The programming codes of GSCS-2015-0157

Bayesian estimation and prediction in the case of the exponential distribution

restart;
$x := sort([3, 19, 23, 26, 27, 37, 38, 41, 45, 58, 84, 90, 99, 109, 138])$:
$\mathrm{n} := 20 : \mathrm{r} := 10 :$
$\mathrm{m} := 10 : \mathrm{s} := 2 :$
$\mathrm{a} := 0.1 : \mathrm{b} := 10 :$
$T1 := 75 : \mathrm{T2} := 110 : \mathrm{D1} := 0$ **:** $D2 := 0$ **:**
for i **from** 1 **to** 15 **do if** $\mathrm{x}[\mathrm{i}] \le \mathrm{T1}$ **then** $\mathrm{D1} := \mathrm{D1} + 1$ **else** $\mathrm{D1} := \mathrm{D1}$
end if end do:
for i2 **from** 1 **to** 15 **do if** $\mathrm{x}[\mathrm{i2}] \le \mathrm{T2}$ **then** $\mathrm{D2} := \mathrm{D2} + 1$ **else** $\mathrm{D2} := \mathrm{D2}$
end if end do:
if $\mathrm{x}[\mathrm{r}] \le \mathrm{T1}$ **then** $\mathrm{T3} := \mathrm{T1} : \mathrm{k1} := \mathrm{D1}$: **else** $\mathrm{T3} := \min(\mathrm{x}[\mathrm{r}], T2) : k1 := \min(r, D2)$ **end if** :

$$\mathrm{I1} := \Gamma(\mathrm{k1} + \mathrm{a}) \cdot \left(\sum_{i1=1}^{k1} x[i1] + (n - k1) \cdot T3 + b \right)^{-k1 - a} :$$

$$f := unapply\left(\frac{\Gamma(k1 + a + 1) \cdot m!}{I1 \cdot (\mathrm{m} - \mathrm{s})!} \cdot \sum_{w=0}^{s-1} \frac{(-1)^w}{w! \cdot (s - w - 1)!} \cdot \left(\sum_{i1=1}^{k1} x[i1] + (n - k1) \cdot T3 + (m - s + w + 1) \cdot t + b \right)^{-k1 - a - 1}, t \right) :$$

$$F := unapply\left(\frac{\Gamma(k1 + a) \cdot m!}{I1 \cdot (\mathrm{m} - \mathrm{s})!} \cdot \sum_{w=0}^{s-1} \frac{(-1)^w}{w! \cdot (s - w - 1)! \cdot (m - s + w + 1)} \cdot \left(\sum_{i1=1}^{k1} x[i1] + (n - k1) \cdot T3 + (m - s + w + 1) \cdot t + b \right)^{-k1 - a}, t \right) :$$

 # The ML estimation of θ

$$\theta ML := evalf\left(\frac{k1}{\left(\sum_{i1=1}^{k1} x[i1] \right) + (n - k1) \cdot T3} \right)$$

0.009372071228

 # The Bayesian estiomation of θ

$$\theta BS := evalf\left(\frac{k1 + a}{\left(\sum_{i1=1}^{k1} x[i1] \right) + (n - k1) \cdot T3 + b} \right)$$

0.009377901578

The point predictor

$$yhat := \frac{\Gamma(k1 + a + 1) \cdot m!}{I1 \cdot (\mathrm{m} - \mathrm{s})!} \cdot \sum_{w=0}^{s-1} \frac{(-1)^{w}}{w! \cdot (s - w - 1)!} \cdot \int_{0}^{\infty} t \cdot \left(\sum_{i1=1}^{k1} x[i1] + (n - k1) \cdot T3 + (m - s + w + 1) \cdot t + b \right)^{-k1 - a - 1} \mathrm{d}t$$

24.98534797

The Bayesian predictive bounds of 95% equi−tailed interval

$L1 := fsolve(F(t) = .975, t, 0..500);\ U1 := fsolve(F(t) = 0.025, t, 6..1000);$

2.627449327

79.00779349

The Bayesian predictive bounds of 95% HPD interval for s=2,...,10

$s1 := fsolve(\{F(L2) - F(U2) = 0.95, f(L2) - f(U2) = 0\}, \{L2 = 0..10, U2 = 0..100\})$

$\{L2 = 0.2872756413, U2 = 64.58332646\}$

The Bayesian predictive bounds of 95% HPD interval for s=2,...,1

$L2 := 0;\ U2 := fsolve(F(t) = 0.05, t, 6..1000);$

0

64.43875982

Bayesian estimation and prediction in the case of the Pareto distribution

$n := 20 : r := 10 : T1 := 8.5 : T2 := 9.5 :$
$m := 10 : s := 2 :$
$\alpha := 3 : \sigma := 6 :$
$a := 1.78 : b := 0.33 : c := 3.48 : d := 11.40 :$
$U := stats[random, uniform[0,1]](n) :$

for i **from** 1 **to** n **do** $y[i] := \sigma \cdot (1 - U[i])^{\frac{-1}{\alpha}}$ **end do**:
$yy := [seq(y[i3], i3 = 1..n)] : x := sort(yy) :$
$D1 := 0 : D2 := 0 :$
for i **from** 1 **to** 15 **do if** $x[i] \le T1$ **then** $D1 := D1 + 1$ **else** $D1 := D1$ **end if end do**:
for i2 **from** 1 **to** 15 **do if** $x[i2] \le T2$ **then** $D2 := D2 + 1$ **else** $D2 := D2$ **end if end do**:
if $x[r] \le T1$ **then** $T3 := T1 : k1 := D1 :$ **else** $T3 := \min(x[r], T2) : k1 := \min(r, D2)$ **end if**:
$L := \min(d, x[1]) :$

$$I1 := evalf\left(\frac{\Gamma(k1 + a)}{n + b} \cdot \left(\sum_{i1=1}^{k1} \ln(x[i1]) + (n - k1) \cdot \ln(T3) - (n + b) \cdot \ln(L) + \ln(c)\right)^{-k1 - a}\right) :$$

$$f1 := unapply\left(\frac{\Gamma(k1 + a + 1) \cdot m!}{I1 \cdot (m - s)!} \cdot \sum_{w=0}^{s-1} \frac{(-1)^w}{w! \cdot (s - w - 1)! \cdot (n + b + m - s + w + 1) \cdot t} \cdot \left(\sum_{i1=1}^{k1} \ln(x[i1]) + (n - k1) \cdot \ln(T3) - (n + b) \cdot \ln(t) + \ln(c)\right)^{-k1 - a - 1}, t\right) :$$

$$f2 := unapply\left(\frac{\Gamma(k1 + a + 1) \cdot m!}{I1 \cdot (m - s)!} \cdot \sum_{w=0}^{s-1} \frac{(-1)^w}{w! \cdot (s - w - 1)! \cdot (n + b + m - s + w + 1) \cdot t} \cdot \left(\sum_{i1=1}^{k1} \ln(x[i1]) + (n - k1) \cdot \ln(T3) - (n + b + m - s + w + 1) \cdot \ln(L) + (m - s + w + 1) \cdot \ln(t) + \ln(c)\right)^{-k1 - a - 1}, t\right) :$$

$$F1 := unapply\left(\frac{\Gamma(k1+a)\cdot m!}{I1\cdot(\mathrm{m}-\mathrm{s})!}\cdot\sum_{w=0}^{s-1}(-1)^{w}\Big/\left(w!\cdot(s-w-1)!\cdot(n+b\right.\right.$$
$$+m-s+w+1)\cdot(m-s+w+1)\cdot(n+b))\cdot\left((n+b+m-s\right.$$
$$+w+1)\cdot\left(\sum_{i1=1}^{k1}\ln(x[i1])+(n-k1)\cdot\ln(T3)-(n+b)\cdot\right.$$
$$\left.\ln(L)+\ln(c)\right)^{-k1-a}-(m-s+w+1)\cdot\left(\sum_{i1=1}^{k1}\ln(x[i1])\right.$$
$$\left.\left.\left.+(n-k1)\cdot\ln(T3)-(n+b)\cdot\ln(t)+\ln(c)\right)^{-k1-a}\right),t\right):$$

$$F2 := unapply\left(\frac{\Gamma(k1+a)\cdot m!}{I1\cdot(\mathrm{m}-\mathrm{s})!}\right.$$
$$\cdot\sum_{w=0}^{s-1}(-1)^{w}\Big/(w!\cdot(s-w-1)!\cdot(n+b+m-s+w+1)\cdot(m$$
$$-s+w+1))\cdot\left(\sum_{i1=1}^{k1}\ln(x[i1])+(n-k1)\cdot\ln(T3)-(n+b\right.$$
$$\left.+m-s+w+1)\cdot\ln(L)+(m-s+w+1)\cdot\ln(t)+\ln(c)\right)^{-k1}$$
$$\left.{}^{-a},t\right)$$

 # The ML estimation of α

$$\alpha hatML := \frac{k1}{\sum_{i1=1}^{k1}\ln(x[i1])+(n-k1)\cdot\ln(T3)-n\cdot\ln(x[1])}$$

2.513212857

 # The ML estimation of σ $\sigma hatML := x[1]$

6.045530718

 # The Bayesian estiomation of α

$$\alpha hatB := \frac{k1+a}{\sum_{i1=1}^{k1}\ln(x[i1])+(n-k1)\cdot\ln(T3)-(n+b)\cdot\ln(L)+\ln(c)}$$

2.540692549

$$\sigma hatB := \frac{\mathrm{L}}{\Gamma(k1+a)} \cdot \left(\int_0^{\infty} \left(z^{(\mathrm{k1}+a)} \Big/ \left(z + \frac{\sum_{i1=1}^{k1} \ln(x[i1]) + (n-k1)\cdot\ln(T3) - (n+b)\cdot\ln(L) + \ln(c)}{n+b} \right) \cdot \mathrm{e}^{-z} \right) \mathrm{d}z \right)$$

$$5.921394837$$

The point predictor $\mathrm{yhat} := \int_0^{\mathrm{L}} \mathrm{t}\cdot \mathrm{f1}(t)\,\mathrm{d}t + \int_{\mathrm{L}}^{\infty} \mathrm{t}\cdot \mathrm{f2}(t)\,\mathrm{d}t$

$$6.496557739$$

\# *The Bayesian predictive bounds of 95% equi−tailed interval*
if $F1(L) < 0.975$ **then** $L1 := fsolve(F1(t) = .975, t, 5..10)$; **else** $L1 := fsolve(F2(t) = .975, t, 6..20)$; **end if**;
if $F1(L) < 0.025$ **then** $U1 := fsolve(F1(t) = 0.025, t, 6..100)$; **else** $U1 := fsolve(F2(t) = 0.025, t, 6..100)$; **end if**;

$$5.875076689$$

$$7.799222024$$

\# *The Bayesian predictive bounds of 95% HPD interval*
$s1 := fsolve(\{F1(L22) - F2(U22) = 0.95, f1(L22) - f2(U22) = 0\}, \{L22 = 5..10, U22 = 0..10\})$

$$\{L22 = 5.758935259, U22 = 7.551985306\}$$