Chapter 8

Section 8.1: Sequences;

Do the following problems from the book;

3, 5, 7, 11, 12, 13, 14, 16, 17, 18, 20, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 41, 42, 45, 46.

Section 8.2: Convergent or Divergent Series;

Do the following problems from the book;

2, 4, 5, 6, 8, 10, 14, 15, 18, 20, 25, 28, 30, 34, 37, 38, 39, 40, 42, 43, 45, 46, 50, 57, 58.

Section 8.3 : Positive term Series;

Do the following problems from the book;

2 - 11, 14, 15, 16, 18, 20, 22, 23, 24, 25, 26, 30, 31, 33, 34, 35, 39, 40, 42, 43, 45, 46, 57, 58.

Section 8.4: The Ratio and Root Test

Do the following problems from the book;

2, 4, 6, 8, 9, 10, 11, 12, 14, 15, 16, 18, 20, 21, 22, 23, 25, 27, 28, 29, 31, 32, 33, 34, 35, 38.

Section 8.5: Alternating Series and Absolute Convergence;

Do the following problems from the book;

2 - 7, 9, 10, 12, 13, 14, 16, 18, 19, 20, 21, 22, 27, 28, 29, 32, 33, 34, 35, 38, 40, 41, 43, 44, 45, 46.

Section 8.6: Power Series;

Do the following problems from the book;

5, 6, 7, 8, 14, 15, 19, 23, 25, 27, 30, 35, 36, 41, 42, 44, 45, 46.

Section 8.7: Power Series Represention of Functions;

Do the following problems from the book;

2, 4, 6, 7, 10, 13, 14, 16, 19, 22, 25, 29, 30, 32, 33, 34, 37.

Section 8.8: Maclaurin and Taylor Series;

Do the following problems from the book;

2, 4, 8, 10, 13, 15, 18, 19, 21, 26, 29, 32, 34, 36, 38, 39, 42.

MATH 201

HOME WORK PROBLEMS

Title of the book used for this course is : **Calculus**, sixth edition, by Swokowski, Olinick and Pence.

In this coures we cover chapters 8, 12 and 13,
section 10.6 should be read by the students.

Chapter 12

Section 12.1 : Functions of Several Variables;
Do the following problems from the book
1, 3, 5, 8, 9, 14, 21, 22, 23, 24, 26, 27, 47, 51.

Section 12.2 : Limits and Continuity;
I- Do the following problems from the book
3, 5, 6, 9, 12, 14, 16, 19, 20, 25, 26, 28, 29, 36, 38, 42.

II- Find the following limits, if they exist:

1) $lim_{(x,y,z)\to(0,0,0)} \dfrac{zy^2}{x^2+y^2+z^2}$, 2) $lim_{(x,y)\to(2,1)} \dfrac{(y-1)(x-2)^2}{(y-1)^3+(x-2)^3}$.

3) $lim_{(x,y)\to(0,0)} \dfrac{xy^3}{x^3+y^6}$, 4) $lim_{(x,y)\to(0,0)} \dfrac{3x^2y}{x^4+y^2}$, 5) $lim_{(x,y)\to(0,0)} \dfrac{10xy}{5x^3+2y^3}$

6) $lim_{(x,y,z)\to(0,0,0)} \dfrac{y^3+x^3 sinz^3}{x^2+y^2+z^2}$, 7) $lim_{(x,y)\to(0,0)} \dfrac{x^3-x^2y+xy^2-y^3}{x^2+y^2}$

8) $lim_{(x,y)\to(0,0)} [\dfrac{4x^2y}{x^4+y^2} + \dfrac{y^4}{x^2+y^2}]$, 9) $lim_{(x,y)\to(1,-1)} \dfrac{2x-y}{x^2+y^2}$

III- Discuss the continuity of the following functions on their domain:

1. $f(x,y) = \begin{cases} \frac{x^2y}{x^4+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

2. $f(x,y,z)=\begin{cases} \frac{x^3+y^3+z^3}{x^2+y^2+z^2}, & (x,y,z)\neq(0,0,0) \\ 0, & (x,y,z)=(0,0,0) \end{cases}$

3. $f(x,y,z)=\begin{cases} \frac{xz-y^2}{x^2+y^2+z^2}, & (x,y,z)\neq(0,0,0) \\ 0, & (x,y,z)=(0,0,0) \end{cases}$

4. $f(x,y)=e^{x^2+5xy+y^3}$.

5. $h(x,y)=sin(\sqrt{y-4x^2})$.

6. $k(x,y,z)=ln(36-4x^2-y^2-9z^2)$.

Section 12.3 : Partial Derivatives;

I- Do the following problems from the book
4, 6, 8, 12, 13, 16, 17, 21, 22, 27, 29, 32, 34, 36, 39, 42, 44, 47.

II- Do the following problems;

1. Using the defintion, find f_x, f_y of the function

$$f(x,y)=3x^2-2xy+y^2.$$

2. Discuss the continuity of the function f at $(0,0)$, where

$$f(x,y)=\begin{cases} \frac{sin\,xy}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$$

Does f_x and f_y exist at $(0,0)$.

3. Let $f(x,y)=\begin{cases} \frac{x^2y}{x^3+y^3}, & x^3+y^3\neq 0 \\ 0, & x^3+y^3=0 \end{cases}$

Find f_x, f_y at $(0,0)$, if they exist.

4. Let $f(x,y)=\begin{cases} \frac{3x^2}{2x-y}+\frac{y^3}{y-2x}, & y\neq 2x \\ 12, & y=2x \end{cases}$

Find f_x, f_y at $(1,2)$, if they exist.

5. Let $f(x,y) = \begin{cases} \frac{x^2-y^2}{x-y}, & x \neq y \\ 0, & x = y \end{cases}$

Find f_x, f_y at $(1,1)$ and at $(0,0)$, if they exist.

6. *Let* $f(x,y) = e^{x-y} sin(x+y)$. Show that

$$(f_x)^2 + (f_y)^2 = \frac{2(f(x,y))^2}{sin^2(x+y)}.$$

Section 12.4 : Increments and Differentials;
Do the following problems from the book
2, 9, 11, 12, 16, 18, 20, 24(b), 32, 38, 42.

1. Use the differential to approximate the change in the function

$$w = f(x,y,z) = x^2 \, ln(z^2 + y^2)$$

as (x,y,z) changes from $(1,2,3)$ to $(0.9, 1.9, 3.1)$.

2. Use the differential to approximate the change in the function

$$w = f(x,y) = y\,x^{2/5} + x\,\sqrt{y}$$

as (x,y) changes from $(52,16)$ to $(35,18)$.

3. Discuss the continuity and the differentiability of the functions in problems 3 and 4 in section 12.3 (part II) at the indicated points.

4. Discuss the differentiability of the function f, where

$$f(x,y) = \begin{cases} \frac{x^3+y^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

at $(0,0)$ and at $(-1,1)$.

5. Discuss the differentiability of the function f, where

$$f(x,y,z)=\begin{cases}\frac{sin(xyz)}{x^3+y^3+z^3}, & x^3+y^3+z^3\neq 0\\ 0, & x^3+y^3+z^3=0\end{cases}$$

at $(0,0,0)$ and at $(1,0,0)$.

6. Let $f(x,y)=\begin{cases}\frac{xy}{\sqrt{x^2+y^2}}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$

Show that f is continuous at $(0,0)$ but not differentiable at $(0,0)$.

7. Let $f(x,y,z)=\begin{cases}\frac{xy^2z}{x^4+y^4+z^4}, & (x,y,z)\neq(0,0,0)\\ 0, & (x,y,z)=(0,0,0)\end{cases}$

1- Show that $f_x(0,0,0), f_y(0,0,0)$ and $f_z(0,0,0)$ exist.

2- Discuss the differentiability of f at $(0,0,0)$.

8. Let $f(x,y)=\begin{cases}\frac{xy^3}{x^2+y^4}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$

1- Show that f_x and f_y exists for all $(x,y)\in R^2$.

2- Show that f_x and f_y are not continuous at point $(0,0)$.

3- Show that f is not differentiable at $(0,0)$.

Section 12.5 : Chain Rules;

Do the following problems from the book
2, 4, 6, 10, 12, 14, 18, 19, 22, 26, 38, 40, 42.

1. If $w=f(x,y)$ such that $x=r\,cos\,\theta$, and $y=r\,sin\,\theta$. Find $g(r,\theta)$ such that the equation below holds

$$(\frac{\partial w}{\partial x})^2+(\frac{\partial w}{\partial y})^2=(\frac{\partial w}{\partial r})^2+g(r,\theta)\,(\frac{\partial w}{\partial \theta})^2.$$

2. If $u=f(x,y)$ where $y=e^t$ and $x=e^k$. Find

$$\frac{\partial^2 u}{\partial k^2}+\frac{\partial^2 u}{\partial t^2}.$$

3. If $w = x^2 + y^2 + z^2$, where $x = rcos\theta$, $y = rsin\theta$ and $z = r$. Use the differential to show that $dw = 4\,r\,dr$.

4. Let $z = f(x, y)$ be determined implicitly by $yx^2 + z^2 + cos(xyz) - 4 = 0$. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. Then show that

$$2\,y\,\frac{\partial z}{\partial y} - x\,\frac{\partial z}{\partial x} = \frac{xyz\,sin(xyz)}{2z - xy\,sin(xyz)}.$$

Section 12.8: Extrema of Functions of Several Variables ;
Do the following problems from the book;
11, 20, 23, 24, 26, 30, 31, 32.

1. Find the extrema of the function $f(x, y) = (x - 4)^2 + y^2$, on the region R bounded by $y = 4\sqrt{x}$ and $y = 4x$.

2. Let $f(x, y) = xy + 1/x + 1/y$, where $x \neq 0$, $y \neq 0$. Find the local extrema and saddle points of f if they exist.

3. Find the maximum and the minimum of the function $f(x, y) = x^3 - 3x + y^2$ on the region bounded by $x^2 - 2x + y^2 = 0$.

Section 12.9: Lagrange Multipliers ;
Do the following problems from the book;
1, 2, 3, 11.

Chapter 13 : Multiple Integral

Section 13.1: Double Integral ;

Do the following problems from the book;
1 - 10, 13, 16, 18, 19, 20, 21, 23, 25, 26, 27, 29, 31, 32, 33, 37, 38, 39, 43, 44, 50.

1. Sketch the region bounded by the graphs of the given equations, and then evaluate the given integral

   a) $y = x, y = \sqrt{x}, x = 0;\quad \int\int_R \sin y^2 dA.$

   b) $y = x^{3/2}, y = 0,\ x = 1\ ;\quad \int\int_R ye^{x^2} dA.$

2. Evaluate the double integral

$$\int_0^2 \int_{y/2}^1 e^{x^2} dx\, dy.$$

Section 13.2: Area and Volume;

Do the following problems from the book;
2, 4, 6, 7, 11, 14, 18, 22, 24, 27, 28, 30, 31, 32.

1. Sketch the region bounded by the graphs of the equation $y = \sin x$, $y = \cos x$, $x = 0$, $x = \pi/4$.. Then use the double intgeral to find its area.

2. Sketch the region bounded by the graphs of the equation $x = -\sqrt{9 - y^2}$, $y = -2x + 9$, and $y = -3$, $y = 3$. Then use the double intgeral to find its area.

Section 13.3: Double Integral by Polar Coordinate;

Do the following problems from the book;
~~1, 12,~~ 13, 15, 17, 18, 19, 21, 23, 24.

1- Use polar coordinate to evaluate the double integral

$$\int_{-3}^{3} \int_0^{\sqrt{9-x^2}} (x^2 + y^2)^{3/2}\, dy\, dx.$$

Section 13.5: Triple Integral;

Do the following problems from the book;
2, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 23, 26, 28.

1. Sketch the region bounded by the graphs of the equations :

   a) $z = x^2 + y^2$, $y + z = 2$.

   b) $z + y^2 = 4$, $x + z = 4$, $x = 0$, $z = 0$.

   c) $z = 9 - y^2$, $z = 0, x = -1$, $x = 2$.

2. Set up a triple integral for the volume of the region in the first octant, bounded above by the cylinder $z = 1 - y^2$ and lying between the vertical planes $x + y = 1$ and $x + y = 3$.