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Accelerated Detector Response Function in Squeezed Vacuum

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Abstract: Casimir/squeezed vacuum breaks Lorentz symmetry, by allowing light to propagate faster than c. We looked at the possible transformation symmetry group such vacuum could obey. By solving the semi-classical Einstein field equation in squeezed vacuum, we have found that the background geometry describes an Anti-deSitter (AdS) geometry. Therefore, the proper transformation symmetry group is the (A)dS group. One can describe quantum field theory in a finite volume as a quantum field theory (QFT) on AdS background, or vice versa. In particular, one might think of QFT vacuum on AdS as a QFT that posses a squeezed vacuum with boundary conditions proportional to R^2_{AdS}. Applying this correspondence to an accelerating detector-scalar field system, we notice at low acceleration the system is at equilibrium at ground state, however if the detector's acceleration (a) is greater than a critical acceleration, the system experience a phase transition similar to Hawking-Page Phase transition at the detector gets excited, with equivalent temperature $\Theta = \frac{\sqrt{a^2 - R'^2_{AdS}}}{2\pi}$.

Keywords: quantum field theory in curved spacetime; anti-deSitter spacetime; Unruh effect; Hawking Effect; first-order phase transition

1. Introduction

Squeezed quantum vacua breaks Lorentz symmetry, calculations of photon propagation between Casimir plates showed it has a superluminal group velocity [1–3], Since Lorentz symmetry is broken for such vacua, one is tempted to ask what spacetime symmetry group these vacua obey, if any? In order to answer this, we turn at the calculations made by Casimir [4] of the energy density of a squeezed vacuum Then plug the energy density term into the Einstein field equations, and study the geometric backreaction [5]. Squeezed vacuum has a negative energy density $\rho_{vac} = -\frac{\pi^2}{720L^4}$, where L is the length of the box forming the boundary conditions [6]. Therefore, the spacetime (locally) under this vacuum can be thought to be Anti-deSitter spacetime. Therefore, the symmetry group for quantum fields in squeezed vacuum is the (A)dS group. This correspondence between boundary conditions forced upon the quantum fields in the vacuum and quantum fields in curved background is seen sometimes in the literature without formally addressing it. We believe this correspondence may help simplifying problems in QFT in curved spacetime, particularly in AdS that has a special interest in string theory or AdS-CFT correspondence. One of the interesting problems in QFT in curved spacetime is accelerating detectors coupled to a field and their thermodynamics (Unruh effect). Using the above correspondence, one can study accelerating detectors in AdS as accelerating detectors in squeezed vacuum. We shall show that the cut-off frequencies due to boundary conditions of squeezed vacuum play a rôle in the thermodynamics of the detector-field system. In addition, Rindler horizon will be modified in accordance to the speed-of-light limit in this vacuum. Later we discuss other effects that could be understood more deeper via this correspondence, like Page-Hawking phase transition.

2. Simple Quantum Field Theory in Squeezed Vacuum

The most interesting vacuum to study its Fock space is an isotropic squeezed (Casimir) vacuum. With critical (IR cut-off) wave number k_c, corresponding to the wavelength of field excitations that is at the maximal length that the boundary conditions allow. A scalar field ϕ can be expanded in mode solutions, with a UV-cutoff k_{max} due to the presence of gravity theory

$$\phi = \sum_{k=k_c}^{k_{max}} \hat{a}_k u_k + \hat{a}_k^\dagger u_k^*, \tag{1}$$

where u_k's are the modes functions and $\hat{a}_k^\dagger, \hat{a}_k$ the creation and annihilation operators respectively. We observe that the corresponding Fock space of the modified vacuum ought to satisfy:

$$\hat{a}_k|0\rangle = 0 \quad \text{and,} \tag{2}$$

$$\hat{a}_k^\dagger|0\rangle = \begin{cases} |1_k\rangle, & \text{if } k > k_c, \\ 0, & \text{if } k < k_c. \end{cases}$$

Now, we turn into determining the background perturbation caused by modifying the vacua . We remain the isotropic and maximally symmetric case to get an analogous spacetime for the modified vacua to Minkowskian spacetime. Writing the semi-classical Einstein Hilbert action:

$$S = \int d^4x \sqrt{-det(g)} \frac{1}{2\kappa} \left(\mathcal{R} + b\right). \tag{3}$$

Were $b = -16\pi\rho_{vac}$. the 'reduced' vacuum density corresponding to the Casimir pressure in this case. The action in (3) yields the following metric for the maximally symmetric (isotropic) case—in Poincarè coordinates:

$$ds^2 = \frac{3}{bz^2}\left(-dt^2 + dz^2 + \sum_{i=2}^{3} dx_i^2\right). \tag{4}$$

This solution is assuming the boundary conditions for b are compatible with the symmetry of the metric. Thus, this metric describes how a detector in the squeezed vacuum would experience the world. We observe that the Ricci scalar is given by $\mathcal{R} = 4b$ That is, the spacetime have a negative curvature proportional to the vacuum density [5]. This is Anti de-Sitter solution with $b = 3/R_{AdS}^2$, R_{AdS} is the AdS radius. This metric will resemble the background for the quantum field discussed above. We clearly notice that Lorentz symmetry is clearly broken, but we have the (Anti)de-Sitter symmetry group, for hyperbolic spaces. We now turn to write the Klein-Gordon equation in curved spacetime of the modified Casimir vacuum; starting from the metric in the conformal form in (4). The normal modes for massive Klein-Gordon equation for conformal spacetime is written as:

$$u_k = (2\pi)^{-3/2}\Omega^{-1}e^{\mathbf{k}\cdot\mathbf{x}}\chi(z), \tag{5}$$

where $\Omega^2 = (bz^2)^{-1}$ is the conformal factor of (4), and the function $\chi(z)$ satisfies the differential equation for conformally coupled field:

$$\frac{d^2}{dz^2}\chi(z) + \omega_k^2(z)\chi(z) = 0, \tag{6}$$

where

$$\omega_k(z) = \Re\left\{\sqrt{k^2 - (\Omega M)^2}\right\}, \tag{7}$$

where M is the mass of the scalar field ϕ.

Before solving Equation (6), we conclude that for certain values of $k < k_c$, there are no excitation of the field, satisfying the conditions set for the squeezed vacuum. However, for wave numbers larger than k_c we expect ordinary field excitations, as they would not be affected by the boundary conditions imposed if boundary effects were ignored. Now, we solve Equation (6) (using the WKB method), we get,

$$\xi(z)_n = A_n e^{(3/2)\Omega^{-1}} \left(J_\nu + B_n Y_\nu\right), \tag{8}$$

where, J_ν and Y_ν are Bessel and Neumann functions, respectively. The constants A_n and B_n depend of the boundary conditions , and the parameter ν is given by:

$$\nu = \sqrt{\frac{9}{4} + \frac{M^2}{b}}. \tag{9}$$

It is called the effective scale for the field [7]. Hereby, we have completed the basic description of quantum field theory in squeezed vacuum, with conformal coupling to gravity.

3. Accelerated Frames

We wish to write a similar metric of (4) but for an accelerating detector, considering only $1+1$ dimensions (only t-z plane), and compactifying the other two spacial dimensions (every 2-sphere is shrunk to a point).

$$ds^2 = \Omega^2 \left(-dt^2 + dz^2\right). \tag{10}$$

Then we need to employ a transformations similar to Rindler transformations for Minkowski spacetime. Nevertheless this is not a straightforward process. We start by investigating the mode solutions for the Klein-Gordon equation in these coordinates (5). The $SO(1,2)$ isometries allow us to write the Hamiltonian, momentum and Lorentz boosts operators in terms of the Killing vector fields ∂_z and ∂_t See [8–10] for details about this technique. Since our spacetime satisfies (Anti)deSiiter group symmetries we can employ the same argument using this group. The Hamiltonian operator for an accelerated field is written as $\hat{H} = a\hat{K}$ where a is the acceleration and $\hat{K}$ is the Lorentz boosts operator. From above we may write the Rindler-like transformations (for small b) as [9]:

$$T = -\sqrt{\frac{3}{b}} + z\sinh at + O(b), \tag{11}$$

$$X = z\cosh at + O(b). \tag{12}$$

With the line element [11], written in terms of the t-z coordinates:

$$ds^2 = -a^2 z^2 \left(1 - \frac{4b}{3} z^2\right) dt^2 + \left(a\sqrt{\frac{b}{3}} z^2\right) dtdz + dz^2. \tag{13}$$

We know that Rindler coordinates have a horizon at $z = 0$ where the metric becomes singular. In particular $\frac{1}{z}\left(\bar{T}^2 - \bar{X}^2\right) = 1$, we use the bar for the Rindler coordinates for accelerated observer in Minkowski space. This defines the null generators as a straight lines with a slope of $\tan^{-1} 1$. We may do the same argument with Rindler coordinates for squeezed vacua.

$$\frac{1}{z}\left(T^2 - X^2\right) = 1 \tag{14}$$

This defines the horizon for a Rindler observer in modified vacua. We may rewrite (14) In terms of Rindler coordinates for the Minkowski spacetime- the barred coordinates- we get :

$$\frac{1}{z}\left(\bar{T}^2 - \bar{X}^2\right) = 1 - \frac{b}{3} \tag{15}$$

As expected, the null generators seemed to be 'rotate' or spread by an angle $\tan^{-1}\frac{b}{3}$ in the conformal diagram Figure 1. Hence, if we immersed the spacetime background of the squeezed vacuum in Minkowski spacetime, we observe how photons in the first will have a superluminal propagation when measured by observed in the Minkowski spacetime.

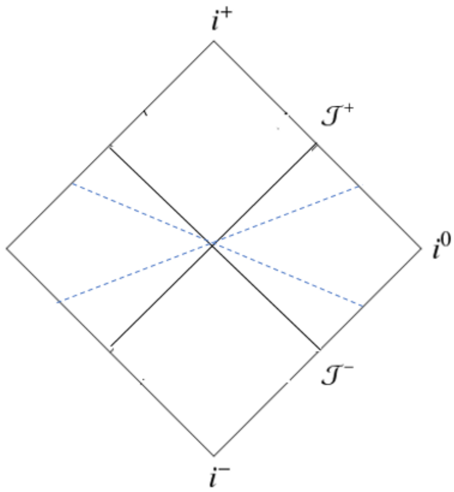

Figure 1. Rindler wedge with the dashed lines resembles the "modifeid" horizons due to modifying the vacuum, and immersing the new geometry in flat spacetime. The new horizons are spread by an angle $\tan^{-1}\frac{b}{3}$ each, corresponding to a superluminal photon propagation.

4. Detector Response Function

We start by considering a detector in the modified vacua coupled to the scalar field ϕ described above via a weak monopole coupling. We care about the coupling term in their Lagrangian $g\hat{\mu}(\tau)\phi$ where g is small coupling constant and $\hat{\mu}$ is the time-dependent monopole operator. The detector has an energy states described by the associated Hilbert space $\mathcal{H}_{\text{detector}}$. The field has an associated Fock space described above $\mathcal{H}_{\text{feild}}$ for the squeezed vacuum. We are interested in the transition amplitude from the initial state $|E_0, 0\rangle$ to the final state $|E, \Psi\rangle$ of the Hilbert space for the detector and the field $\mathcal{H}_{\text{detector}} \otimes \mathcal{H}_{\text{feild}}$. The transition amplitude shall refer to excitation of the detector energy state above initial ground state due to particle creation in the scalar field. Hence it is rather natural to assume the final state in the Fock space would be $|\Psi\rangle = |1_\omega\rangle, \omega > \omega_0$ since we have only weak coupling. Writing the first order perturbation term for the transition amplitude $\langle E, 1_\omega|g\hat{\mu}\phi|E_0, 0\rangle$:

$$-ig\int_{-\infty}^{+\infty} d\tau \langle E, 1_\omega|g\hat{\mu}\phi|E_0, 0\rangle. \tag{16}$$

where τ is the proper time of the detector. We may use Heisenberg equation to rewrite the operator $\hat{\mu}(\tau)$ as:

$$\hat{\mu}(\tau) = e^{i\hat{H}\tau}\hat{\mu}e^{-i\hat{H}\tau} \tag{17}$$

Substituting (17) into (16) to get:

$$-ig\langle E|\hat{\mu}|E_0\rangle \int_{-\infty}^{+\infty} d\tau e^{-i(E-E_0)\tau}\langle 1_\omega|\phi(x(\tau))|0\rangle. \tag{18}$$

In order to calculate the probability, we square the term and sum over the energies:

$$P = g^2\sum_E |\langle E|\hat{\mu}|E_0\rangle|^2\mathcal{F}(E), \tag{19}$$

where $\mathcal{F}(E)$ is the detector's response function which is given by:

$$\mathcal{F}(E) = \int_{-\infty}^{+\infty} d(\Delta\tau) e^{-i(E-E_0)\Delta\tau} G^{+}(\Delta\tau). \tag{20}$$

It could be interpreted as the Fourier transform of the two-point correlation (Wightman) function $G^{+}(\Delta\tau) = \langle\phi(x)\phi(x')\rangle$. The dependence on $\Delta\tau$ rather on the initial and final times the detector was adiabatically turned on; is due to the assumption that the detector and the field it is coupled to are in thermal equilibrium. The task now is to calculate the correlation function, which depends on the path the detector follows in spacetime. Hence we need to write it in terms of the detector's proper time instead of coordinate time:

$$G^{+} = \frac{-1}{4\pi^2} \left(\frac{1}{(t - t' - i\epsilon)^2 - |\mathbf{x} - \mathbf{x}'|^2} \right). \tag{21}$$

Since the spacetime is no longer flat, and observers cannot be inertial. It is needed to specify the path of the particle. The proper time for the Rindler observer, with acceleration a in the squeezed vacuum can be written as

$$t = \varpi^{-1} \sinh(\tau\varpi), \tag{22}$$

here $\varpi = \sqrt{a^2 + \xi^2}$ and $\xi = i\sqrt{3/|b|}$. Note that it is more helpful to write ϖ as $\sqrt{a^2 - \xi'^2}$. The primed term is $\Im(\xi)$, this is merely a convention that seems to help reading the results better. We now substitute t in (21), and expand around zero, we obtain correlation function in terms of the detector's proper time:

$$G^{+} = \frac{1}{2\pi}^{2} \sum_{n\in\mathbb{Z}} \left(\Delta\tau - 2i\epsilon - 2\pi i\varpi^{-1} n \right)^{-2}. \tag{23}$$

It is, in fact, expansion of detector's excitation modes n. Substituting in (20), we get,

$$\mathcal{F} = \int_{\gamma} d(\Delta\tau) \sum_{n\in\mathbb{Z}} \frac{e^{-i(E-E_0)\Delta\tau}}{4\pi^2 \left(\Delta\tau - 2i\epsilon - 2\pi i\varpi^{-1} n\right)^2}. \tag{24}$$

The contour γ runs through the entire lower half of the complex plane. We can use the methods of residues to calculate the response function; we have the following cases:

- When the detector's acceleration is ($a \leq \xi'$) The poles all lie in the upper half, therefore the integral vanishes and thereby the transition probability is vanishing . Hence, particle creation is not observed.
- Moreover, for ($a > \xi'$) The poles lie in the lower half, therefore we can sum the resides and have the following transition probability :

$$P = g^2 \sum_{E} \left(\frac{|\langle E|\hat{\mu}|E_0\rangle|^2}{e^{2\pi\varpi(E-E_0)} - 1} \right). \tag{25}$$

The Planckian distribution indicates that the detector and the field are at thermal equilibrium at temperature:

$$\Theta = \frac{\sqrt{a^2 - \xi'^2}}{2\pi}. \tag{26}$$

5. Discussion

We have started by solving the semi-classical Einstein field equations for squeezed quantum vacua, that are known to possess negative energy density. The backreaction of geometry is assumed to be of first order, as the energy perturbation above the existing geometry of (flat) spacetime. The solution yields a curved spacetime, for isotropic boundary conditions the solution yields an anti-deSitter spacetime. The correspondence between QFT with boundary conditions and QFT in the curved AdS is starting to appear. One can use this correspondence to move from one picture to another in order to simplify calculations or clarify physical pictures.The n-dimensional AdS spacetime is a conformally flat spacetime, with a symmetry group $\mathrm{AdS}_n = \frac{\mathrm{O}(2,n-1)}{\mathrm{O}(1,n-1)}$. It plays the isometry group of transformations, instead of the Lorentz group of Minkowski spacetime. This explains the superluminal propagation of photons in squeezed vacuum (Scharnhorst effect), as the latter calculations are made with QFT's with Lorentz symmetry in mind (as if the AdS patch was immersed in flat space). Applying the above correspondence to a *massive* scalar field with boundary conditions. then, solving the Klein-Gordon equation in AdS background. We observe that the solution for the wave number $k < k_c$ predicts an exponential suppression of field fluctuations at low frequencies. Whilst for higher frequencies the differential Equation (6) behaves like a harmonic oscillator of z-dependent normal modes (7). A careful look at the formulation of QFT on squeezed vacuum/AdS reveals that if the field in study is a conformally-symmetric field (CFT), this field theory is insensitive to the boundary conditions imposed/AdS background geometry.

We turn to the main focus of this work, the thermal equilibrium conditions of an accelerated detector-scalar field on squeezed vacuum. We observe that: (a) The detector-field system are in equilibrium in the ground state if the detector is weakly accelerating, $a \leq \xi$. (b) The detector is excited if it is acceleration $a > \xi$. Unruh temperature registered by that accelerator is given by (26). (c) The (modified) Rindler horizon is at the null generators of AdS, this horizon would be at the velocity limit of photons in the squeezed vacuum when measured by observers in the unbounded vacuum, viz light velocity in the squeezed vacuum is the speed limit for observers there. The previous observations indicates that for the accelerating detector-field system the acceleration $a = \xi$ form a critical point, at which the thermodynamic behaviour changes.This was seen Hawking-Page phase transition [12], when a blackhole in an AdS reaches a critical mass. The same logic underlies both phenomena. Moreover, by the correspondence mentioned above, we conclude that Hawking-Page transition could be deduced from putting a blackhole in a box (unphysical thought experiment), since the latter would already be in an AdS by the geometric backreaction mentioned above.

6. Conclusions

An accelerated detector coupled to a field in squeezed vacuum will not detect particle production/thermal radiation if its acceleration is below a critical acceleration, this result is expected when the acceleration is weak such that most thermal radiation produced by (ordinary) Unruh effect would lies in the large wavelengths region of the spectrum, where the boundary condition of the squeezed vacuum prohibits their production. Using the correspondence between the QFT in AdS and QFT having squeezed vacuum we could show this expected result. For the limit $a >> \xi'$, the temperature in (26) approaches Unruh temperature for ordinary vacuum, as the maximal intensity gets higher and wavelengths move towards the shorter wavelengths as Plank's law predicts. These results indicates the usefulness of the above correspondence, and could be helpful in more complicated calculations as well. Moreover deepens understanding of critical phenomena in both QFT's in AdS or with boundary conditions like blackholes in AdS. It would be interesting to invistigate the rô le of this IR cut-off assumed in renormalisation of the thermodynamics of fields in curved space, in a more general setting. Moreover, the calculations made in this paper would provide a useful technique for computations in AdS/CFT correspondence.

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