"from the first page and from page 1 to 3"

لا يكتب في هذا الهامش

① a) $S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$
$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),$
$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),$
$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),$
$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$
$= \{1,2,3,4,5,6\} \times \{1,2,3,4,5,6\}$

\# of green die \# of red die

b) $S = \{(x,y) : x = \#$ of green die and $y = \#$ of red die$\}$

② a) $A = \{(3,6),(4,5),(4,6),(5,4),(5,5),(5,6),(6,3),(6,4),(6,5),(6,6)\}$

b) $B = \{(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),$
$(1,2),(3,2),(4,2),(5,2),(6,2)\}$

c) $C = \{(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),$
$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

d) $A \cap C = \{(5,4),(5,5),(5,6),(6,3),(6,4),(6,5),(6,6)\}$

e) $A \cap B = \emptyset$

f) $B \cap C = \emptyset$

g) [illegible]

لا يكتب في هذا الهامش

③ $S=\{c,s,N,P,U,O,3\}$

$A=\{c,s,3\}$, $B=\{s,N,P\}$, $C=\{O\}$

a) $A'=S-A=\{N,P,U,O\}$

b) $AUC=\{c,s,3,O\}$

c) $B'=S-B=\{c,U,O,3\}$

$C'=S-C=\{c,s,N,P,U,3\}$

$A\cap B'=\{c,3\}$

$\therefore (A\cap B')\cup C'=\{c,s,N,P,U,3\}$

d) $B'\cap C'=\{O\}$

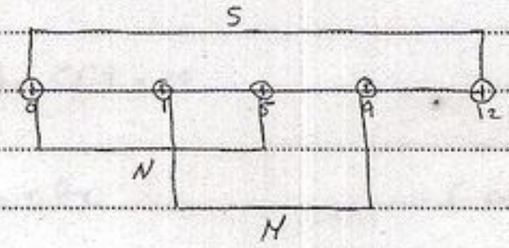
e) $A\cap B\cap C=\emptyset$

f) $A'\cup B'=\{c,U,O,3,N,P\}$

$A'\cap C'=\{c,s,N,P,U,3\}$

$\therefore (A'\cup B')\cap(A'\cap C')=\{c,U,3,N,P\}$

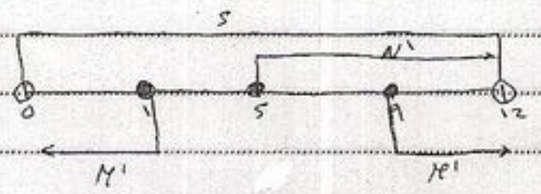
④



a) $M\cup N=(0,9)$ or $M\cup N=\{x: 0<x<9\}$

b) $M\cap N=(1,5)$ or $M\cap N=\{x: 1<x<5\}$

c)



$M'=(-\infty,1]\cup[9,12)$

$N'=[5,12)$

$\therefore M'\cap N'=[9,12)$

لا يكتب في
هذا الهامش

"Combinations"

Q1 $\binom{n}{r} = nCx$ (هذا هو الرمز المستخدم في الآلة الحاسبة)

a) $\binom{6}{2} = 6C2 = 15$

b) $\binom{6}{4} = 6C4 = 15$

Q2 $L.H.S = \binom{n}{x} = \frac{n!}{x!(n-x)!}$

$R.H.S = \binom{n}{n-x} = \frac{n!}{(n-x)!(n-(n-x))!} = \frac{n!}{(n-x)!(n-n+x)!}$

$= \frac{n!}{(n-x)!\,x!} = \frac{n!}{x!\,(n-x)!}$

$\therefore L.H.S = R.H.S$

Q3 a) $\binom{n}{0} = \frac{n!}{0!(n-0)!} = \frac{n!}{0!\,n!} = \frac{1}{0!} = \frac{1}{1} = 1$

b) $\binom{n}{1} = \frac{n!}{1!(n-1)!} = \frac{n(n-1)(n-2)\cdots\times2\times1}{(n-1)(n-2)\cdots\times2\times1} = \frac{n}{1} = n$

c) $\binom{n}{n} = \frac{n!}{n!(n-n)!} = \frac{n!}{n!\,(0)!} = \frac{1}{0!} = \frac{1}{1} = 1$

Q4 $\binom{6}{3} = 6C3 = 20$

Q5 $\binom{5}{2} = 5C2 = 10$

Q6 $P^n_r = nPr$ (هذا هو الرمز المستخدم في الآلة الحاسبة)

$\therefore P^5_2 = 5P2 = 20$

لا يكتب في هذا الهامش

"Probability, Conditional probability and independent"

Q.1 $P(A)=.5$, $P(B)=.4$, $P(C\cap A^c)=.6$, $P(C\cap A)=.2$

$P(A\cup B)=.9$

a) $P(C)=P(C\cap A)+P(C\cap A^c)$ ✓

or $=P(C\cap B)+P(C\cap B^c)$

$\therefore P(C)=P(C\cap A)+P(C\cap A^c)=.6+.2=.8$

b) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow .9=.5+.4-P(A\cap B) \Rightarrow P(A\cap B)=.9-.9=0$

c) $P(C|A)=\frac{P(A\cap C)}{P(A)}=\frac{.2}{.5}=\frac{2}{5}=.4$

d) $P(A^c\cap B^c)=P(A\cup B)^c=1-P(A\cup B)=1-.9=.1$

Q.2 $S=\{H,T\}\times\{H,T\}\times\{H,T\}$

$=\{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$

a) $n(S)=2\times2\times2=8$

b) A : exactly two heads

$A=\{HHT, HTH, THH\} \Rightarrow P(A)=\frac{n(A)}{n(S)}=\frac{3}{8}=.375$

c) B : exactly three heads, $B=\{HHH\}$

$\because A$ and $B=A\cap B=\emptyset \Rightarrow \because A$ and B are disjoint

d) C : the first coin is heads, $C=\{HHH, HHT, HTH, HTT\} \Rightarrow P(C)=\frac{4}{8}=\frac{1}{2}$

D : the second and third coin are tails

$D=\{HTT, TTT\} \Rightarrow P(D)=\frac{2}{8}=\frac{1}{4}$

$C\cap D=\{HTT\} \Rightarrow P(C\cap D)=\frac{1}{8}$

as $P(C\cap D)\neq 0$ $\because$ C and D are not disjoint

and as $P(C)\neq P(D)$ $\because$ C and D are not equally likely

and as $P(C\cap D)=P(C).P(D)=\frac{1}{8}$ $\because$ C and D are independent

لا يكتب في هذا الهامش

Q3 $S = \{1,2,3,4,5,6\} \times \{1,2,3,4,5,6\}$

$= \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$

$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),$

$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),$

$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$

$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),$

$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\} \Rightarrow n(S) = 6 \times 6 = 36$

1) A : the sum of numbers of two dice ≤ 4

$A = \{(1,1),(1,2),(1,3),(2,1),(2,2),(3,1)\} \Rightarrow P(A) = \frac{6}{36} = \frac{1}{6} = .167$

2) B : at least one of the die shows 4

$B = \{(1,4),(2,4),(3,4),(4,4),(4,5),(4,6),$

$(4,1),(4,2),(4,3),(5,4),(6,4)\} \Rightarrow P(B) = \frac{11}{36} = .3056$

3) C : the sum of numbers of two dice = 4 and one die shows 1

$C = \{(1,3),(3,1)\} \Rightarrow P(C) = \frac{2}{36} = .0556$

4)

D : the sum of two dice = 4, $D = \{(1,3),(3,1),(2,2)\} \Rightarrow P(D) = \frac{3}{36} = .0833$

E : one die shows 2

$E = \{(2,2),(2,1),(2,3),(2,4),(2,5),(2,6),$

$(3,2),(4,2),(5,2),(6,2)\} \Rightarrow P(E) = \frac{10}{36} = .2778$

$D \cap E = \{\} = \emptyset \Rightarrow P(D \cap E) = 0$ $\therefore$ D and E are disjoint

Q4 $P(A) = .3$, $P(B) = .4$, $P(A \cap B \cap C) = .03$, $P(\overline{A \cap B}) = P(A \cap B)' = .88$

1) $P(A \cap B) = 1 - P(A \cap B)' = 1 - .88 = .12 \neq 0$

$\therefore$ A and B are not disjoint

$P(A) \cdot P(B) = .3 \times .4 = .12 = P(A \cap B)$

$\therefore$ A and B are independent

2) $P(C \mid A \cap B) = \frac{P(C \cap (A \cap B))}{P(A \cap B)} = \frac{P(C \cap A \cap B)}{P(A \cap B)} = \frac{P(A \cap B \cap C)}{P(A \cap B)} = \frac{.03}{.12} = .25$

Q5 A : it will rain tomorrow $\Rightarrow P(A) = .23$

$\therefore P(A') = P(\bar{A}) = 1 - P(A) = 1 - .23 = .77$

لا يكتب في هذا الهامش

Q6 A : factory open a branch in Riyadh $\Rightarrow P(A) = .7$

B : factory open a branch in Jeddah $\Rightarrow P(B) = .4$

$P(A \cup B) = .8$

	A	A^c	Sum
B	$P(A \cap B) = .3$	$P(A^c \cap B) = .1$	$P(B) = .4$
B^c	$P(A \cap B^c) = .4$	$P(A^c \cap B^c) = .2$	$P(B^c) = .6$
Sum	$P(A) = .7$	$P(A^c) = .3$	1

where $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow .8 = .7 + .4 - P(A \cap B) \Rightarrow P(A \cap B) = .3$ (من القيمة المستخرجة للجدول)

1) $P(A \cap B) = .3$

2) $P(A^c \cap B^c) = .2$

Q7 A : the lab specimen is contaminated $\Rightarrow P(A) = .1$

$\therefore P(A^c) = 1 - P(A) = 1 - .1 = .9$

and we have Three samples of lab specimen checked independent

sample space ← $S = \{A, A^c\} \times \{A, A^c\} \times \{A, A^c\}$

$= \{AAA, AAA^c, AA^cA, A^cAA, A^cAA^c, A^cA^cA, AA^cA^c, A^cA^cA^c\}$

1)

B : none of lab specimen is contaminated

$B = \{A^cA^cA^c\} \Rightarrow P(B) = P(\{A^cA^cA^c\}) = P(A^c)P(A^c)P(A^c) = (.9)(.9)(.9) = .729$

2) C : exactly one of lab specimen is contaminated

$C = \{A^cAA^c, A^cA^cA, AA^cA^c\}$

$\Rightarrow P(C) = P(\{A^cAA^c\}) + P(\{A^cA^cA\}) + P(\{AA^cA^c\})$

$= P(A^c)P(A)P(A^c) + P(A^c)P(A^c)P(A) + P(A)P(A^c)P(A^c)$

$= 3[P(A^c)P(A^c)P(A)] = 3[(.9)(.9)(.1)] = .243$

لا يكتب في هذا الهامش

Q8

	M	$M^c = F$	Sum
E	$n(M \cap E) = 28$	$n(F \cap E) = 50$	$n(E) = 78$
S	$n(M \cap S) = 38$	$n(F \cap S) = 45$	$n(S) = 83$
C	$n(M \cap C) = 22$	$n(F \cap C) = 17$	$n(C) = 39$
Sum	$n(M) = 88$	$n(F) = 112$	200

1) $P(M) = \frac{n(M)}{200} = \frac{88}{200} = .44$

2) $P(M|S) = \frac{P(M \cap S)}{P(S)} = \frac{n(M \cap S)}{n(S)} = \frac{38}{83} = .4575$

3) $P(C^c|F) = \frac{P(C^c \cap F)}{P(F)} = \frac{n(C^c \cap F)}{n(F)} = \frac{n(C^c \cap F)}{112}$ to find it

	F	F^c	Sum
C	$n(F \cap C) = 17^*$	$n(F^c \cap C) = 22$	$n(C) = 39^*$
C^c	$n(F \cap C^c) = 95$	$n(F^c \cap C^c) = 66$	$n(C^c) = 161$
Sum	$n(F) = 112^*$	$n(F^c) = 88$	200^*

$\Rightarrow \therefore n(C^c \cap F) = 95$

$\therefore P(C^c|F) = \frac{95}{112} = .475$

4) we want to see that $P(M \cap E) \overset{!?}{=} P(E)P(M)$

$L.H.S = P(M \cap E) = 28/200 = .14$

$R.H.S = P(M)P(E) = \frac{88}{200} \times \frac{78}{200} = .1716$ $\therefore L.H.S \neq R.H.S$

$\therefore$ M and E are not independent (dependent)

لا يكتب في هذا الهامش

Q9

	M	$M^c = F$	Sum
D	$n(M \cap D) = 300$	$n(F \cap D) = 50$	$n(D) = 350$
O	$n(M \cap O) = 200$	$n(F \cap O) = 50$	$n(O) = 250$
N	$n(M \cap N) = 100$	$n(F \cap N) = 300$	$n(N) = 400$
Sum	$n(M) = 600$	$n(F) = 400$	1000

1) $P(F) = \frac{n(F)}{1000} = \frac{400}{1000} = .4$

2) $P(F \cap D) = \frac{n(F \cap D)}{1000} = \frac{50}{1000} = .05$

3) $P(F \mid D) = \frac{P(F \cap D)}{P(D)} = \frac{n(F \cap D)}{n(D)} = \frac{50}{350} = .1429$

4) we want to see that $P(F \cap D) \overset{?}{=} P(F) \cdot P(D)$

L.H.S $= P(F \cap D) = .05$

R.H.S $= P(F) \cdot P(D) = \frac{400}{1000} \times \frac{350}{1000} = .14$

$\therefore$ L.H.S $\neq$ R.H.S $\therefore$ F and D are dependent

Q10 A: the first engine start $\Rightarrow P(A) = .4$

B: the second engine start $\Rightarrow P(B) = .6$

and $\because$ the two engine operate independent

$\therefore P(A \cap B) = P(A) \cdot P(B) = (.4)(.6) = .24$