Chapter 3: Probability The Basis of Statistical Inference

Probability:

$$P(E) = \frac{n(E)}{n(\Omega)} \quad , \quad 0 \leq P \leq 1$$

Some Operations on Events:

- **Union** → Or , $\cup$
- **Intersection** → And , $\cap$
- **Complemen** → Not \ No , c , $'$, $\overline{\ }$

Rules of Probability :

Addition Rule:	Conditional Probability:	Complement Probability:
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ = احتمال الاول + احتمال الثاني - احتمال التقاطع	Known - Given $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$ = احتمال التقاطع / احتمال مابعد الشرط	$P(\bar{A}) = 1 - P(A)$ = 1 − الاحتمال الاساسي

Table of number of elements in each event:

العدد للتقاطعات

	B_1	B_2	B_3	Total	
A_1	50	30	70	150	$n(A_1)$
A_2	20	70	10	100	$n(A_2)$
A_3	30	100	120	250	$n(A_3)$
Total	100	200	200	500	
	$n(B_1)$	$n(B_2)$	$n(B_3)$	$n(\Omega)$	

calculating the probability by using the probability rule

$$P(E)=\frac{n(E)}{n(\Omega)}$$

Table of probabilities of each event:

احتمال التقاطع

	B_1	B_2	B_3	Marginal Probability	
A_1	0.1	0.06	0.14	0.3	$P(A_1)$
A_2	0.04	0.14	0.02	0.2	$P(A_2)$
A_3	0.06	0.2	0.24	0.5	$P(A_3)$
Marginal Probability	0.2	0.4	0.4	1	$P(\Omega)$
	$P(B_1)$	$P(B_2)$	$P(B_3)$		

Calculating the probability directly from the table mutually exclusive (disjoint)

$A \cap B = \emptyset$

$P(A \cap B) = 0$

exhaustive

$A \cup B = \Omega$

$P(A \cup B) = 1$

Independent

$P(A \cap B) = P(A)P(B)$

$P(A|B) = P(A)$

$P(B|A) = P(B)$

3.5 Bayes Theorm

True status of the disease

The result of the test

	Has the disease (D)	Dose not have the disease ($\bar{D}$)	Total
Positive (T)	Correct decision Sensitivity $P(T\mid D)=\frac{n(T\cap D)}{n(D)}$	False decision false positive + $P(T\mid \bar{D})=\frac{n(T\cap \bar{D})}{n(\bar{D})}$	n(T)
Negative ($\bar{T}$)	False decision false negative- $P(\bar{T}\mid D)=\frac{n(\bar{T}\cap D)}{n(D)}$	Correct decision Specificity $P(\bar{T}\mid \bar{D})=\frac{n(\bar{T}\cap \bar{D})}{n(\bar{D})}$	n($\bar{T}$)
Total	n(D)	n($\bar{D}$)	n(Ω)

Predictive value Positive :	Predictive value Negative :
$P(D\mid T) = \frac{Sensitivity * P(D)}{same\ numerator + (1-Specificity) * P(\bar{D})}$	$P(\bar{D}\mid \bar{T}) = \frac{SpecificityS * P(\bar{D})}{same\ numerator + (1-Sensitivity) * P(D)}$

P(D)= probability of the relevant disease in the general population (معطاه بالسؤال)