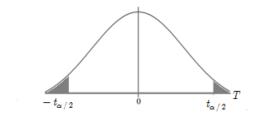
$$H_0\text{: }\mu = \mu_0 \quad vs \qquad \quad H_1\text{: }\mu \neq \mu_0 \quad (\ \mu \geq \mu_0 \text{ or } H_1\text{: }\mu \leq \mu_0 \ )$$

**Step-2**: Test Statistic

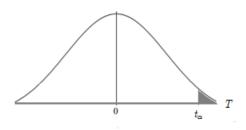
$$T_0 = \frac{\bar{X} - \mu_0}{s / \sqrt{n}}$$

**Step-3**: The critical region(s)

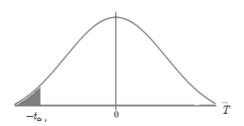
- when  $H_1$ :  $\mu \neq \mu_0$ 



- when  $H_1$ :  $\mu > \mu_0$ 



- when  $H_1$ :  $\mu \le \mu_0$ 



**Step-4**: Decision: When the calculated statistics T0 belongs under the shaded areas, we reject H0, otherwise, we cannot reject H0.

Or one can use p-value approach (reject H0 in p-value  $\leq \alpha$ )

## **Example**

In the previous example, suppose that you do not know  $\sigma$ . To test if the population mean is 5 and to obtain a 90% confidence interval for the mean, you use a t-procedure.

- 1 Open the worksheet EXH STAT.MTW.
- 2 Choose Stat > Basic Statistics > 1-Sample t.
- 3 In Samples in columns, enter Values.
- 4 Check Perform hypothesis test. In Hypothesized mean, enter 5.
- 5 Click Options. In Confidence level, enter 90. Click OK in each dialog box.

Session window output

One-Sample T: Values

Test of mu = 5 vs not = 5

Variable N Mean StDev SE Mean 90% CI T P

Values 9 4.7889 0.2472 0.0824 (4.6357, 4.9421) -2.56 0.034

From the results, we can write down the four steps as follows:

**Step-1**: Setup the hypotheses

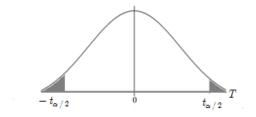
$$H_0$$
:  $\mu = 5$  vs  $H_1$ :  $\mu \neq 5$ 

Step-2: Test Statistic

$$T_0 = \frac{\bar{X} - \mu_0}{s / \sqrt{n}} = -2.56$$

**Step-3**: The critical region(s)

- when  $H_1$ :  $\mu \neq 5$ 



Where at 49 degrees of fredom we have  $-t_{\alpha/2}=-t_{0.05}=-1.86$ 

**Step-4**: Decision: When the calculated statistics T0 belongs under the shaded areas, we reject H0

Or one can use p-value approach (reject H0 in p-value=.0.034  $\leq \alpha = 0.1$ )

A 90% confidence interval for the population mean,  $\mu$ , is (4.6357,4.9421). This interval is slightly wider than the

# 3.3) 2-sample t test

To perform this test, select

### **Stat > Basic Statistics > 2-Sample t**

Use the 2-sample t-test to two compare between two population means, when the variances are unknowns

#### For 1-Sample t, the hypotheses are:

#### **Null hypothesis**

 $H_0$ :  $\mu_1 = \mu_2$  The population means ( $\mu_1$  and  $\mu_2$ ) are equal

#### **Alternative hypothesis**

#### Choose one:

H₁: μ₁ ≠ μ₂	The population mean ( $\mu$ ) differs from the hypothesized mean ( $\mu$ 0).
H <sub>1</sub> : μ <sub>1</sub> > μ <sub>2</sub>	The population mean $(\mu_1)$ is greater than the mean $(\mu_2)$ .
H <sub>1</sub> : μ <sub>1</sub> < μ <sub>2</sub>	The population mean $(\mu_1)$ is less than the mean $(\mu_2)$ .

In general, the test can be done through 4 steps, or p-values approach

#### **Step-1**: Setup the hypotheses

$$H_0: \mu_1 = \mu_2 \text{ vs } H_1: \mu_1 \neq \mu_2$$

Step-2: p-value = ?, then (reject H0 in p-value  $\leq \alpha$ )

#### Example:

A study was performed in order to evaluate the effectiveness of two devices for improving the efficiency of gas home-heating systems. Energy consumption in houses was measured after one of the two devices was installed. The two devices were an electric vent damper (Damper=1) and a thermally activated vent damper (Damper=2). The energy consumption data (BTU.In) are stacked in one column with a grouping column (Damper) containing identifiers or subscripts to denote the population. Suppose that you performed a variance test and found no evidence for variances being unequal.

Now you want to compare the effectiveness of these two devices by determining whether or not there is any evidence that the difference between the devices is different from zero.

- 1 Open the worksheet FURNACE.MTW.
- 2 Choose Stat > Basic Statistics > 2-Sample T.
- 3 Choose **Samples in one column**.
- 4 In Samples, enter 'BTU.In'.
- 5 In **Subscripts**, enter *Damper*.
- 6 Check Assume equal variances. Click OK.

Session window output

Two-Sample T-Test and CI: BTU.In, Damper

Two-sample T for BTU.In

Damper N Mean StDev SE Mean

1 40 9.91 3.02 0.48

2 50 10.14 2.77 0.39

Difference = mu(1) - mu(2)

Estimate for difference: -0.235

95% CI for difference: (-1.450, 0.980)

T-Test of difference = 0 (vs not =): T-Value = -0.38 P-Value = 0.701 DF = 88

Both use Pooled StDev = 2.8818

#### Interpreting the results

Minitab displays a table of the sample sizes, sample means, standard deviations, and standard errors for the two samples.

Since we previously found no evidence for variances being unequal, we chose to use the pooled standard deviation by choosing **Assume equal variances**. The pooled standard deviation, 2.8818, is used to calculate the test statistic and the <u>confidence intervals</u>.

A second table gives a confidence interval for the difference in population means. For this example, a 95% confidence interval is (-1.450, 0.980) which includes zero, thus suggesting that there is no difference. Next is the <u>hypothesis test</u> result. The test statistic is -0.38, with p-value of 0.701, and 88 <u>degrees of freedom</u>.

Since the <u>p-value</u> is greater than commonly chosen  $\underline{\alpha\text{-levels}}$ , there is no evidence for a difference in energy use when using an electric vent damper versus a thermally activated vent damper.