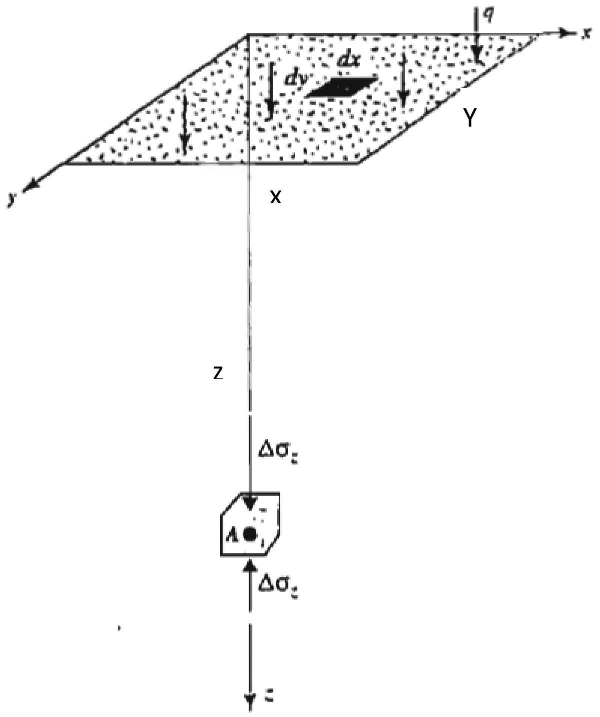
Vertical Stresses Caused by a Rectangular Loaded Area:

$\Delta\sigma_z = qI_2$ This Eqn. gives $\Delta\sigma_z$ **at the corner of the Rectangular Loaded Area**

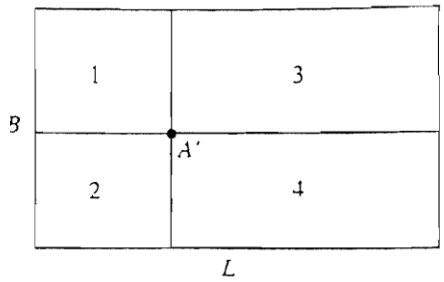
I_2 Depends on m, n from table

$m = \dfrac{x}{z}$, $n = \dfrac{y}{z}$



Vertical stress below the corner of a uniformly loaded flexible

Ex:



For Point A':

$$\Delta\sigma_z = q\,\{I_{2_{(1)}} + I_{2_{(2)}} + I_{2_{(3)}} + I_{2_{(4)}}\}$$

Vertical stress Below the Center of Uniformity Loaded Circular Area:

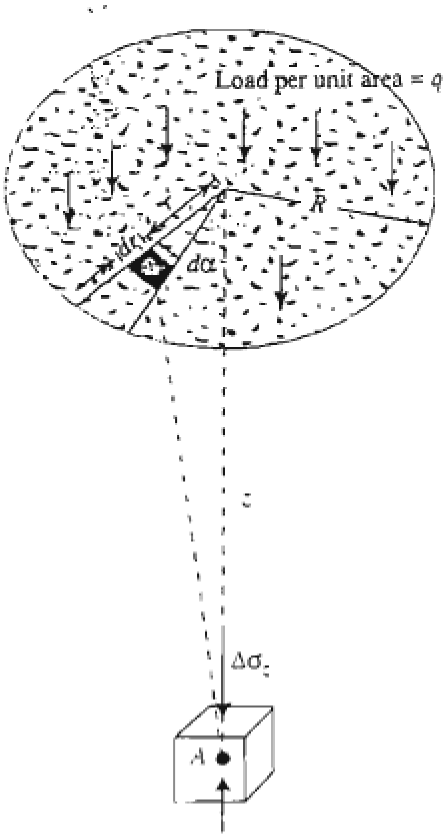
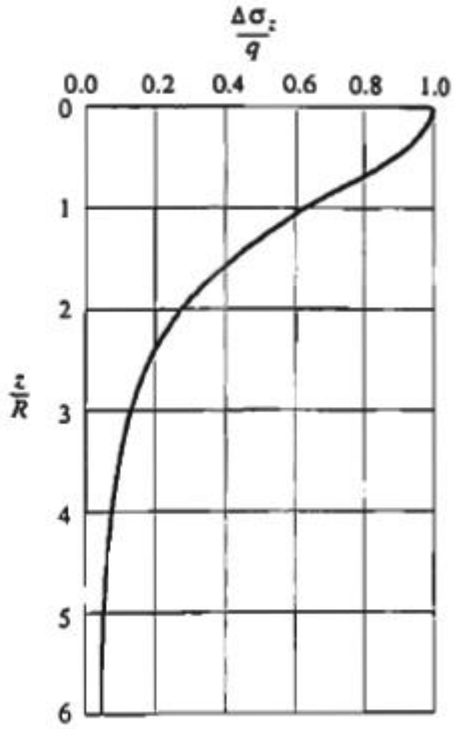


Figure 9.13
Vertical stress below the center of a uniformly loaded flexible circular area

$$\Delta\sigma_z = q\left\{1 - \frac{1}{[(R/z)^2 + 1]^{3/2}}\right\}$$

Variation of $\Delta\sigma_z/q$ with z/R

z/R	$\Delta\sigma_z/q$
0	1
0.02	0.9999
0.05	0.9998
0.10	0.9990
0.2	0.9925
0.4	0.9488
0.5	0.9106
0.8	0.7562
1.0	0.6465
1.5	0.4240
2.0	0.2845
2.5	0.1996
3.0	0.1436
4.0	0.0869
5.0	0.0571



Stress under the center of a uniformly loaded flexible circular area

The variation of $\Delta\sigma_z/q$ with z/R as obtained from Eq. (9.24) is given in Table 9.4. A plot of this is also shown in Figure 9.14. The value of $\Delta\sigma_z$ decreases rapidly with depth, and at $z = 5R$, it is about 6% of q, which is the intensity of pressure at the ground surface.

Influence Chart for Vertical Pressure:

The procedure for obtaining vertical pressure at any point below a loaded area is as follows:

1. Determine the depth z below the uniformly loaded area at which the stress increase is required.
2. Plot the plan of the loaded area with a scale of z equal to the unit length of the chart ($\overline{AB}$).
3. Place the plan (plotted in step 2) on the influence chart in such a way that the point below which the stress is to be determined is located at the center of the chart.
4. Count the number of elements (M) of the chart enclosed by the plan of the loaded area.

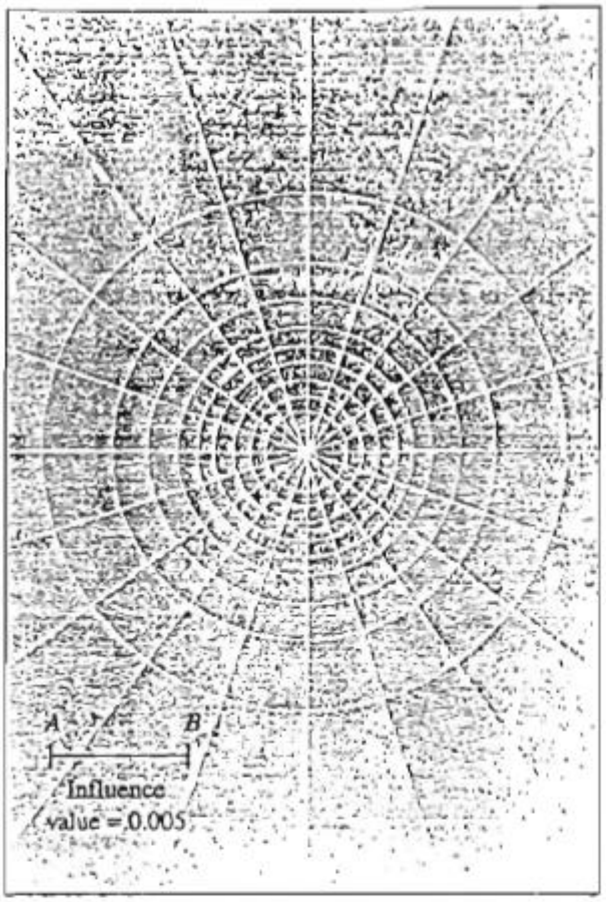


Figure 9.21 Influence chart for vertical pressure based on Boussinesq's theory (after Newmark, 1942)

The increase in the pressure at the point under consideration is given by

$$\Delta\sigma_z = (IV)qM \tag{9.39}$$

where IV = influence value
q = pressure on the loaded area

Problems:

1- **Consider a Circularly loaded flexible area on the ground surface. Given: R = 6ft, q=3500 lb/ft^2.**
Calculate the vertical stress increase $\Delta\sigma_z$ at points 1.5ft, 3ft, 6ft, 9ft, 12ft located below the ground surface.

Sol.

From Eqn. (5.32) and Table (5.3), q= 3500 lb/ft^2

R (ft)	z (ft)	$\frac{z}{R}$	$\frac{\Delta\sigma_z}{q}$	$\Delta\sigma_z$ (lb / ft^2)
6	1.5	0.4	0.9488	**3321**
6	3	0.5	0.9106	**3187**
6	6	1.0	0.6465	**2263**
6	9	1.5	0.4240	**1484**
6	12	2.0	0.2845	**996**

2- **Refer to figure 1. The circular flexible area is uniformly loaded. Given: q=300 kN/m2, and using Newmark's chart, determine the vertical stress increase $\Delta\sigma_z$ at point A.**

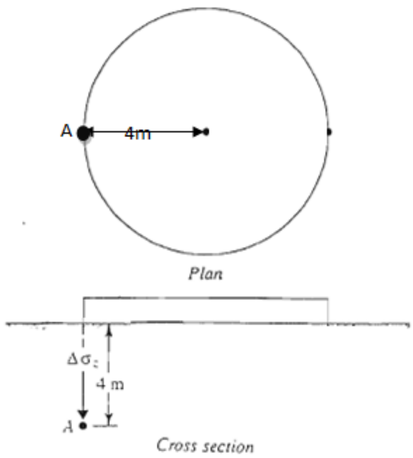


Figure 1

Sol.

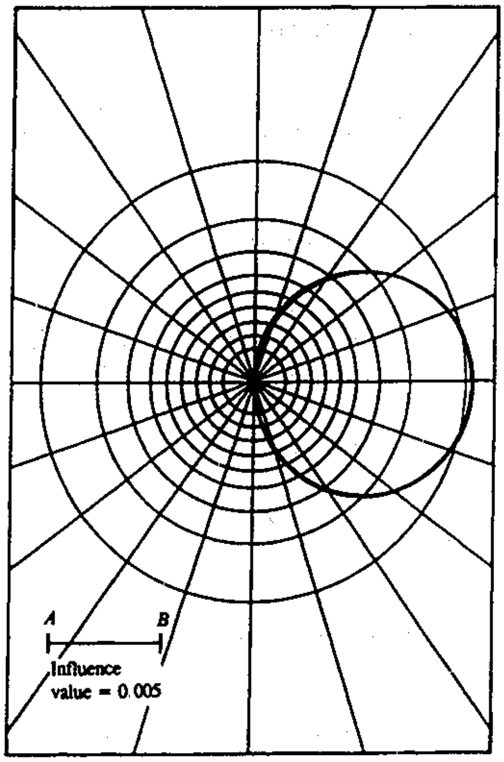
Refer to the Newmark's chart.

The plan is drawn to scale.

$\overline{AB} = 4$ m. $M \approx 65$.

$\Delta\sigma_z = (IV)qM = (0.005)(300)(65)$

$= \mathbf{97.5\ kN / m^2}$



3- The plan of a flexible rectangular loaded area shown in figure 2. q=1800 lb/ft^2 determines the vertical stress increase $\Delta\sigma_z$ at point at depth z= 5ft below.

a-Point A

b- Point B

c- Point C

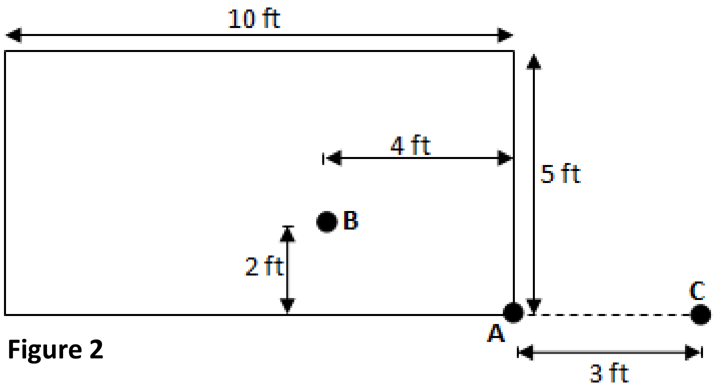


Figure 2

Sol.

a-

$$n = \frac{L}{z} = \frac{10}{5} = 2; \quad m = \frac{B}{z} = \frac{5}{5} = 1$$

$$\Delta\sigma_z = qI_4; \; I_4 = 0.1999$$

$$\Delta\sigma_z = (1800)(0.1999) = \mathbf{359.8\ lb/ft^2}$$

b. Refer to the figure below.

① 6 ft × 3 ft	③ 4 ft × 3 ft
② 6 ft × 2 ft	④ 4 ft × 2 ft

For rectangle 1: $m = \frac{3}{5} = 0.6; \quad n = \frac{6}{5} = 1.2; \quad I_4 = 0.1431$

For rectangle 2: $m = \frac{2}{5} = 0.4; \quad n = \frac{6}{5} = 1.2; \quad I_4 = 0.1063$

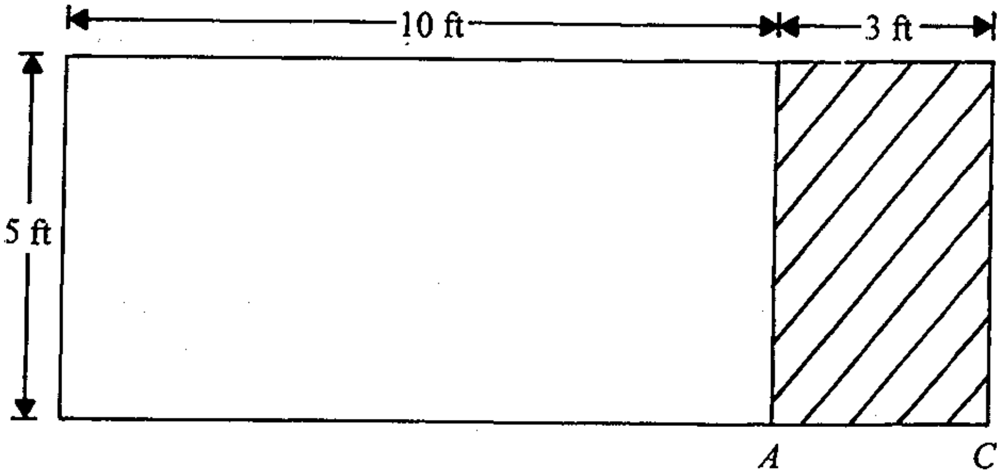
For rectangle 3: $m = \frac{3}{5} = 0.6; \quad n = \frac{4}{5} = 0.8; \quad I_4 = 0.1247$

For rectangle 4: $m = \frac{2}{5} = 0.4; \quad n = \frac{4}{5} = 0.8; \quad I_4 = 0.0931$

$$\Delta\sigma_z = q[I_{4(1)} + I_{4(2)} + I_{4(3)} + I_{4(4)}] = (1800)(0.1431 + 0.1063 + 0.1247 + 0.0931)$$

$$= \mathbf{841\ lb/ft^2}$$

c. Refer to the figure.



$$\Delta\sigma_z = \left\langle \begin{array}{l} \text{stress at } C \text{ due to rectangular area 13 ft} \times \text{5 ft} \\ \quad - \text{ stress at } C \text{ due to rectangular area 3 ft} \times \text{5 ft} \end{array} \right.$$

For rectangular area 13 ft × 5 ft: $m = \dfrac{5}{5} = 1;\quad n = \dfrac{13}{5} = 2.6;\quad I_4 = 0.202$

For rectangular area 3 ft × 5 ft: $m = \dfrac{3}{5} = 0.6;\quad n = \dfrac{5}{5} = 1;\quad I_4 = 0.1361$

$$\Delta\sigma_z = q(0.202 - 0.1361) = (1800)(0.202 - 0.1351) = \mathbf{118.6\ lb / ft^2}$$