Chapter Six

Actuarial Mathematics of Life Insurance

What did you study?

After study, the basic concepts of risk and insurance in the first part, the fundamentals of life insurance had been studied in the last chapter. These fundamentals comprise characteristics of both life risks and life policies, classifications of life insurance policies and annuities. Moreover life insurance program for a family.

What are you going to study in this chapter?

In this chapter, the reader will study the following points:

- A general introduction, about net premium and gross premium.
- The requirements elements for calculating the net premium
- Mortality table and how can you construct it
- Expectation of life (curtate – complete)
- Probabilities of death and survival for a person.

Objectives of studying this chapter

After study this chapter, the reader has to be able to answer the following questions:

- What is the difference between net premium and gross premium?
- What is the ratemaking in life insurance?
- How can you construct a mortality table?

- What is expectation of life and what are its types?
- What is expectation of life and what are its types?
- How can you calculate any probability for a specific policy?

Chapter Six

Actuarial Mathematics of Life Insurance

6.1-Introduction

It is known, the price of any product, in all firms, is to be calculated after determining, the cost that is charged by the company over the different stages of production. Likewise, insurance company calculates the premiums for the various types of policies after determining the cost of the policy. The latter is used to provide funds for the payment of benefit of insurance policy. This cost is called ***net premium***. In addition to the net premium, other amounts of money that are charged by insurance company are called *loadings* costs, which include *the profit* element and *operating expenses* of the company (i.e. commissions of agents – salaries of employees and workers – rent – telephone and mail expenses … etc). The summation of the net premium and loading cost is called ***the gross premium***.

Since, the risks that are covered by life insurance policies (death risk and survival risk) are probable risks. Also, the benefits are to be paid to policyholders depending upon a

future occurrence of the risks. Furthermore, the premiums are collected in advance from insureds and are invested in income producing assets. So, insurance company will make allowance for the interest that will be earned on the premiums, when the company invests them.

Hence, <u>the ratemaking of life insurance policies</u> (i.e. calculation premiums) is depending upon three elements, they are:

i) *Mortality rates*: These rates mean probabilities of death and survival of life insurance policies. They can be calculated by Mortality table.
ii) *Interest* (i.e. interest rate that by which the premiums are invested)
iii) *Sum insured* (i.e. the face value of the policy)

Using the foregoing elements, the fundamental relationship between <u>the net premium</u> and <u>the future benefits</u> for any type of life insurance policies may be expressed on the purchase date as follows:

Present value of <u>*net premiums*</u> = present value of <u>*future benefits*</u> (on purchase date)

In other words, if we have **n** policies from any type of life policies (for example endowment policy) and of these **m** policies each have one claim (i.e. benefit), the relationship between the net premium for these policies and their claims (i.e. benefits) may be written on the purchase date as below:

$$\sum_{i=1}^{n} P_i = \sum_{j=1}^{m} C_j \quad \text{(on purchase date)}$$

That is: **Insureds' obligations = insurance company's obligations (*on purchase date*)**

Where:

P = net premium

C = Claim (i.e. benefit)

Life insurance policies may be purchased either by paying a single amount, called *the net single premium* on purchase date or by paying equal annual payments, called *the net annual premiums*[*].

---

[*] The net premium may be paid monthly, quarterly or semiannually. Premiums other than the single premium may continue until the death of policyholder or the maturity of

A notice: Net single premiums are generally paid for annuities policies, while net annual premiums are usually paid for life insurance policies as pointed out earlier.

Insurance companies invest the collected premiums in various fields to earn interest, so the interest plays a vital role in actuarial calculations of premiums.

Hence, if the interest income and the collected premiums exceed both payments of claims, admunstrative expenses and the reserve reuirements, the insurance company may use the excess as ***a dividend*** payable to policyholders.

Also, sum isured has a forceful relationship with net premium. The large sum insured the large net premium and vice verse.

Inconclusion: The three preceding elements (mortality rates – interest – sum insured) should be taken into account during calculation of the net premium (single premium – annual

---

the policy or may be limited to a define number of years (for example 20 payment life policy).

premium) for the various life insurance policies as illustrated below:

6.2-Mortality table:

It is notable, calulation of the net premiums for various life insurance policies (whether policies cover deat risk or policies cover survival risk) is based on *the mortality table*.

Henc, the following question may be raise.

What is the mortality table?

The answer:

The mortality table is "*a statistical table by means of which the probabilities of death and survival may be measred at any given age for a group of individual".* In other words, it represents a record of mortality observed in the past and is arranged in a form to show the probabilities of death and survival at each separate age.

It is known, if you asked somebody two questions. *First question*: How long do you live?

and *second question*: When will you die?

The answer

Nobody can predict how many years can live. Also, the person does not know when will die. However, studies of mortality individuals (i.e. a group of insureds) based on years of experience, enable insurance companies to make a reasonably accurate predication of the death rate of any particular age group.

So, the following question may be raised

How can insurance companies predict the death rate of a particular age group?

The answer:

The actuary, in insurance company, determines the rate of death at each given age (e.g. the number dying per thousand at age 1, 2, 3, 4, and so on) by study a large number of ndividuals, realizes the law of large numbers. This number is called the ***radix***, usually policyholders, is included in the study. The number of individuals living and dying in each age group is recorded and the findings are tabulated. In other words, a group of individuals (e.g. policyholders)entering upon a certain

age and traces the history of the entire group year by year untill all have died (i.e. the 1958 CSO Table)

Many mortality tables, have been constructed in developed contries in particular USA and Europe, to meet the current needs of life insurance business. For example:

- Commissioners' 1958 Standard Ordinary Table of Mortality based on experience 1950-54.
- Commissioners' 1980 Standard Ordinary Table of Mortality based on experience 1970-75.
- United states total population Morality Table based on experience 1979-81.

A notice: the calcutation of premiums of life insurance policies is not affected by the use of a particular mortality table.

In this text, we will use The 1958 CSO Table and its related table of commutations columns for calculating the premiums of life insurance policies (see tables in appendix)

6.3-Construction of a Mortality Table

Mortality tables may be constructed either based upon the experience of the general census of population or based upon the experience of life insurance companies. The mortality rates in a table had constructed from the general census of population are always higher than their conterpart in a table had constructed based on the experience of life insurance companies. That is due to, the population table includes many individuals in poor health and others in hazardous or unhealthy occuptions. Hence, the computed premiums according to the tables based upon the experience of life insurance companies ar more equity rather than computed based upon the general census of population.

6.3.1-Basic Data of Mortality Table:

before you know, how can you construct a mortality table. It is preferable to explain what is the shape of the table and its contents by looking at the 1958 CSO table as follows:

Table (6.1) Commissioners 1958 standard Ordinary CSO table of mortabity

Age		Number of living (L_x)	Number of dying (d_x)	Probability of death (q_x)	Probability of survival (P_x)
Male	Femal				
0		10000000	70800	0.00708	0.99292
1		9929200	17475	0.00176	0.99824
2		9911725	15066	0.00152	0.99848
3		9896659	14449	0.00146	0.99854
4		9882210	13835	0.0014	0.9986
5		9868375	13322	0.00135	0.99865
6		9855053	12812	0.0013	0.9987

From table (6.1) we may notice the follwing:

i) *Radix*: The table starts with 10 millions lives. This number is known as the radix .It is completely an arbitrary number, it is usually a round number such as 100,000 , 1,000,000 or 10,000,000 ... etc

ii) *Contents of mortabity table*: The 1958 CSO table may contain the following columns:

a) *age (X):* This column contains all the ages over the mortabity table at the beginning of age 0 to age 99. It should be integer number. This column may start with 1 or 10 or 20 but it ends either at age 85 (e.g. Egypt) or 100 (e.g. America)

b) *Number of living (L_x):* This number means the number of individuals living at age X from individuals who have age less than X. **For example**, L_4 is the number of individuals living at age 4 from individuals who have age less than age 4.

Thus, if we look at the 1958 CSO table, it is assumed that a group of individuals 10 millions males comes under obervation at exactly the same moment as they enter

upon the first year of life (age 0). Of this group 70800 die during the year (10000000 ×0.00708) , leaving 9929200 lives to begin the econd year. Also,the table proceeds in this manner to record the number of dying during each year of life and the number living at the beginning of each succeeding year, until only 6415 of the original group (radix number) are found to enter upon the one hundredth year of life (age 99), and these 6415 die during that year.

Hence, the number of individual living at age 1 may be concluded from the following relationship

$\mathbf{L_1 = L_0 - D_0 = 10.000.000 – 70800 = 9929200}$

Likewise $\mathbf{L_2 = L_1 – D_1 = 9929200 – 17475 = 9911725}$ **and so on That is:**

$$L_{x+n} = L_x - d_x \qquad (6.1)$$

__c)Number of dying (d_x):__ This number means the number od dying between the current age and the next age. For example:

$\mathbf{d_0 = L_0 – L_1 = 10.000.000 – 9929200 = 70800}$

That is, 70800 individuals die from 10,000,000 individuals during the first year i.e. between age 0 and age 1 likewise

$d_{50} = L_{50} – L_{51}$

So $dx = L_x – L_{x+1}$ (6.2)

It is worthwhile to mentioned that, number of living at age 0 equals summation of number of dying in the table ,that is

$L_0 = d_0 + d_1 + d_2 + ... + d_{n-1}$ (where n = 100)

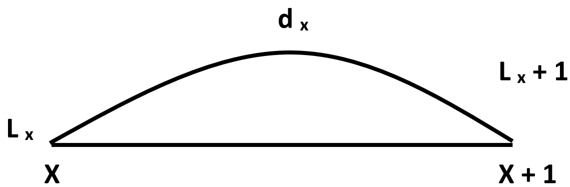
That is,

$L_0 = d + d_1 + d_2 + ... + d\ 99$ (6.3)

d)Probability of death (q_x): q_x means the probability that an individual his age (x) will die before the attainment of age (x + 1) that is (using probabilities theory)

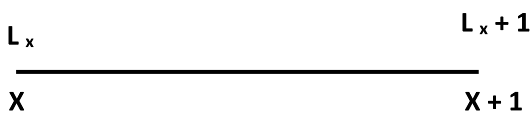
$$q_x = \frac{d_x}{L_x} = \frac{L_x - L_{x+1}}{L_x}$$

(6.4)



for example: $\mathbf{q}_{30} = \frac{\mathbf{d}_{30}}{\mathbf{L}_{30}} = \frac{\mathbf{20193}}{\mathbf{9484358}}$

__e)Probability of survival (P_x):__ P_x means the probability that an individual his age (x) will survive to age (x + 1)



That is

$$\mathbf{P_x = \frac{L_{x+1}}{L_x}} \qquad (6.5)$$

For example: $\mathbf{P_{30}} = \frac{\mathbf{L_{31}}}{\mathbf{L_{30}}} = \frac{\mathbf{9480358}}{\mathbf{9484358}}$

since $P_{30} + q_{30} = 0.99787 + 0.00213 = 1$

So, we conclude that

$$q_x + p_x = 1 \text{ and } q_x = 1 - p_x \quad (6.6)$$

Also $\quad p_x = 1 - q_x$

Other relationships among L_x , d_x , q_x , and P_x

i)Relationship between L_x and q_x

since $\mathbf{q_x} = \frac{\mathbf{d_x}}{\mathbf{L_x}}$

So, we conclude that: $\mathbf{L_x} = \frac{\mathbf{d_x}}{\mathbf{q_x}}$ **and** $\mathbf{d_x} = \mathbf{L_x} . \mathbf{q_x}$

ii) Relationship between L_x and P_x

since $\mathbf{P_x} = \dfrac{\mathbf{L_{x+1}}}{\mathbf{L_x}}$

So, we conclude that $\mathbf{L_x} = \dfrac{\mathbf{L_{x+1}}}{\mathbf{P_x}}$ and $\mathbf{L_{x+1}} = \mathbf{P_x} \,.\mathbf{L_x}$

iii) $L_x - L_{x+n} = d_x + d_{x-1} \ldots\ldots + d_{x+n-1}$

For example: $L_{10} - L_{98} = d_{10} + d_{11} + \ldots. + d_{98}$

iv) $L_{x-1} = d_{x-1}$ i.e. $L_{99} = d_{99}$

6.3.2-Construction The Mortality Table

After study of mortabity table and its contents the following question may be raised:

<u>How can we construct the mortabity table?</u>

The answer:

The mortality table, may be constructed according to the following steps:

- Determining the *<u>radix</u>* of the table.

- Using the foregoing relationships, in particular, the relationships $q_x = 1 - p_x$ and $p_x = 1 - q_x$ for calcutating probabilities of death and survival
- Finding number of living for all ages using the relationships $L_{x+1} = L_x \cdot P_x$

  , $L_{x+2} = L_{x+1} \cdot P_{x+1}$ …. and so on

- Finding number of dying for all ages using the relationships $d_x = L_x - L_{x+1}$

  , $d_{x+1} = L_{x+1} - L_{x+2}$ …. and so on

Other Method: The mortabity table may be constructed by finding both number of dying and number of living in the first age, then the second age …. and so on up to the last age.

6.4-Expectation of life

As pointed out earlier, the person does not predict ,how many year can live. However, he can get an estimate using *the expectation of life* for persons of his age and his sex. Hence, we may ask the following questions:

What is the expectation of life?

Expectation of life at age (x) is meant*," the average number of year to be lived in future by persons now aged(x)"*

Expectation of life is classified into two types, they are:

i)*Curtate expectation of life (e_x):*

For calculating the curtate expectation (e_x) we assume that:

1) We have a group of individuals (L_x) aged (x) as indicated in the following diagram.

L_x	L_{x+1}	L_{x+2}	....	L_{w-1}	L_w
X	**X + 1**	**X + 2**		**W-1**	**w**

2) All deaths that occur for the individuals in any year take place at the beginning of that year (i.e. fractional parts of years are neglected).

Hence, L_{x+1} individuals out of the original group L_x will survive to the end of the first year

and consequently, the number of total years that L_{x+1} survive for 1 year = $L_{x+1} \times 1 = L_{x+1}$ years ***likewise*** L_{x+2} individuals (out of

L_{x+1}) will survive to the end of the second year and consequently, the number of total years that L_{x+2} survive for 1 year = $L_{x+2} \times 1 = L_{x+2}$ years and

so ----- on up to age w – 1 (where w is the terminal age of the mortality table)

L_{w-1} individuals (out of L_{w-2}) will survive to the end of the year. Consequently, the number of total years that L_{w-1} survive.

for 1 year = $L_{w-1} \times 1 = L_{w-1}$

By adding these whole years together and dividing by the number of individuals L_x in the original group, we get the average number of years to be lived in the future by individuals now aged x, that is:

$$\mathbf{e_x} = \frac{\mathbf{L_{x+1} + L_{x+2} + L_{x+3} + \dots\dots\dots\dots + L_{w-1}}}{\mathbf{L_x}} \qquad (6.7)$$

$\mathbf{e_x}$ is *<u>The curtate expectation</u>* with negligence of the fractional parts of years.

<u>ii)The complete expectation of life ($\mathring{e}_x$)</u>

Given that deaths that occur for individuals in any year take place at the end of year, Hence

$$\boxed{e_x = \frac{L_x + L_{x+1} + \ldots\ldots\ldots\ldots + L_{w-1}}{L_x} \qquad (6.8)}$$

However, the deaths do not take place neither at the beginning of year nor at the end of the year but they are distributed throughtt the year. So, *the complete expectation of life* will equal the arithmatic average of formulas (6.7) and (6.8), as follows:-

$$\overset{\circ}{e}_x = \frac{1}{2} x \left[\frac{L_{x+1} + L_{x+2} + \ldots\ldots\ldots\ldots + L_{w-1}}{L_x} + \frac{L_x + L_{x+1} + L_{x+2} + \ldots\ldots\ldots\ldots + L_{w-1}}{L_x} \right]$$

$$\overset{\circ}{e}_x = \frac{1}{2} x \left[\frac{L_x + 2L_{x+1} + 2L_{x+2} + \ldots\ldots\ldots\ldots + 2L_{w-1}}{L_x} \right]$$

$$\overset{\circ}{e}_x = \frac{1}{2}\left(\frac{L_x}{L_x} \right) + \frac{1}{2} x2 \left(\frac{L_{x+1} + L_{x+2} + \ldots\ldots\ldots\ldots + L_{w-1}}{L_x} \right)$$

$$\overset{\circ}{e}_x = \frac{1}{2} + \left(\frac{L_{x+1} + L_{x+2} + \ldots\ldots\ldots\ldots + L_{w-1}}{L_x} \right)$$

That is :

$$\boxed{\overset{\circ}{e}_x = \frac{1}{2} + e_x = \frac{1}{2} + \frac{1}{L_x} \sum_{t=1}^{w-x-1} L_{w+t} \qquad (6.9)}$$

A Notice: *The complete expectation of life* takes into consideration the fractional parts of years. Moreover, it is useful in making comparison between the various of mortality tables.

Solved problems

Example 1:

Given that mortality rates of population in TANTA over the ages 30,31,32,33,34 and 35 as follows:

$q_{30} = 0.0155$, $q_{31} = 0.0156$, $q_{32} = 0.0158$

$q_{33} = 0.0159$, $q_{34} = 0.0160$, $q_{35} = 0.0162$

Required: Construction of a mortality table in TANTA, if you know $L_{30} = 1000,000$ individuals

solution

A life table may be constructed according to the following steps.

First step: Finding survival rates by relationship $P_x = 1 - q_x$,

$P_x = 1 - q_x$

P_{30} = 1 – 0.0155 = 0.9845 and so on for the next ages as indicated in the following table:

x	***L_x***	***d_x***	***q_x***	***P_x***
30	1000,000		0.0155	0.9845
31	984500		0.0156	0.9844
32			0.0158	09842
33			0.0159	0.9841
34			0.0160	0.9840
35			0.0162	0.9838

<u>Second step</u>: Finding number of living (L_x) by relationship L_{x+1} = L_x . P_x

L_{31} = L_{30} . P_{30}

For next age as indicated int the following table:

x	***L_x***	***d_x***	***q_x***	***P_x***

30	1000,000		0.0155	0.9845
31	984500		0.0156	0.9844
32	696142		0.0158	09842
33	953829		0.0159	0.9841
34	938663		0.0160	0.9840
35	923645		0.0162	0.9838

Third step: Finding number of dying (dx) by relationship $d_x = L_x - L_{x-1}$

$D_{30} = L_{30} - L_{31}$ = 1000,000 – 984500 = 15500 and so on for next ages as indicated in the following table

x	L_x	d_x	q_x	P_x
30	1000,000	15500	0.0155	0.9845
31	984500	15358	0.0156	0.9844
32	696142	15313	0.0158	09842
33	953829	15166	0.0159	0.9841

34	938663	15018	0.0160	0.9840
35	923645	14963	0.0162	0.9838

<u>Exampl 2</u>: Complete the following mortality table:

x	L_x	d_x	q_x	P_x
40		370		
41	99630			
42	99231			
43		454		
44	98350			0.9950

Solution

Using the relashinships that have already studied, we may complete the previous table, age by age, as follows:

1) *<u>age 40</u>*:

$L_x = L_{x+1} + d_x$

$L_{40} = L_{41} + d_{40} = 99630 + 370 = 100{,}000$

Also $q_{40} = \dfrac{d_{40}}{L_{40}} = \dfrac{370}{100000} = 0.00370$

So, $P_{40} = 1 - q_{40} = 1 - 0.00370$

2-) *age 41:*

$d_{41} = L_{41} - L_{42} = 99630 - 99231 = 399$

$q_{41} = \dfrac{d_{41}}{L_{41}} = \dfrac{399}{99630} = 0.00400$

$P_{41} = 1 - 0.00400 = 0.99600$

$L_{43} = L_{42} - d_{42} = 99231 - 427 = 98804$

3) *age 42:*

$d_{43} = L_{44} + d_{43} = 98350 + 454 = 98804$

$q_{42} = L_{41} - L_{43} = 99231 - 98804 = 427$

$q42 = \dfrac{d_{42}}{L_{42}} = \dfrac{427}{99231} = 0.00430$

4)*age 43 and 44*:

$$q_{43} = \frac{d_{43}}{L_{43}} = \frac{454}{98804} = 0.0046$$

$P_{43} = 1 - 0.0046 = 0.99540$

$q_{44} = 1 - 0.995 = 0.005$

$d_{44} = L_{44} \times q_{44} = 98350 \times 0.005 = 492$

Hence, the mortality table will be completed as follows:

x	***L_x***	***d_x***	***q_x***	***P_x***
40	1000000	370	0.0037	0.99630
41	99630	399	0.0040	0.9960
42	99231	427	0.0043	0.9957
43	98804	454	0.0046	0.9954
44	98350	492	0.0050	0.9950

A notice: The preceding table may be constructed by other method by completing L_x first, then d_x, then q_x and p_x .. *try by yourself*.

<u>Example 3</u>: Calculate both the curtate expectation of life and the complete expectation of life for the ages 95 – 100 in the following table:

x	L_x	e_x	$\overset{\circ}{e}_x$
95	59		
96	35		
97	12		
98	7		
99	3		
100	1		
101	0		

Solution

The curtate expection of life (e_x) can be constructed by the following relationship:

$$e_x = \frac{L_x + L_{x+1} + \dots\dots\dots\dots + L_{w-1}}{L_w}$$

Then, *the complete expectation of life* can be constructed by the relationship $\overset{\circ}{e}_x = e_x + \frac{1}{2}$ as shown below:

$$e_{95} = \frac{L_{96} + L_{97} + L_{98} + L_{98} + L_{99} + L_{100} + L_{101}}{L_{95}}$$

$$= \frac{35 + 12 + 7 + 3 + 1 + 0}{59} = 0.9830$$

$$\overset{\circ}{e}_{95} = 0.5 + 0.9830 = 1.4830$$

Also :

$$e_{96} = \frac{L_{97} + L_{98} + L_{98} + L_{99} + L_{100} + L_{101}}{L_{96}}$$

$$= \frac{12 + 7 + 3 + 1 + 0}{35} = 0.6571$$

$$\overset{\circ}{e}_{95} = 0.5 + 0.6571 = 1.1571$$

and so on for the next ages (97 – 100), then the table will become as follows:

x	**L_x**	**e_x**	$\overset{\circ}{e}_x$
95	59	0.9830	1.4830
96	35	0.6571	1.1571
97	12	0.61661	1.4166
98	7	0.5714	1.0714
99	3	0.3333	0.8333
100	1	0	0.500

101	0	-	-

6.5-Probabilities of Death and Survival for a Person

As we have already seen ,the heart of the morality table is, $\mathbf{q_x}$, that is called *__probability of death__*. This probability has a vital role in calulating *__the net premium__* for any life insurance policy where, The premium equals the probability (q_x or p_x) multiplied by sum insured.

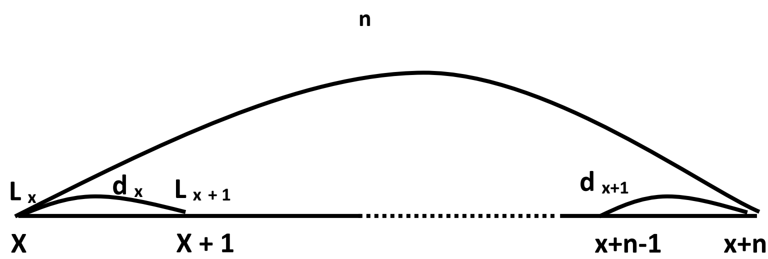
Also the probability (q_x or p_x) is calculuted just only for one year, in spite of all life insurance policies are long-term policies.

Hence, probabilities of death and survival for ***__n years__*** are very necessary for calculating the premiums of various life insurance policies that are issued for n years. These probabilities may be shown as below:

6.5.1-Probability of death for a person aged (x) over

n year($^{n}q_x$)

The symbol $^{n}q_x$ means the probability that a person aged (x) will die before reaching age (x+n). *In other words*, the probability that a person will die over n years. That is between age (x) and age (x+n) as indicated in following diagram



Consquently, $$n_{q_x} = \frac{d_x + d_{x+1} + d_{x+2} + \ldots\ldots\ldots\ldots + + d_{x+n-1}}{L_x} = \frac{L_x - L_{x+n}}{L_x}$$

(6.10)

6.5.2-Probability of survival for a person aged (x) n years ($^n p_x$)

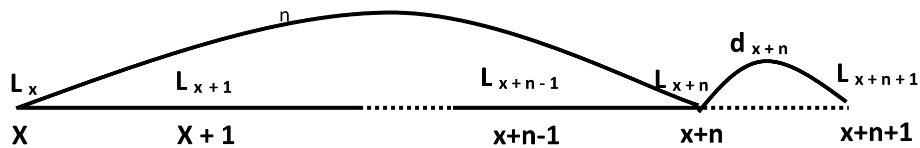
The $^n p_x$ means the probability that a person aged (x) will live to reach age (x+n). That is out of the L_x persons alive at age (x) there are L_{x+n} survivors at age (x+n) as indicated in the preceding diagram. Hence,

$$nP_x = \frac{L_{x+n}}{L_x} \qquad (6.11)$$

6.5.3- Probability of survival of a person n years

and his death over 1 year ($^n/q_x$)

The sympol $^{n}/q_{x}$ means the probability that a person aged (x) will live to reach age (x+n), they die between age (x+n) and age (x+n+1) as indicated in the following diagram.



Hence,

$$^{n}/q_{x} = \frac{L_{x+n} - L_{x+n-1}}{L_{x}} \qquad (6.12)$$

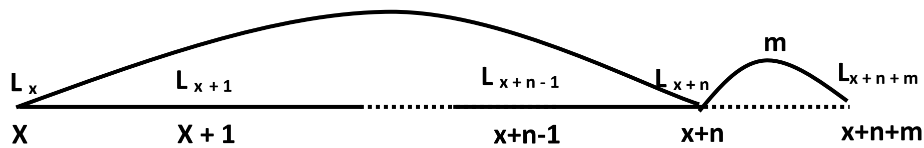
or

$$^{n}/q_{x} = \frac{d_{x+n}}{L_{x}} \qquad \text{(6.12) repeated}$$

2.5.4-probability of survival of a person n year and

his death over m years ($^{n}/{}^{m}q_{x}$)

This symbol ${}^{n}/{}^{m}q_x$ means the probability that a person aged (x) will live to reach age (x+n) then die between age (x + n) and age (x + n + m) as indicated in the following diagram:



Hence

$$ {}^{n}/{}^{m}q_x = \frac{d_{x+n} + d_{x+n+1} + d_{x+n+2} + \ldots\ldots + d_{x+n+m-1}}{L_x} \qquad (6.13)$$

or

$$ {}^{n}/{}^{m}q_x = \frac{L_{x+n} - L_{x+n+m}}{L_x} = {}^{n}p_x - {}^{n+m}p_x \quad (6.13) \text{ repeated}$$

<u>In conclusion</u> by contemplating the preceding notation, it should be noted that:

a) The letter ***P*** with the proper subscripts is used to denote the probability of a person living a given period

b) The letter ***q*** is used to denote the probability of a person dying during a given period

Solved problem

Example 4

Interpret in words the following symbols, then calculate their values using the American life table (1958 CSO)

a) q_{25} , P_{60}

b) ${}^{5}P_{30}$, ${}^{7}q_{27}$

c) ${}^{6}/q_{35}$, ${}^{7}/{}^{5}q_{25}$

solution

a) **q_{25}**: means a probability that a person aged (25) will die over one year. That is between age (25) and age (26)

$$q_{25} = \frac{L_{25} - L_{26}}{L_{25}} = \frac{9575636 - 9557155}{9575636} = 0.00193$$

or

$$q_{25} = \frac{d_{25}}{L_{25}} = \frac{18981}{9575636} = 0.00193$$

- **P_{60}:** means a probability that a person aged (60) will live to reach age (61)

$$P_{60} = \frac{L_{61}}{L_{60}} = \frac{7542106}{7698698} = 0.9797$$

<u>b)$^{5}p_{30}$</u>: means a probability that a person aged (30) will live to reach age (35)

$$^{5}P_{60} = \frac{L_{35}}{L_{30}} = \frac{9373807}{9480358} = 0.9888$$

- $^{7}q_{25}$: means probability that a person aged (25) will die over seven years. That is between age 25 and 1ge 32

$$^{7}q_{25} = \frac{L_{25} - L_{32}}{L_{25}} = \frac{9575636 - 9439447}{9575636} = 0.0142$$

<u>c)$^{6}/q_{35}$</u>: means a probability that a person aged (35) will live to reach age (41), then die between age (41) and age (42):

$$6/q_{35} = \frac{L_{41} - L_{42}}{L_{35}} = \frac{9208737 - 9173375}{9373807} = 0.0037$$

or

$$6/q_{35} = \frac{d_{41}}{L_{35}} = \frac{35362}{9373807} = 0.0037$$

- $^{7}/^{5}q_{25}$: means a probability that a person aged (25) will live to reach age (32) then die between age (32) and age (37)

$$^{7}/^{5}\mathbf{q}_{25} = \frac{\mathbf{L}_{32} - \mathbf{L}_{37}}{\mathbf{L}_{25}} = \frac{\mathbf{9439447 - 9325594}}{\mathbf{9575636}} = \mathbf{0.01189}$$

or

$$^{7}/^{5}\mathbf{q}_{25} = \frac{\mathbf{d}_{32} + \mathbf{d}_{33} + \mathbf{d}_{34} + \mathbf{d}_{35} + \mathbf{d}_{36}}{\mathbf{L}_{25}}$$

$$= \frac{\mathbf{21239 + 21850 + 22551 + 23528 + 24685}}{\mathbf{9575636}}$$

$$= \mathbf{0.01189}$$

Example 5

Using Mortality table (1958 CSO) calculate the following probabilities:

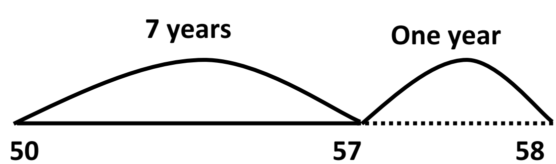
i) A probability that a peson aged (50) will die at age (58)

ii) A probability that a peson aged (50) will die between age (60) and age (70)

Solution

i) A probability that a person aged (50) will die

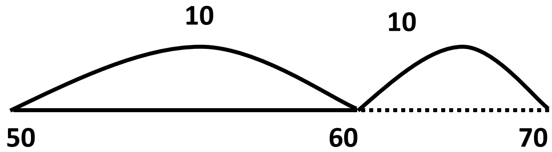
at age (58) means the person will live to reach age (57), then die between age (57) and age (58) i.e. he will die over one year, as shown below:



Hence:

The probability = $^{7}/q_{50} = \frac{d_{57}}{L_{50}} = \frac{125970}{8762306} = 0.0144$

ii)A probability that a person aged (50) will die between age (60) and age (70) means the person will live to reach age 60 then die between age (60) and age (70) as shown below:



Hence: The probability

$$= {}^{10}/{}^{10}q_{50} = \frac{L_{60} - L_{70}}{L_{50}} = \frac{7698698 - 5592012}{8762307} = 0.0240$$

Exercises

1- Given that L_{20} = 1000000 and mortality rates at ages 20,21,22,23,25,and 26 are 0.0030,0.0032,0.0034 , 0.0036 , 0.0038 and 0.0040 respectively,

Required:

i) Construct a mortality table over ages 20-26

ii) Calculate $\overset{\circ}{e}_{22}$

2- Given that: $P_{x+1} = 0.997 - 0.001\,(x - 30)$ and the number living at age of 44 = 100000

Required:

i) Constitute a mortality label over age 40 to 43

ii) $\overset{\circ}{e}_{40}$

3- Complete the following mortality table:

x	L_x	d_x	q_x	P_x	$\mathring{e}_x$
30			0.022		
31		379		0.9977	
32			0.027		
34				0.9922	
35	50000				

4- Given that number of birth in Egypt at year 2005 was 1250000 child. Compute using Mortality Table (1958 CSO) the following:

i)Number of children who will admit in the primary schools (6 years)

ii)Number of pupils who graduated from secondary schools (18 years)

iii)Number of people who retired (60 years)

iv)Number of dying between age 20 and age 57

v)Number of living in 2010

5- Verify in the table 1958 CSO that :

$L_{30} - L_{35} = D_{30} + d_{31} + d_{32} + d_{33} + d_{34}$

6- Given that a probability that a person aged (35) will live to reach age (40) = 0.18 and number of living over ages 35 – 45 as indicated in the following table

X	35	36	37	38	39	40	41	42	43	44	45
L_x	10000	9000	8500	8000	7000	65000	6000	5400	5000	4800	4500

Find the following probabilities using the preceding table.

i)A probability that a person aged (35) will live to reach age (40) but he not live to reach age (45).

ii)A probability that a person aged (36) will live to reach age (39).

iii)A probability that a person aged (37) will die before to reach age (45).

v)A probability that a person aged (37) will

die after to reach age (42).

vi)A probability that a person aged (37) will live at least 8 years.

vii)Number of persons who live to reach age (40)

7- Write only the proper symbol of the following probabilities

   i) A person aged (40) will die in the year following the attainment of age (55).

   ii) A person aged (30) will live to reach age (45) but he not live to age 50.

   iii) A person aged (25) will live at least 15 years more.

   iv) A person aged (35) will live 15 years then die.

   v) A person aged (42) will die between ages 50 and 60.

   vi) A person aged (45) will die within 15 years.

8- Interpret in words the probabilities represented by the following symbols $^{7}/q_{20}$, $^{11}P_{22}$, $^{18}/^{3}q_{30}$, $^{10}/^{10}q$

9- Prove the following indentities:

   i) $^{n+m}P_x + {}^{n/m}q_x = {}^{n}P_x$

ii) ${}^{n+1}p_x + {}^{n}/q_x = {}^{n}P_x$

iii) ${}^{n}P_x = = \dfrac{{}^{n}P_x - {}^{n+1}P_x}{q_{x+n}}$

10- Prove that $e_x = P_x + 2P_x + 3P_x + \ldots\ldots$

11- Show that:

$1 + e_x = q_x + P_x\,(1 + q_{x+1}) + 2\,P_x\,(1 + q_{x+2})$

12- From a group of 100000 now age 20, how many will probably:

a) be alive at age 40

b) die after reaching age 40 but before reaching 42

Important concepts and terminologies to remember:

Net premium

Complete expectation of life

Curtate expectation of life

Expection of life

Gross premium

Interest

Level premiun

Loading

Mortality table

Natural premium

Net level premium

Single premium

The sympols ${}^{n}q_{x}$, ${}^{n}P_{x}$, ${}^{n}/q_{m}$, ${}^{n}/{}^{m}q_{x}$, e_{x} , $\mathring{e}_{x}$, L_{x} , d_{x} , P_{x} , q_{x}