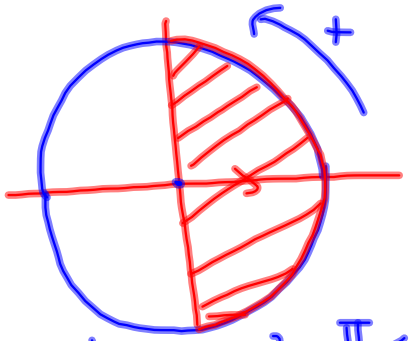
أسئلة

الفصل الأول 36/37

س3: 2. استعمل الاحداثيات القطبية، احسب التكامل

$I=\iint_R \frac{1}{x^2+y^2+2}\,dA$ حيث $R=\{(x,y): x\geq 0,\ x^2+y^2\leq 4\}$

الحل:



$R=\{(r,\theta): 0\leq r\leq 2,\ -\frac{\pi}{2}\leq\theta\leq\frac{\pi}{2}\}$

$$I=\iint_R \frac{1}{r^2+2}\,r\,dr\,d\theta$$

$$=\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\int_0^2 \frac{1}{r^2+2}\,2r\,dr\right)d\theta$$

$$=\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \ln\left|r^2+2\right|\Big|_0^2\,d\theta$$

$$=\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\ln(6)-\ln(2)\right)d\theta=\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\ln\left(\frac{6}{2}\right)d\theta$$

$$=\frac{1}{2}\ln(3)\,\theta\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}=\frac{\ln 3}{2}\left(\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right)$$

$$\boxed{I=\frac{\pi\ln 3}{2}}$$

$$I=\int_0^3\left(\int_{x^2}^{3x}(2xe^y+3y^2)\,dy\right)dx$$

$$=\int_0^3 \left(2xe^y+y^3\right)\Big|_{x^2}^{3x}\,dx$$

$$I=\int_0^3\left(2xe^{3x}+(3x)^3-2xe^{x^2}-(x^2)^3\right)dx$$

$$=\int_0^3\left(2xe^{3x}+27x^3-2xe^{x^2}-x^6\right)dx$$

$$\int_0^3 2xe^x\,dx=2xe^x\Big|_0^3-\int_0^3 2e^x\,dx$$

$$=2xe^x-2e^x\Big|_0^3=6e^3-2e^3+2e^0$$

$$=4e^3-2$$

$$I=4e^3-2+\left(27\frac{x^4}{4}-e^{x^2}-\frac{x^7}{7}\right)\Big|_0^3=4e^3-2+\frac{27}{4}3^4-e^9-\frac{3^7}{7}+e^0$$

$$I=4e^3-1-e^9+\left(\frac{27}{4}\cdot 81-\frac{27\cdot 81}{7}\right)$$

$$=4e^3-1-e^9+27\cdot 81\left(\frac{7-4}{28}\right)$$

$$\boxed{I=4e^3-e^9-1+\frac{81^2}{28}}$$

$$I=\iint_R(2xe^y+3y^2)\,dA$$

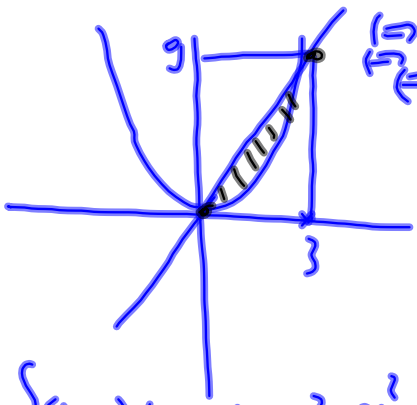
$y=3x$; $y=x^2$

$x^2=3x$

$\Leftrightarrow x^2-3x=0$

$\Leftrightarrow x(x-3)=0$

$x=0$; $x=3$



$$R=\{(x,y):0\le x\le 3,\ x^2\le y\le 3x\}$$

$R_1=$

$R_2=$

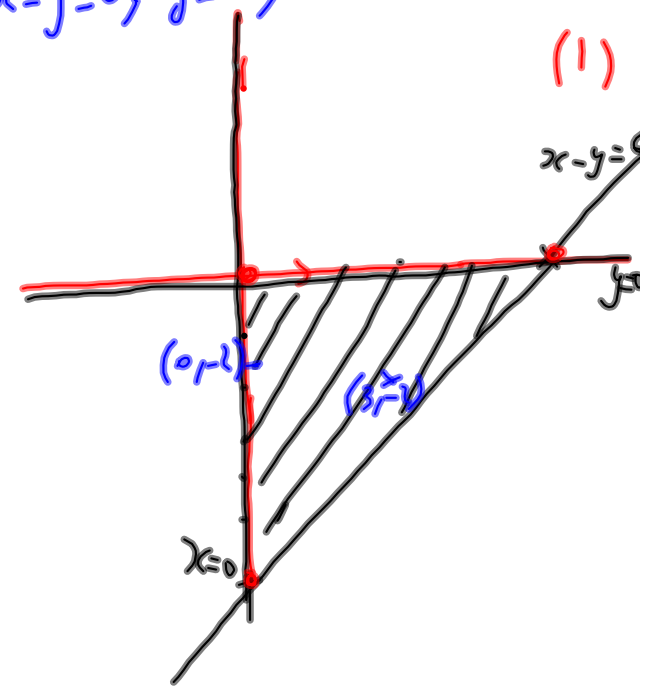
الفصل الثاني 33/34

$f(x,y) = x^2 - 6x + y^2 + 4y + 4$

R :

$x-y=6$, $y=0$, $x=0$

(1)



2. نقطة حرجة (x,y)

$\frac{\partial f}{\partial x}(x,y) = \frac{\partial f}{\partial y}(x,y) = 0$

$\Leftrightarrow \begin{cases} 2x-6=0 \\ 2y+4=0 \end{cases} \Leftrightarrow \begin{cases} x=3 \\ y=-2 \end{cases}$

$f(3,-4)=-9$ هي قيمة قصوى محلية.

3. لدينا $f(6,0)=4$, $f(0,0)=4$

$f(0,-6)=16$

$f(0,y) = y^2+4y+4$, $x=0$ ←

$g(y) = y^2+4y+4$

بما أن: $g'(y)=0$ تكافئ $2y+4=0$ بالتالي $y=-2$

$f(0,-2)=g(-2)=0$

$f(x,0) = x^2-6x+4$; $y=0$ ←

$h(x) = x^2-6x+4$

لدينا $h'(x)=0$ تكافئ $2x-6=0$ بالتالي $x=3$

$f(3,0)=h(3)=-5$

$x-y=6 \Leftrightarrow x = y+6$ ← بما أن

$k(y) = f(y+6,y) = (y+6)^2 - 6(y+6) + y^2 + 4y + 4$

$= 2y^2 + 10y + 4$

بما أن $k'(y)=0$ تكافئ $4y+10=0$ بالتالي $y=\frac{5}{2}$

$k(\frac{5}{2}) = f(\frac{17}{2}, \frac{5}{2}) = 2\cdot(\frac{5}{2})^2 + 10\,\frac{5}{2} + 4 = \frac{25}{2} + \frac{50}{2} + \frac{8}{2} = \frac{83}{2}$

4.

(x,y)	$(0,0)$	$(6,0)$	$(0,-6)$	$(3,-4)$	$(0,-2)$	$(3,0)$	$(\frac{17}{2},\frac{5}{2})$
$f(x,y)$	4	4	16	-9	0	-5	$\frac{83}{2}$