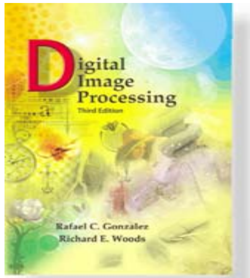
Intensity Transformations, Spatial Filtering, Histogram Processing

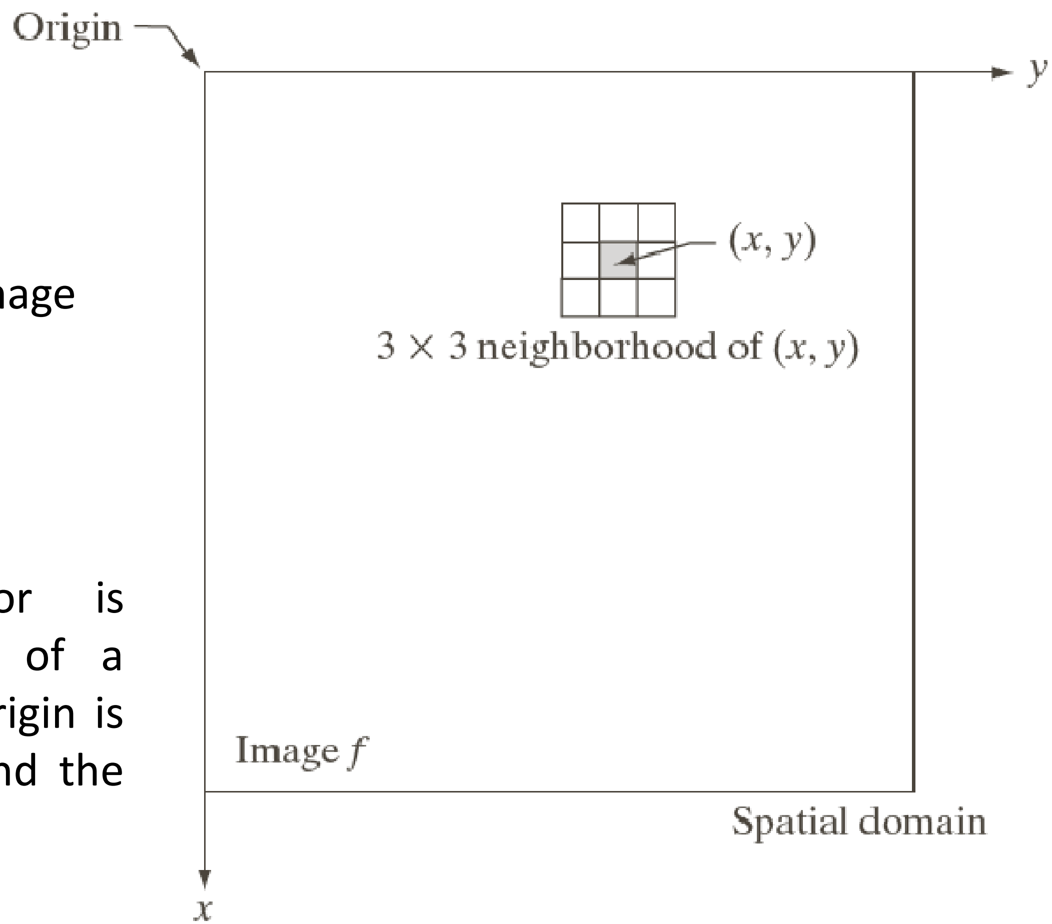


$$g(x, y) = T[f(x, y)]$$

Output image

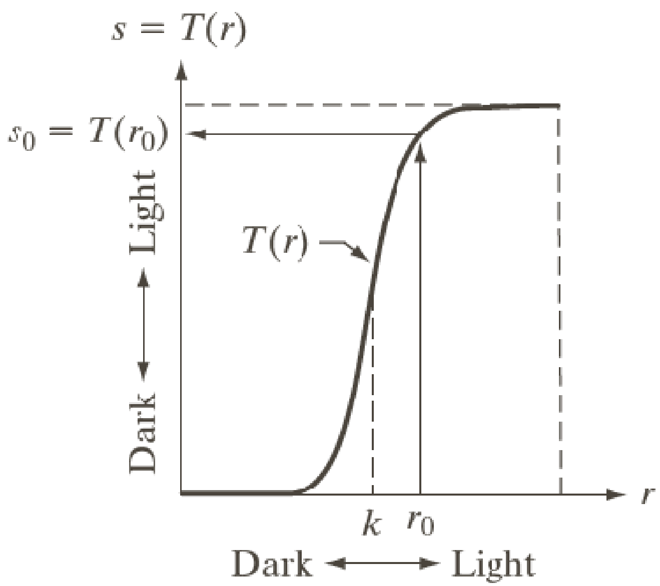
Operator

Input image

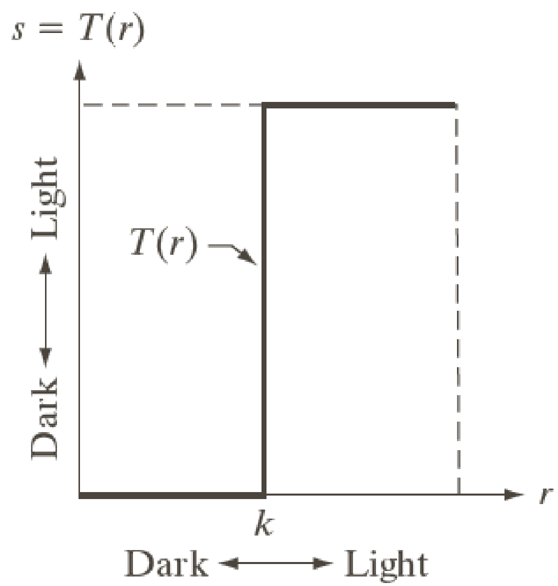


Spatial filtering: Operator is applied on the neighbors of a location (origin); then the origin is moved to a new location and the operation is repeated.

Intensity Transfer Functions



Contrast stretching function.



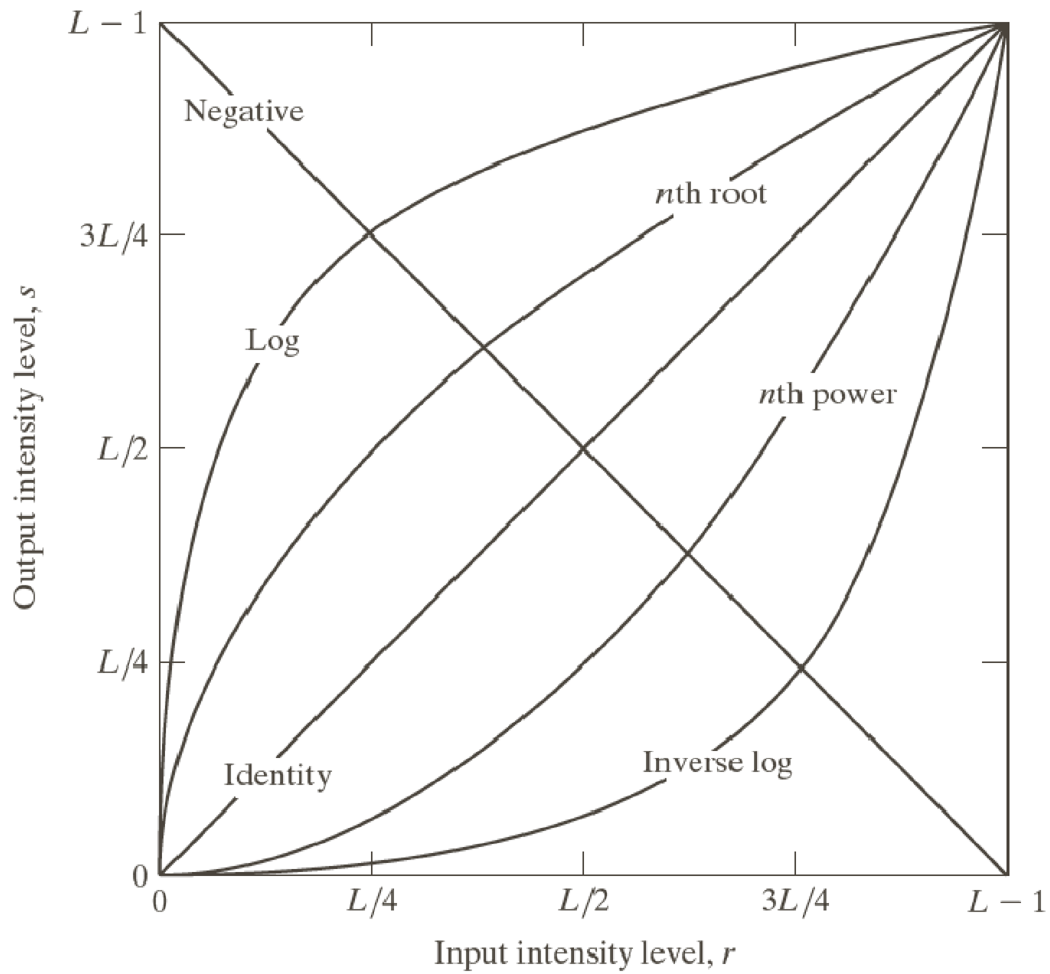
Thresholding function.

r: intensity variable of input image.

s: intensity variable of output image.

Basic Intensity Transfer Functions

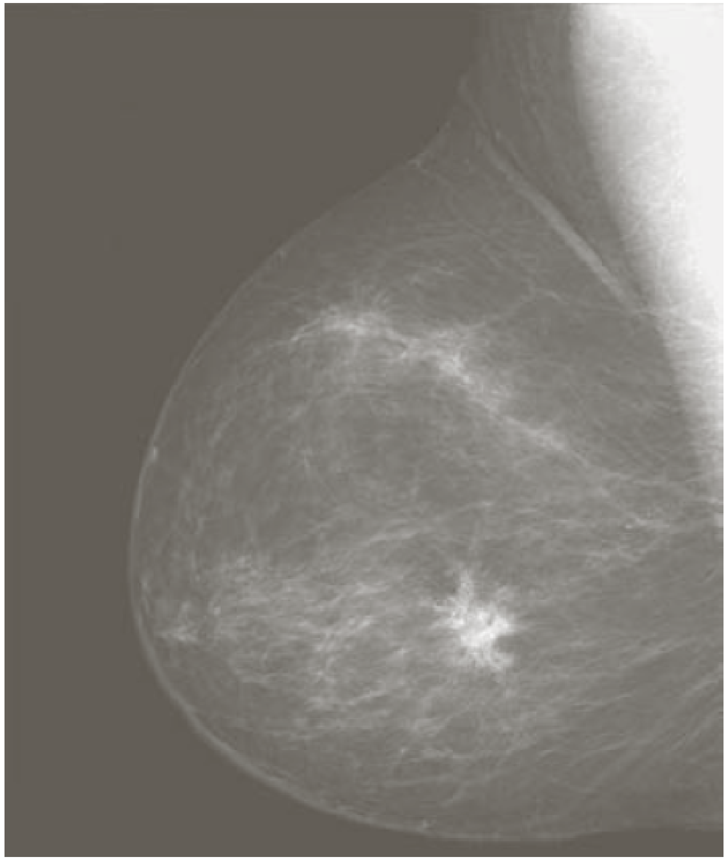
- Linear
- Logarithmic
- Power-law



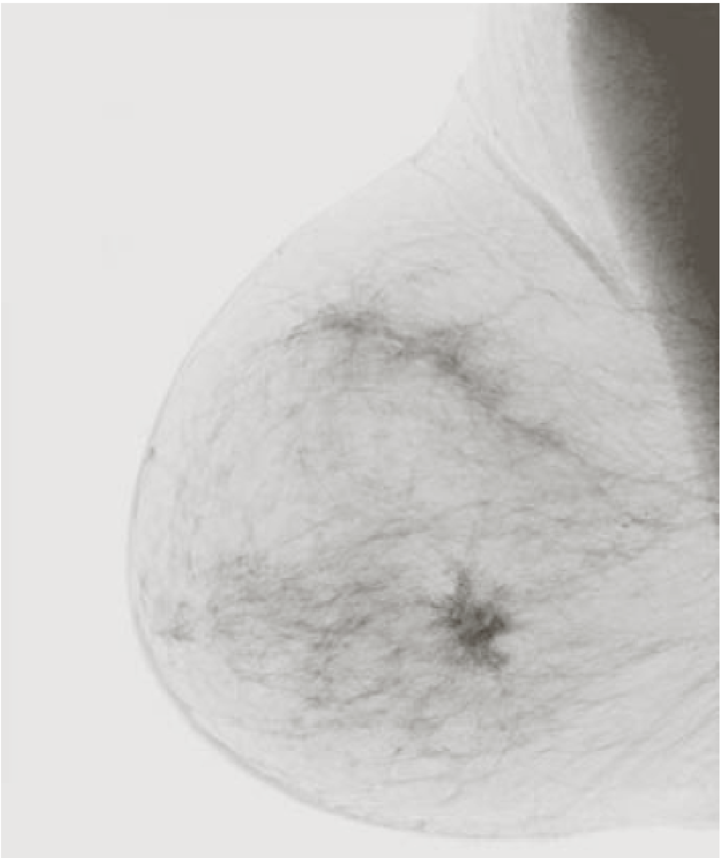
Negative Transformation

Intensity level: [0, L - 1]

$$s = L - 1 - r$$



Original Image



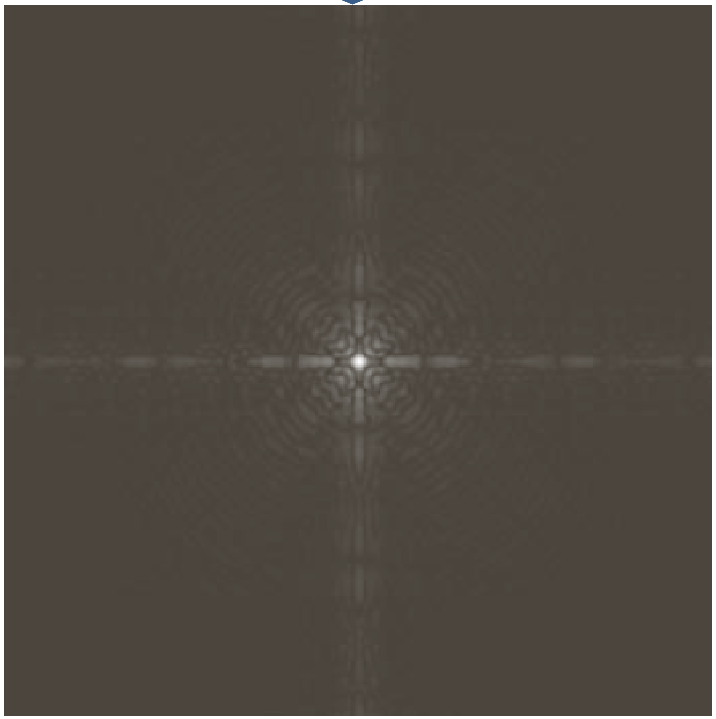
Negative Image

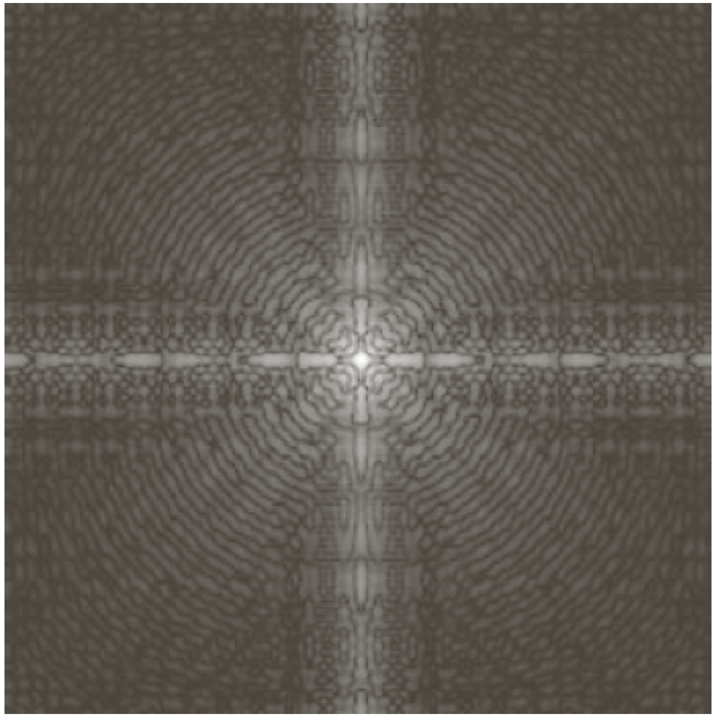
Log Transformation

$$s = c \log(1 + r)$$

Compresses the dynamic range of images with large variations in pixel values.

Loss of details in low pixel values

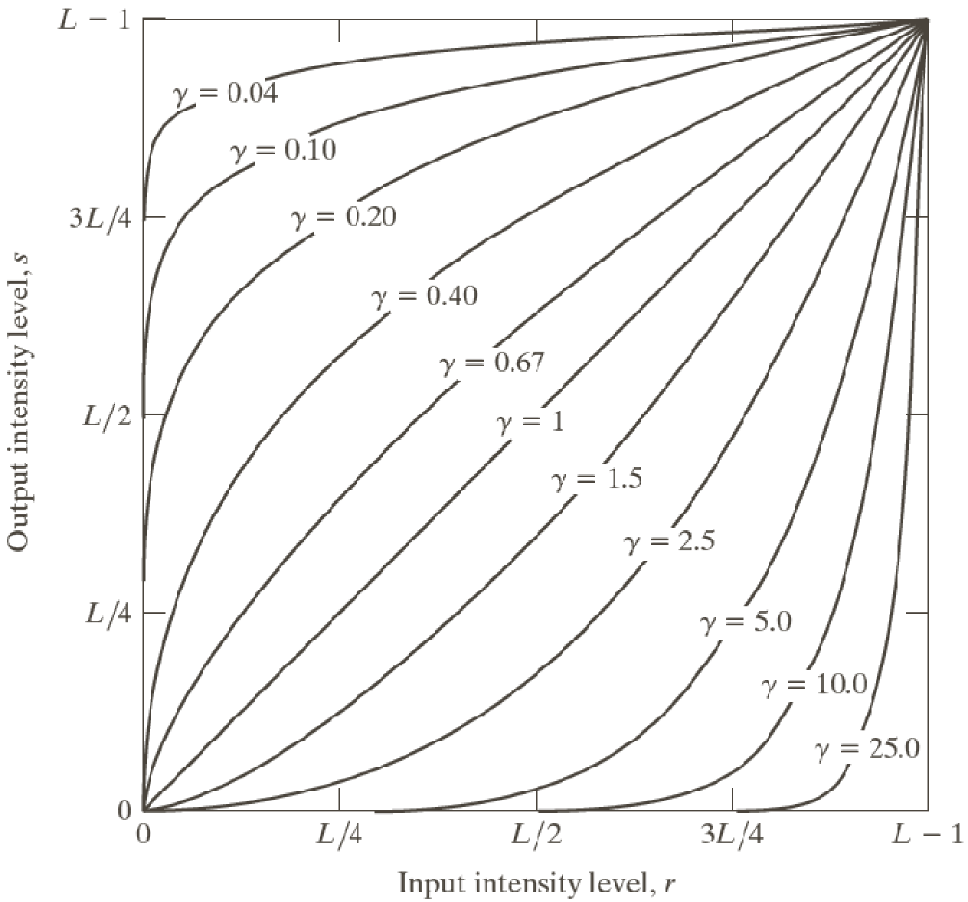




Original Image (Fourier Spectrum) Log Transformed Image

Power-Law (Gamma) Transformation

$$s = cr^{\gamma}$$



Gamma Correction

Process used to correct power-law response phenomena.

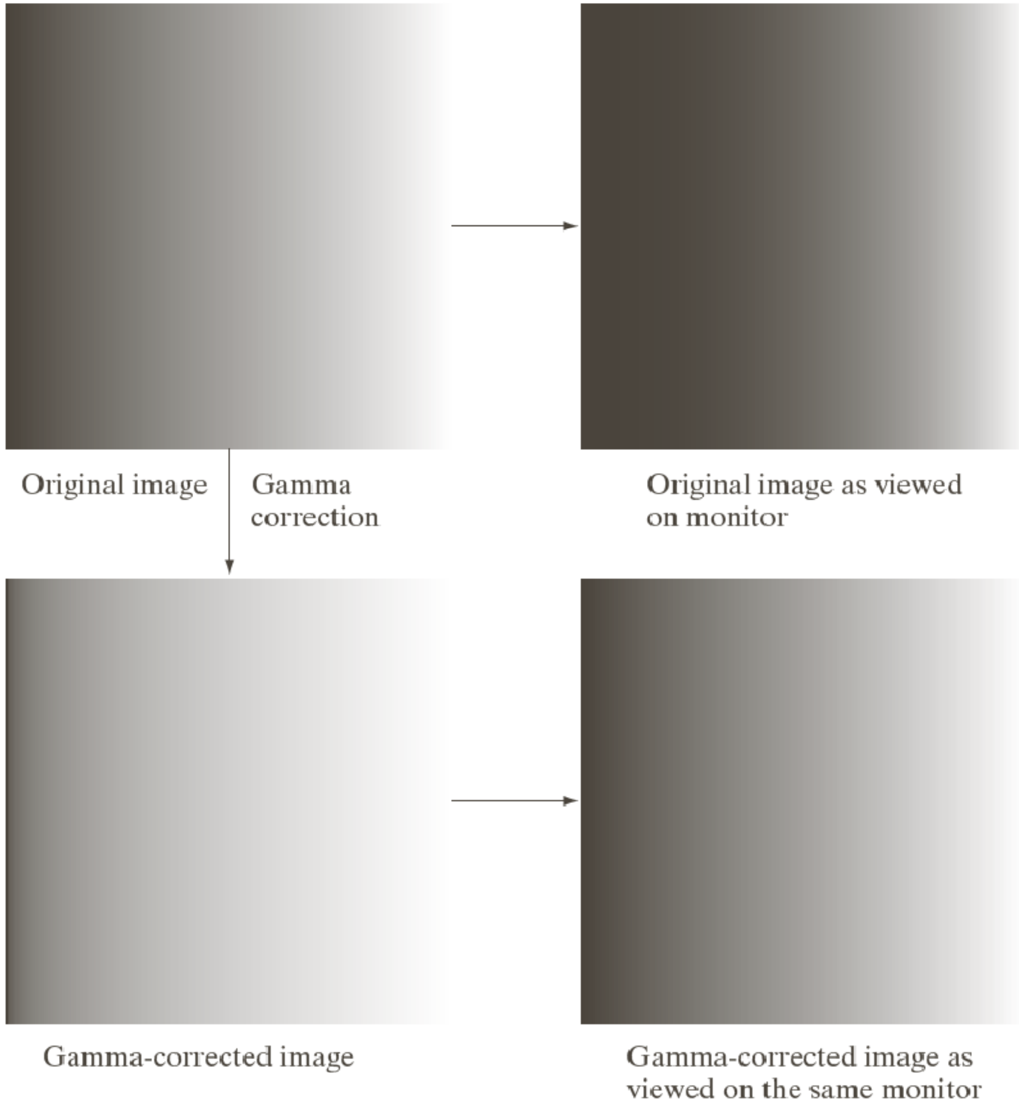
Example: CRT monitor.

Tends to produce images darker.

Solution

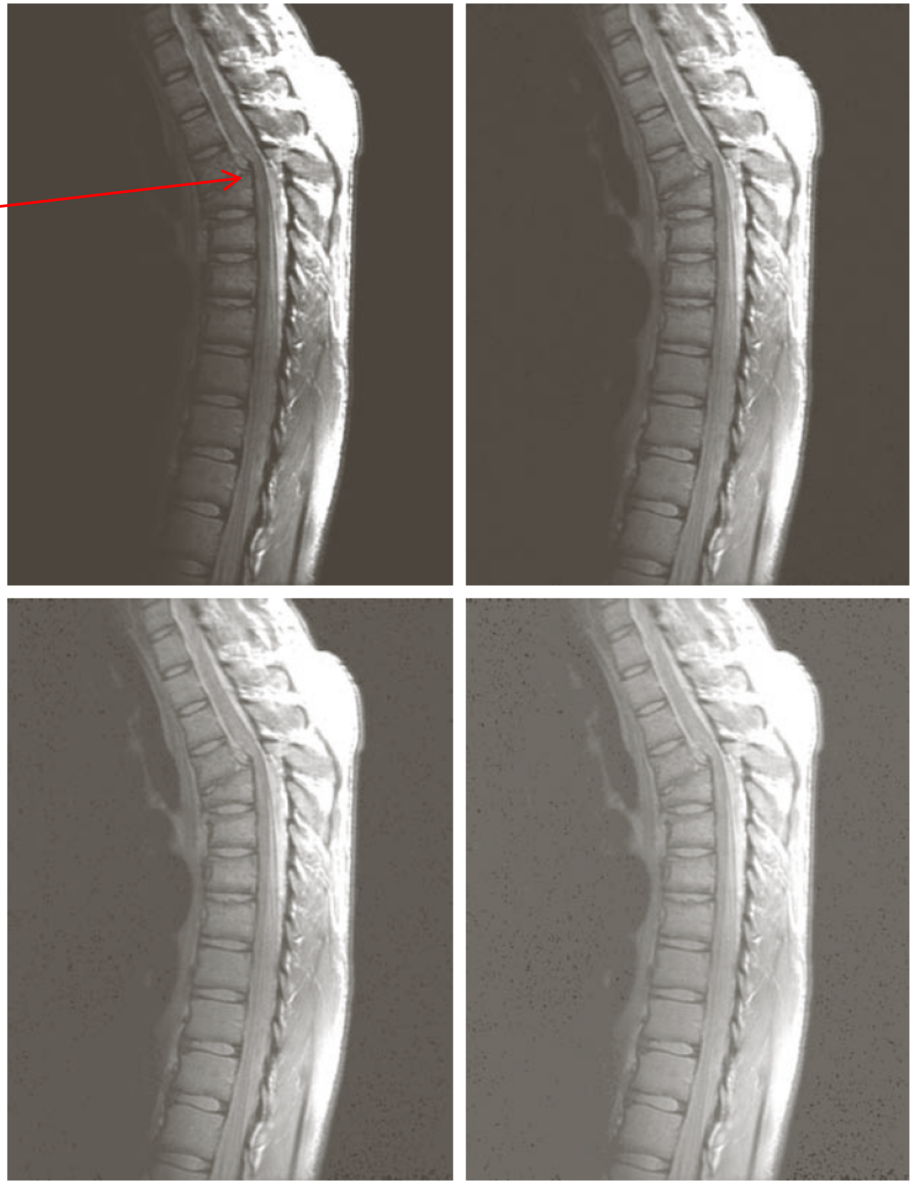
Decrease value of γ

Example: storing images in web sites.



Contrast manipulation: Power-Law

MRI of a <u>fractured</u> spine.



$\lambda = 0.6$

$\lambda = 0.4$

$\lambda = 0.3$

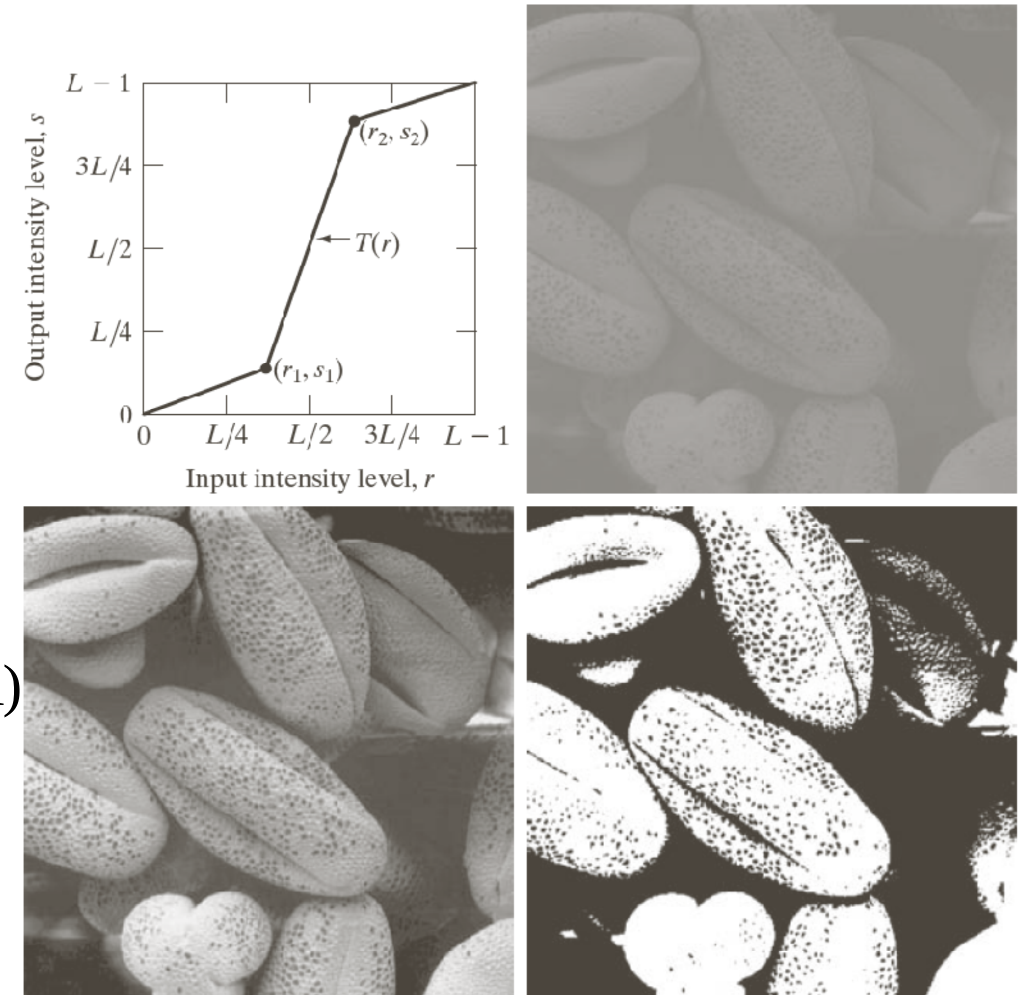
Best contrast

Washed out

Piecewise-Linear Transform Functions

Contrast Stretching

Expands range of intensity level of an image to full intensity range of a device.



Low-contrast Image

$$(r_1, s_1) = (r_{min}, 0)$$
$$(r_2, s_2) = (r_{max}, L-1)$$

Contrast stretched image

$$(r_1, s_1) = (m, 0)$$
$$(r_2, s_2) = (m, L-1)$$

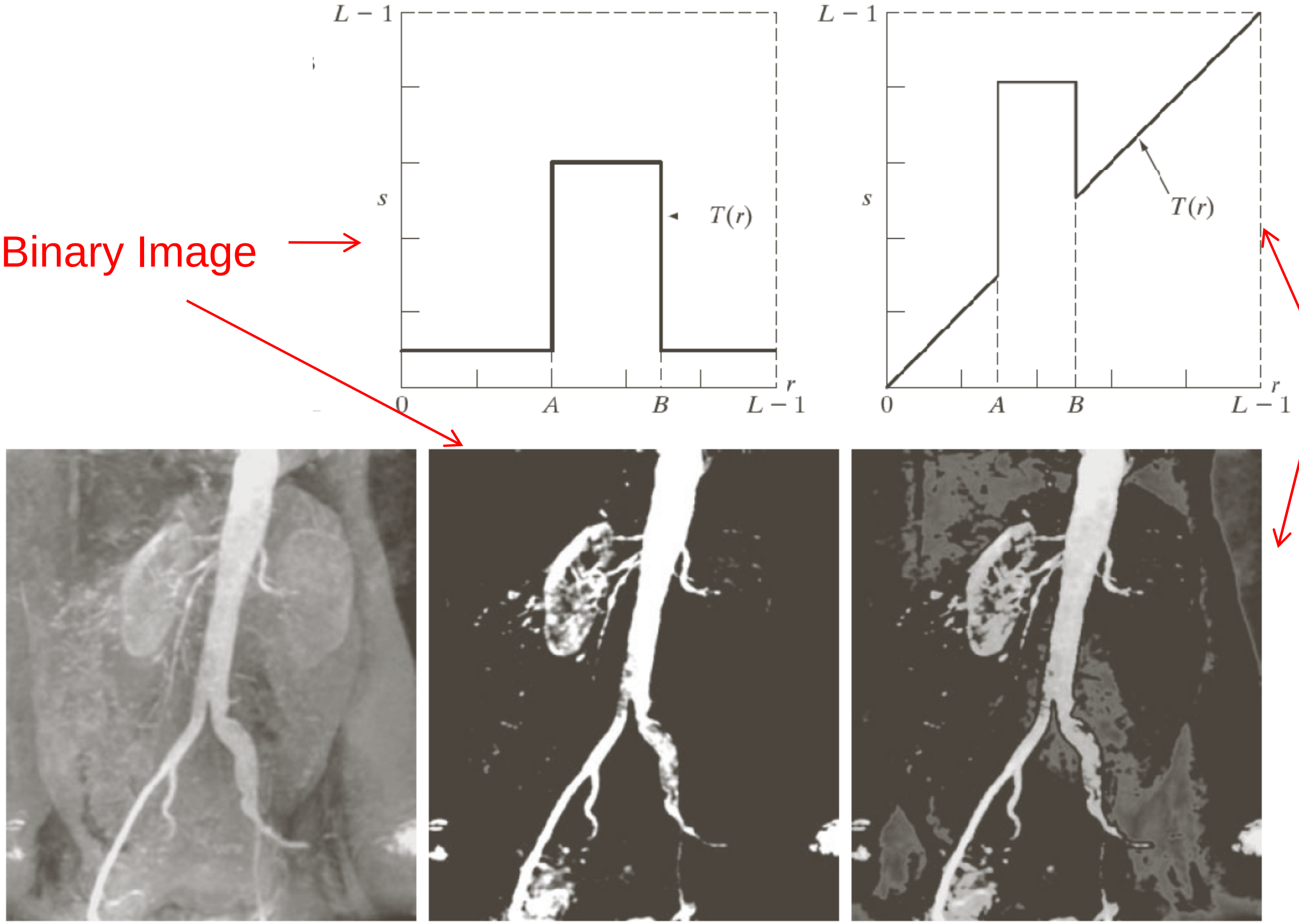
Thresholding

Piecewise-Linear Transform Functions

Intensity-Level Slicing

Highlighting a specific range of intensities.

Binary Image



Brightens (darkens) desired range

Piecewise-Linear Transform Functions

Bit-Plane Slicing

Gray (194) border:

1 1 0 0 0 0 1 0

LSB

MSB

a	b	c
d	e	f
g	h	i

FIGURE 3.14 (a) An 8-bit gray-scale image of size 500 × 1192 pixels. (b) through (i) Bit planes 1 through 8, with bit plane 1 corresponding to the least significant bit. Each bit plane is a binary image.

Bit-Plane Slicing: Image Compression

a b c

FIGURE 3.15 Images reconstructed using (a) bit planes 8 and 7; (b) bit planes 8, 7, and 6; and (c) bit planes 8, 7, 6, and 5. Compare (c) with Fig. 3.14(a).

The four highest-order bits are sufficient to reconstruct the original image in acceptable detail. 50% less storage.

Histogram Processing

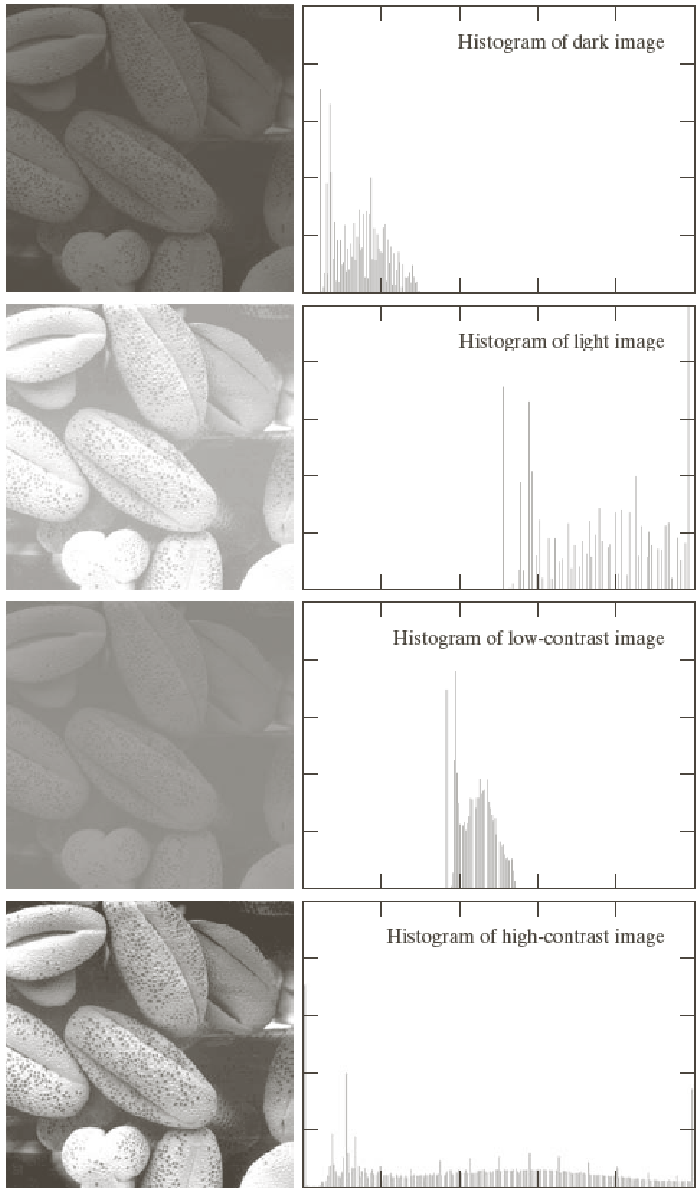
Histogram, $h(r_k) = n_k$

Normalized Histogram, $p(r_k) = \dfrac{n_k}{MN}$

Usefulness:

- Image Enhancement
- Image Compression
- Image Segmentation, etc.

Histogram of a high-contrast image has a high dynamic range.



Histogram Equalization - I

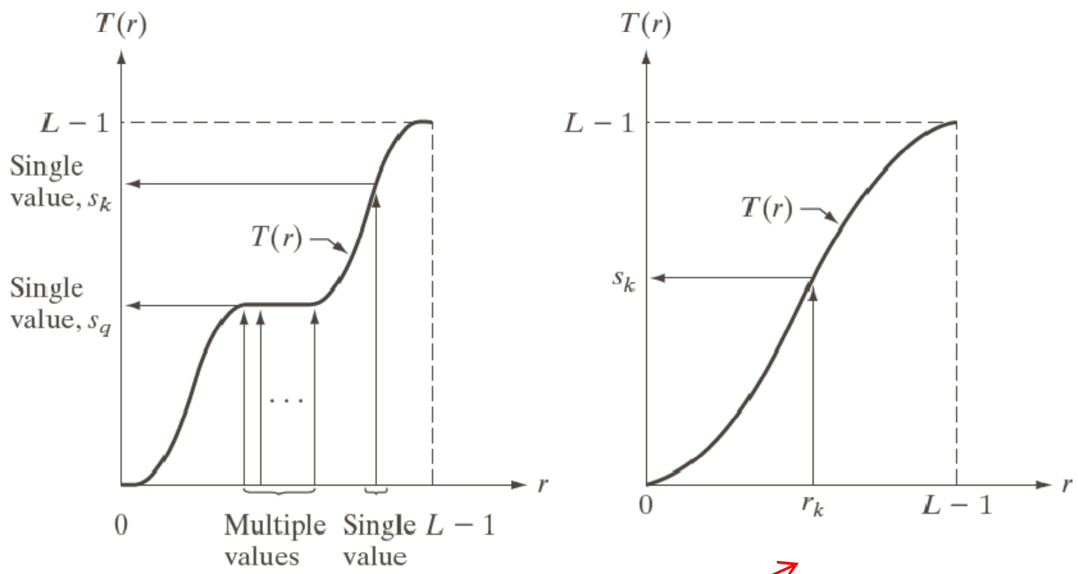
$$s = T(r) \quad 0 \le r \le L-1$$

(a) T(r) is a monotonically increasing function in

$$0 \le r \le L-1$$

and

(b) $0 \le T(r) \le L-1 \quad \text{for } 0 \le r \le L-1$



For inverse:

(a) Is changed to

(a') T(r) is strictly monotonically increasing function in $0 \le r \le L-1$

Histogram Equalization - II

If p_r (r) and T(r) are known, and T(r) is continuous and differentiable, then

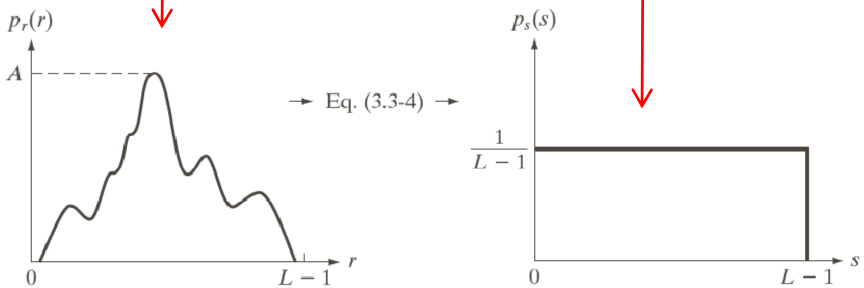
$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$$

A common transformation function in image processing: $s = T(r) = (L-1)\int_0^r p_r(w)dw$

$$\frac{ds}{dr} = \frac{dT(r)}{dr} = (L-1)\frac{d}{dr}\left[\int_0^r p_r(w)\right] = (L-1)p_r(r)$$

Uniform probability density function

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right| = p_r(r)\left|\frac{1}{(L-1)p_r(r)}\right| = \frac{1}{L-1} \qquad 0 \le s \le L-1$$



Histogram Equalization - III

Let, $p_r(r) = \begin{cases} \dfrac{2r}{(L-1)^2} & \text{for } 0 \le r \le L-1 \\ 0 & \text{otherwise} \end{cases}$

$$s = T(r) = (L-1)\int_0^r p_r(w)dw = \frac{2}{L-1}\int_0^r wdw = \frac{r^2}{L-1}$$

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right| = \frac{2r}{(L-1)^2}\left|\left[\frac{ds}{dr}\right]^{-1}\right|$$

$$= \frac{2r}{(L-1)^2}\left|\left[\frac{d}{dr}\frac{r^2}{L-1}\right]^{-1}\right| = \frac{2r}{(L-1)^2}\left|\frac{(L-1)}{2r}\right| = \frac{1}{L-1}$$

Uniform distribution

Discrete form:

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

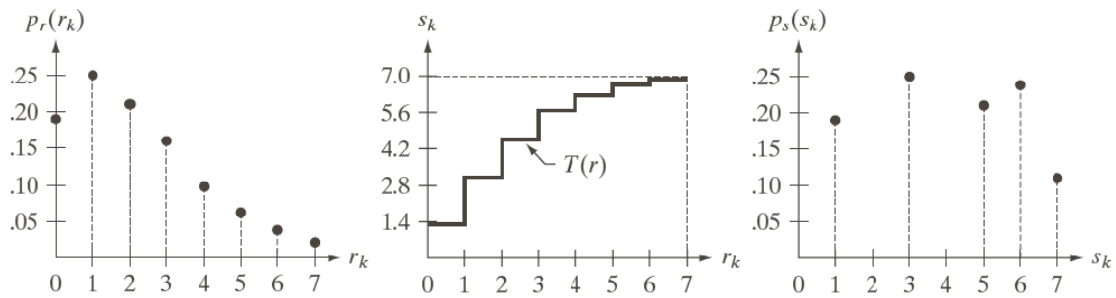
$$= \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j \quad k = 0,1,\ldots L-1$$

Histogram Equalization - Example

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

3-bit image (L=8) of size 64X64 (MN=4096) pixels.

$$s_1 = T(r_1) = 7\sum_{j=0}^{1} p_r(r_j) = 7p_r(r_0) + 7p_r(r_1) = 3.08$$



We get:

$s_0 = 1.33 \rightarrow 1$ $s_4 = 6.23 \rightarrow 6$

$s_1 = 3.08 \rightarrow 3$ $s_5 = 6.65 \rightarrow 7$

$s_2 = 4.55 \rightarrow 5$ $s_6 = 6.86 \rightarrow 7$

$s_3 = 5.67 \rightarrow 6$ $s_7 = 7.00 \rightarrow 7$

Equalized histogram for r = 7: $\frac{245+122+81}{4096} = 0.11$

Histogram Matching

Specified Histogram

Let, $$p_r(r) = \begin{cases} \dfrac{2r}{(L-1)^2} & \text{for } 0 \le r \le L-1 \\ 0 & \text{otherwise} \end{cases}$$

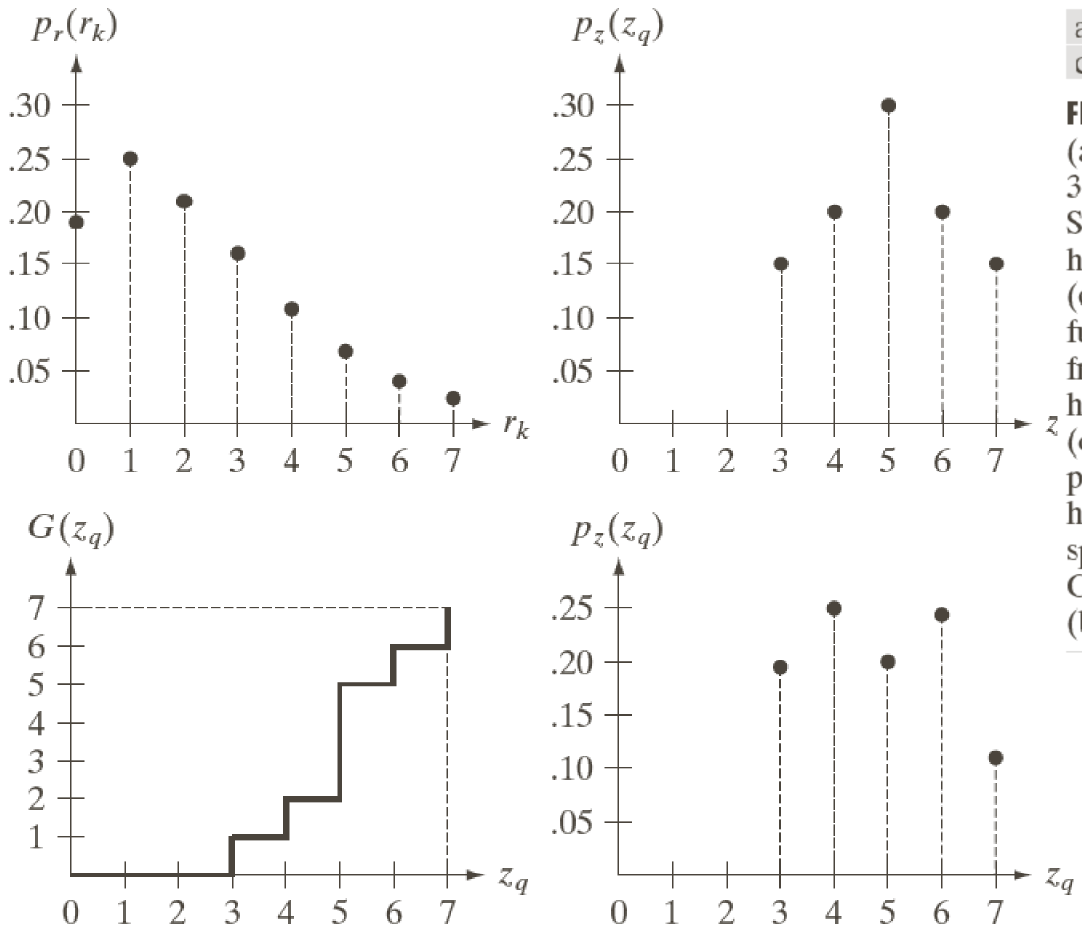
Desired image whose intensity PDF: $p_z(z) = \dfrac{3z^2}{(L-1)^3}$ for $0 \le z \le L-1$, otherwise 0.

$$s = T(r) = (L-1)\int_0^r p_r(w)dw = \frac{2}{L-1}\int_0^r w\,dw = \frac{r^2}{L-1}$$

$$G(z) = (L-1)\int_0^z p_z(w)dw = \frac{3}{(L-1)^2}\int_0^z w^2 dw = \frac{z^3}{(L-1)^2} = s$$

$$\Rightarrow \quad z = \left[(L-1)^2 s\right]^{1/3} = \left[(L-1)^2 \frac{r^2}{(L-1)}\right]^{1/3} = \left[(L-1)r^2\right]^{1/3}$$

Histogram Matching: Example (1)



a b
c d

FIGURE 3.22
(a) Histogram of a 3-bit image. (b) Specified histogram. (c) Transformation function obtained from the specified histogram. (d) Result of performing histogram specification. Compare (b) and (d).

From previous example:

$$s_0 = 1 \quad s_1 = 3$$
$$s_2 = 5 \quad s_3 = 6$$
$$s_4 = 6 \quad s_5 = 7$$
$$s_6 = 7 \quad s_7 = 7$$

Histogram Matching: Example (2)

z_q	Specified $p_z(z_q)$	Actual $p_z(z_k)$
$z_0 = 0$	0.00	0.00
$z_1 = 1$	0.00	0.00
$z_2 = 2$	0.00	0.00
$z_3 = 3$	0.15	0.19
$z_4 = 4$	0.20	0.25
$z_5 = 5$	0.30	0.21
$z_6 = 6$	0.20	0.24
$z_7 = 7$	0.15	0.11

$$G(z_i) = 7\sum_{j=0}^{i} p_z(z_j)$$

$$G(z_0) = 0.00 \rightarrow 0 \qquad G(z_4) = 2.45 \rightarrow 2$$

$$G(z_1) = 0.00 \rightarrow 0 \qquad G(z_5) = 4.55 \rightarrow 5$$

$$G(z_2) = 0.00 \rightarrow 0 \qquad G(z_6) = 5.95 \rightarrow 6$$

$$G(z_3) = 1.05 \rightarrow 1 \qquad G(z_7) = 7.00 \rightarrow 7$$

Find the smallest value of z_q so that $G(z_q)$ is the closet to s_k.

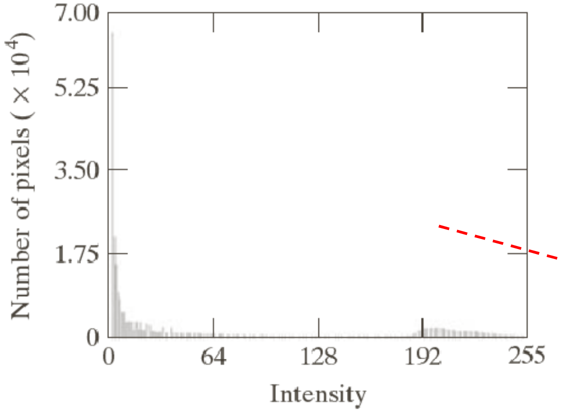
z_q	$G(z_q)$
$z_0 = 0$	0
$z_1 = 1$	0
$z_2 = 2$	0
$z_3 = 3$	1
$z_4 = 4$	2
$z_5 = 5$	5
$z_6 = 6$	6
$z_7 = 7$	7

s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7

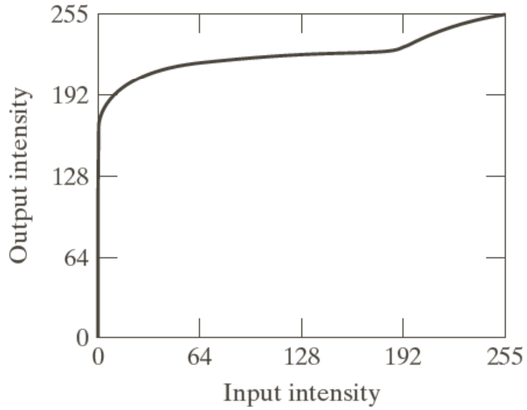
$$p_z(z_3) = \frac{790}{4096} = 0.19$$

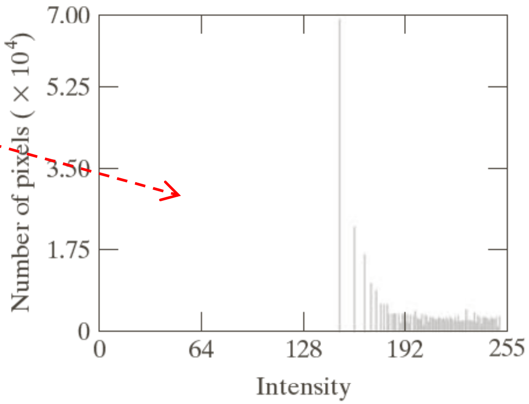
Histogram Equalization vs Histogram Matching - I

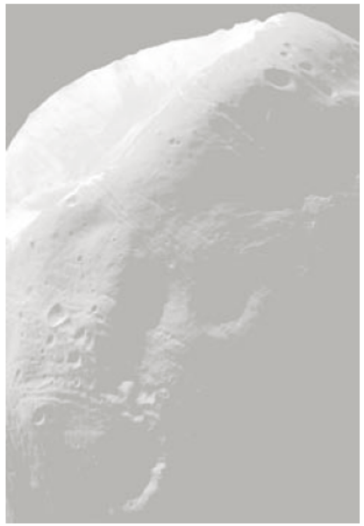
Histogram Equalization:



Original image and its histogram





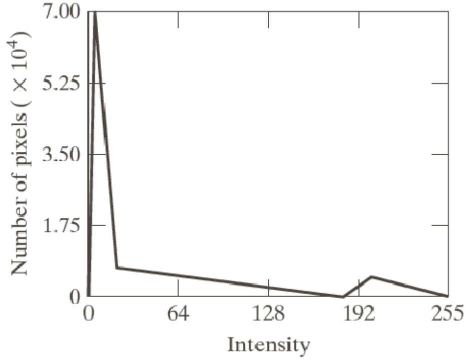


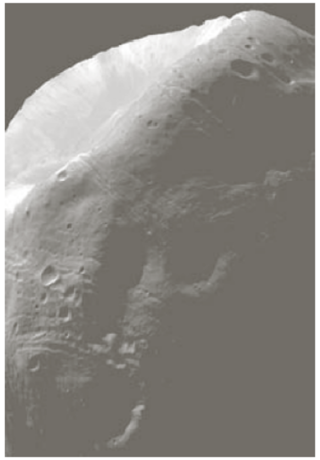
Histogram-equalized image

Histogram Equalization vs Histogram Matching - II

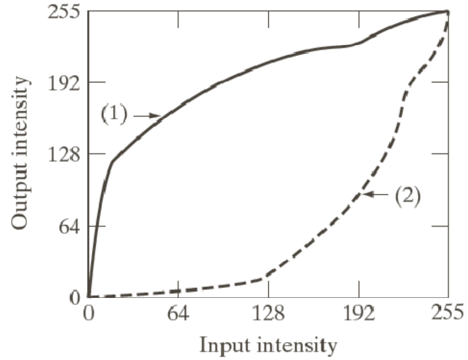
Histogram Matching:

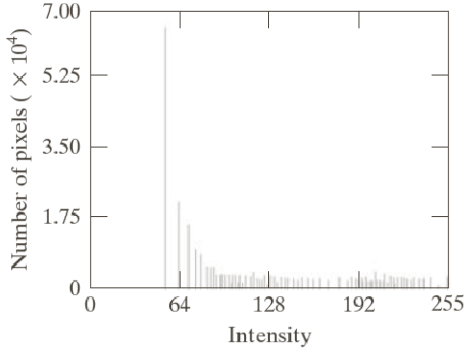
Specified histogram:





Transformation:





Histogram Statistics

Mean (average intensity): $m = \sum_{i=0}^{L-1} r_i p(r_i)$

Intensity variance (second moment): $\mu_2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i)$

Drill: Example 3.11

Without histogram:

$$m = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) \quad \text{and} \quad \sigma^2 = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[f(x,y) - m \right]^2$$

Learn about local statistics.

Spatial Filtering

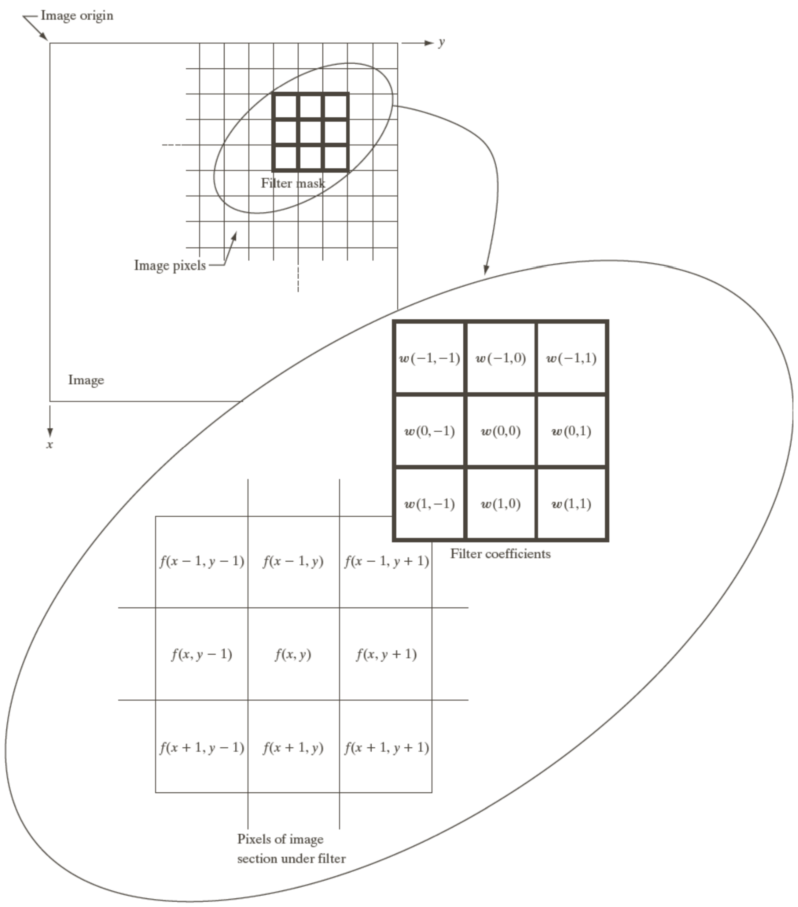
Consists of: (1) A neighborhood.
(2) A predefined operation on that neighborhood.

Filter response:

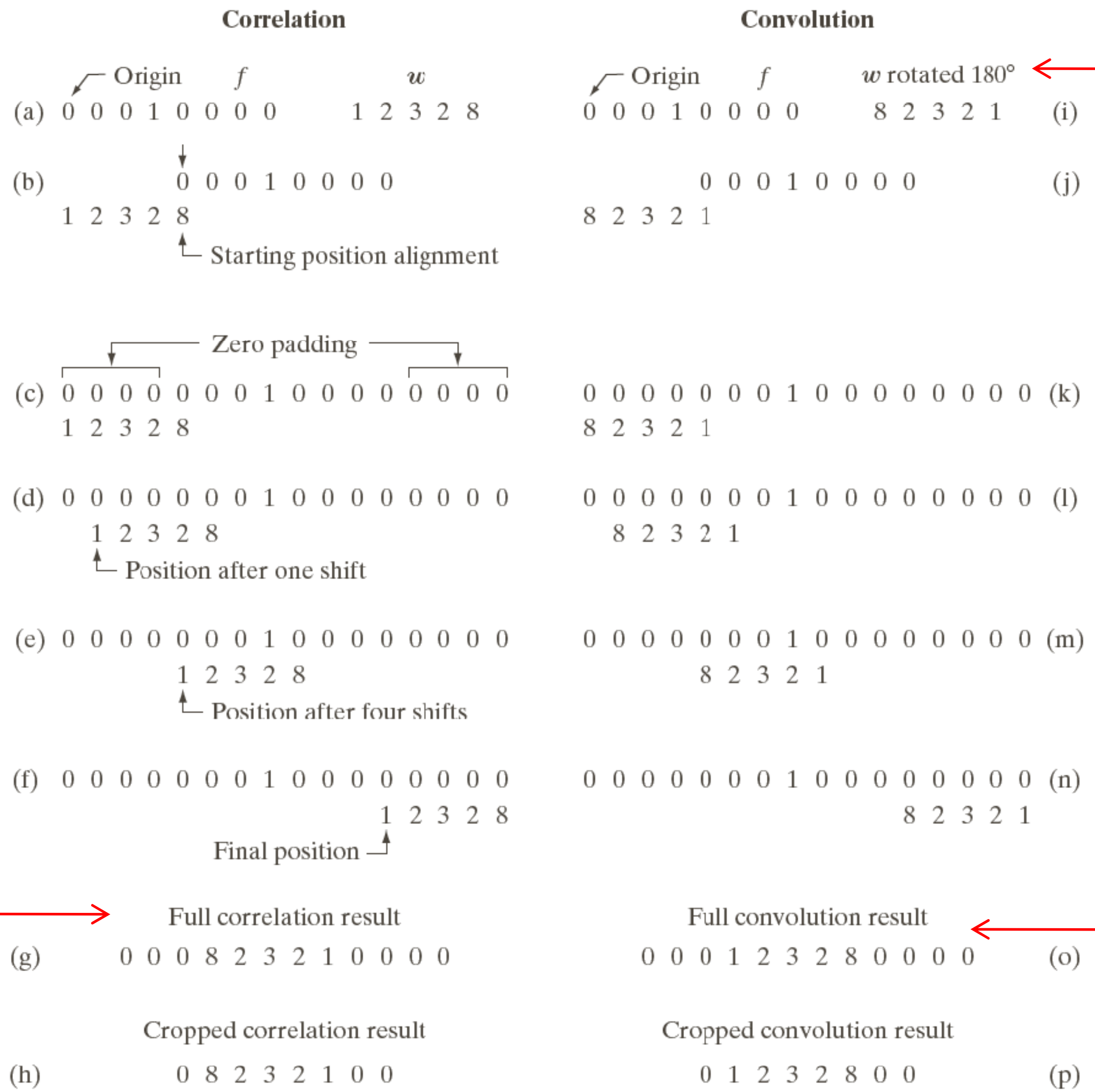
$$g(x,y) = w(-1,-1)f(x-1,y-1) + w(-1,0)f(x-1,y) + \dots$$
$$+ w(0,0)f(x,y) + \dots + w(1,1)f(x+1,y+1)$$

For a mask of size m x n, m = 2a+1, n = 2b+1:

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$



Spatial Correlation & Convolution



Check

Check

Check

FIGURE 3.29 Illustration of 1-D correlation and convolution of a filter with a discrete unit impulse. Note that correlation and convolution are functions of *displacement*.

Spatial Correlation & Convolution

2-D

Origin $f(x, y)$

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

$w(x, y)$

1	2	3
4	5	6
7	8	9

(a)

Padded f

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(b)

Initial position for w

1	2	3	0	0	0	0	0	0
4	5	6	0	0	0	0	0	0
7	8	9	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(c)

Full correlation result

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	9	8	7	0	0	0
0	0	0	6	5	4	0	0	0
0	0	0	3	2	1	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(d)

Cropped correlation result

0	0	0	0	0
0	9	8	7	0
0	6	5	4	0
0	3	2	1	0
0	0	0	0	0

(e)

Rotated w

9	8	7	0	0	0	0	0	0
6	5	4	0	0	0	0	0	0
3	2	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(f)

Full convolution result

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	1	2	3	0	0	0
0	0	0	4	5	6	0	0	0
0	0	0	7	8	9	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(g)

Cropped convolution result

0	0	0	0	0
0	1	2	3	0
0	4	5	6	0
0	7	8	9	0
0	0	0	0	0

(h)

Filter Mask

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

$$R = w_1 z_1 + w_2 z_2 + \ldots + w_9 z_9 = \sum_{k=1}^{9} w_k z_k = \mathbf{w}^T \mathbf{z}$$

Smoothing Filter (low pass) mask:

$\frac{1}{9} \times$

1	1	1
1	1	1
1	1	1

Equal weight

$\frac{1}{16} \times$

1	2	1
2	4	2
1	2	1

Weighted average

$$g(x,y) = \frac{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)}{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)}$$

Normalization factor

Smoothing Effect

Original image

3 X 3 mask

Noise is less pronounced.

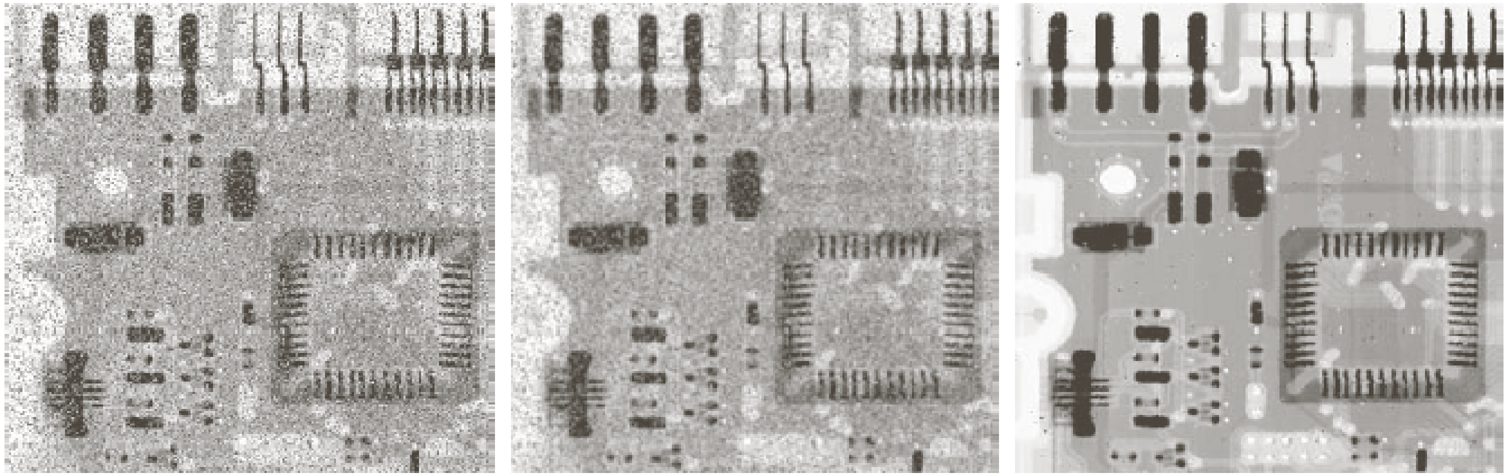
5 X 5 mask

35 X 35 mask

Completely blurred!

Median Filtering

Find the median in the neighborhood, then assign the center pixel value to that median.



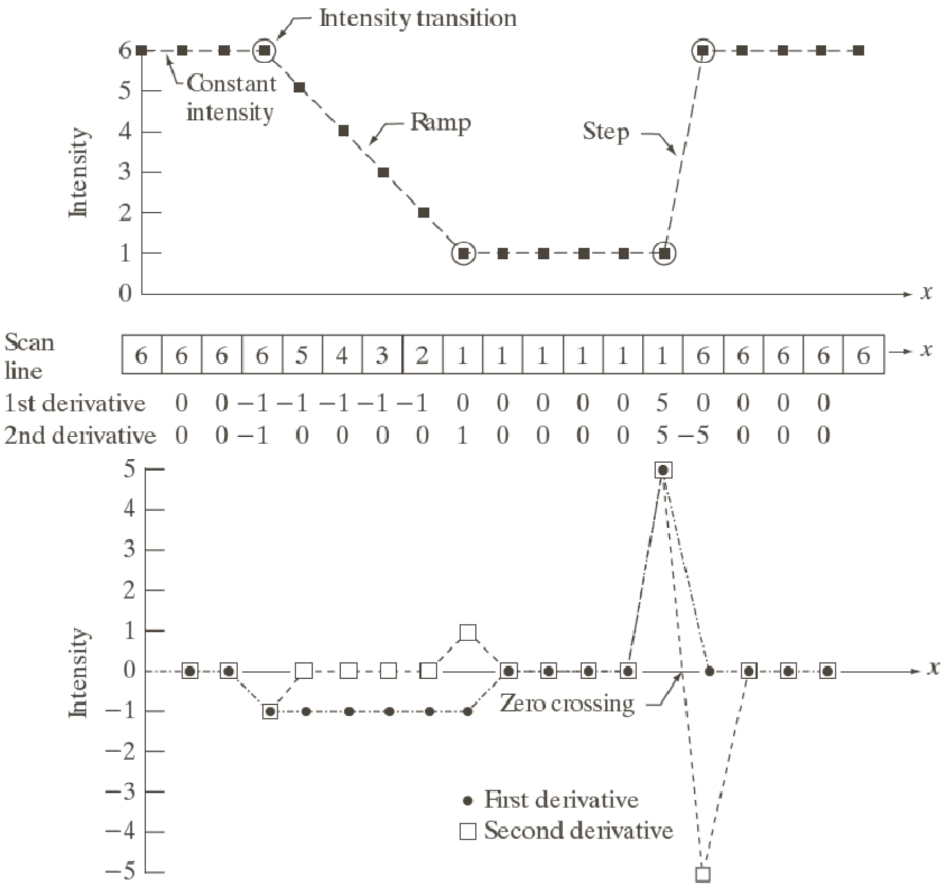
a b c

FIGURE 3.35 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3 × 3 averaging mask. (c) Noise reduction with a 3 × 3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Sharpening Spatial Filters

First-order derivative: $$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

Second-order derivative: $$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$



First derivative:

1. Zero at constant area.
2. Nonzero at onset.
3. Nonzero along ramp.

Second derivative:

1. Zero at constant area.
2. Nonzero at onset & end.
3. Zero along constant slope ramp.

Laplacian Mask - I

Second derivative is more useful in edge detection.

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

$$\therefore \nabla^2 f(x, y) = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

x, y, and two diagonal directions

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

New image:

$$g(x, y) = f(x, y) + c\left[\nabla^2 f(x, y)\right]$$

Laplacian Mask - II

Original blurred image

Using Laplacian mask along x, y axes

Using Laplacian mask along x, y axes, and two diagonal

Unsharp Masking & Highboost Filtering

To sharpen an image

1. Blur the original image.
2. Subtract the blurred image from the original (result is mask).
3. Add the mask to the original.

$$g_{mask}(x,y) = f(x,y) - \overline{f}(x,y)$$
$$g(x,y) = f(x,y) + k \times g_{mask}(x,y)$$

If $k = 1$, unsharp mask.
If $k > 1$, highboost filtering.