Time Rate of Consolidation

m_v = coefficient of volume compressibility = $a_v/(1 + e_O)$

$a_v = coefficient\ of\ compressibility = \frac{\Delta e}{\Delta \sigma'}$

c_v = coefficient of consolidation = $k/(\gamma_w m_v)$

$$c_v = \frac{k}{\gamma_w m_v} = \frac{k}{\gamma_w\left(\frac{a_v}{1 + e_O}\right)}$$

$$T_v = \frac{c_v t}{H_{dr}^2} = \text{time factor}$$

$H_{dr} = avg. longest\ drainge\ path\ durring\ consolidation$

$$U(\%) = degree\ of\ consolidation\ (\%) = \frac{S_{c(t)}}{S_c}$$

$S_{c(t)} = settlement\ of\ the\ layer\ at\ time\ t$

$S_c = ultimate\ settlement\ of\ the\ layer\ from\ primary\ consolidation$

$$T_v = f(U)$$

The table below gives the variation of T_v with U

U (%)	T_v	U (%)	T_v	U (%)	T_v
0	0	34	0.0907	68	0.377
1	0.00008	35	0.0962	69	0.390
2	0.0003	36	0.102	70	0.403
3	0.00071	37	0.107	71	0.417
4	0.00126	38	0.113	72	0.431
5	0.00196	39	0.119	73	0.446
6	0.00283	40	0.126	74	0.461
7	0.00385	41	0.132	75	0.477
8	0.00502	42	0.138	76	0.493
9	0.00636	43	0.145	77	0.511
10	0.00785	44	0.152	78	0.529
11	0.0095	45	0.159	79	0.547
12	0.0113	46	0.166	80	0.567
13	0.0133	47	0.173	81	0.588
14	0.0154	48	0.181	82	0.610
15	0.0177	49	0.188	83	0.633
16	0.0201	50	0.197	84	0.658
17	0.0227	51	0.204	85	0.684
18	0.0254	52	0.212	86	0.712
19	0.0283	53	0.221	87	0.742
20	0.0314	54	0.230	88	0.774
21	0.0346	55	0.239	89	0.809
22	0.0380	56	0.248	90	0.848
23	0.0415	57	0.257	91	0.891
24	0.0452	58	0.267	92	0.938
25	0.0491	59	0.276	93	0.993
26	0.0531	60	0.286	94	1.055
27	0.0572	61	0.297	95	1.129
28	0.0615	62	0.307	96	1.219
29	0.0660	63	0.318	97	1.336
30	0.0707	64	0.329	98	1.500
31	0.0754	65	0.304	99	1.781
32	0.0803	66	0.352	100	∞
33	0.0855	67	0.364		

Coefficient of Consolidation (C_v):

For a given load increment on a specimen, two graphical methods is commonly for determine C_v from laboratory one- dimensional consolidation test:

Logarithm-of-Time Method

For a given incremental loading of the laboratory test, the specimen deformation against log-of-time plot is shown in Figure 10.25. The following constructions are needed to determine c_v:

1. Extend the straight-line portions of primary and secondary consolidations to intersect at A. The ordinate of A is represented by d_{100}— that is, the deformation at the end of 100% primary consolidation.
2. The initial curved portion of the plot of deformation versus log t is approximated to be a parabola on the natural scale. Select times t_1 and t_2 on the curved portion such that $t_2 = 4t_1$. Let the difference of specimen deformation during time $(t_2 - t_1)$ be equal to x.

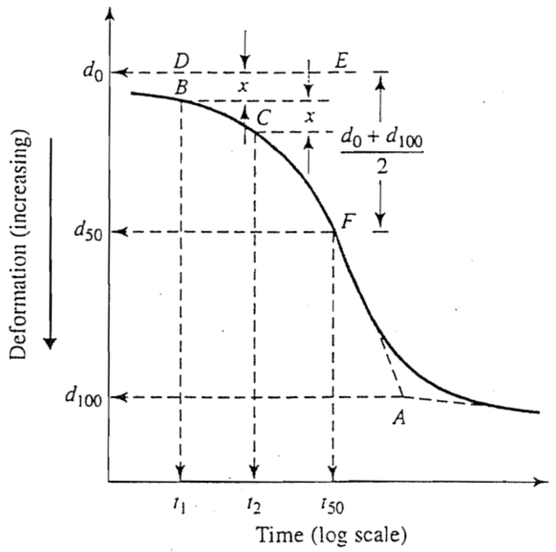

Figure 10.25 Logarithm-of-time method for determining coefficient of consolidation

3. Draw a horizontal line DE such that the vertical distance BD is equal to x. The deformation corresponding to the line DE is d_0 (that is, deformation at 0% consolidation).
4. The ordinate of point F on the consolidation curve represents the deformation at 50% primary consolidation, and its abscissa represents the corresponding time (t_{50}).
5. For 50% average degree of consolidation, $T_v = 0.197$ (see Table 10.5), so,

$$T_{50} = \frac{c_v t_{50}}{H_{dr}^2}$$

or

$$c_v = \frac{0.197 H_{dr}^2}{t_{50}} \tag{10.54}$$

Square-Root-of-Time Method

In the square-root-of-time method, a plot of deformation against the square root of time is made for the incremental loading (Figure 10.26). Other graphic constructions required are as follows:

1. Draw a line AB through the early portion of the curve.
2. Draw a line AC such that $\overline{OC} = 1.15\overline{OB}$. The abscissa of point D, which is the intersection of AC and the consolidation curve, gives the square root of time for 90% consolidation ($\sqrt{t_{90}}$).

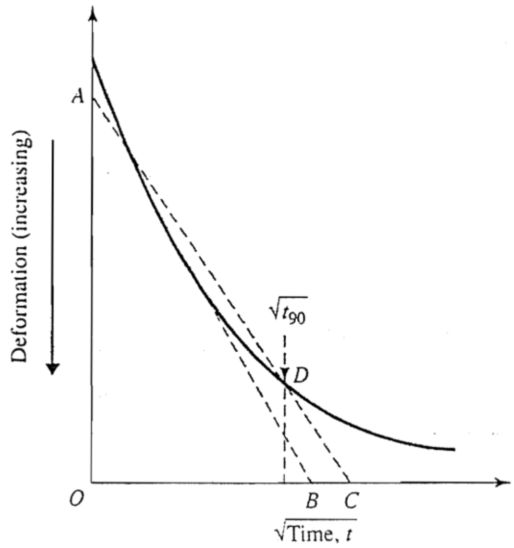

Figure 10.26
Square-root-of-time fitting method

3. For 90% consolidation, $T_{90} = 0.848$ (see Table 10.5), so

$$T_{90} = 0.848 = \frac{c_v t_{90}}{H_{dr}^2}$$

or

$$c_v = \frac{0.848 H_{dr}^2}{t_{90}} \tag{10.55}$$

Problems:

The coordinates of two points on a virgin compression curve are as follows:

- e_1=1.82 $\sigma'_1 = 200\ KN/m^2$
- e_2= 1.54 $\sigma'_2 = 400\ KN/m^2$

a- *Find the coefficient of volume compressibility for the pressure range stated above.*

b- *If the coefficient of consolidation for the pressure range is 0.003cm^2/sec, find the coefficient of permeability (cm/sec) of the clay corresponding to the average void ratio.*

Sol.

a. $$m_v = \frac{a_v}{1+e_{av}} = \frac{\frac{\Delta e}{\Delta \sigma'}}{1+e_{av}}$$

$$\Delta e = e_1 - e_2 = 1.82 - 1.54 = 0.28$$

$$\Delta \sigma' = \sigma'_2 - \sigma'_1 = 400 - 200 = 200 \text{ kN / m}^2$$

$$e_{av} = \frac{1.82 + 1.54}{2} = 1.68$$

$$m_v = \frac{\left(\frac{0.28}{200}\right)}{1+1.68} = \mathbf{0.000522\ m^2 / kN}$$

b. $$c_v = \frac{k}{m_v \gamma_w} = 0.003 \text{ cm}^2 / \text{sec}$$

$$= \frac{\text{k}}{(0.000522 \times 100^2 \text{ cm}^2 / \text{kN})\left(\frac{9.81}{100^3}\right)\text{kN / cm}^3}$$

$$\boldsymbol{k = 1.53 \times 10^{-7}\ \text{cm / sec}}$$

6.6 Laboratory test on 25mm thick clay sample drained at the top and bottom show that 50% consolidation takes place in 11minutes.

a- How long will it take for a similar clay layer in the field, 4m thick and drained at the top only, to undergo 50% consolidation?

b- Find the time required for the clay layer in the field as described in Part a to reach 70% consolidation.

Sol.

a-

$$\frac{t_{lab}}{H^2_{dr(lab)}} = \frac{t_{field}}{H^2_{dr(field)}}$$

$$\frac{11\ min}{\left[\frac{0.025m}{2}\right]^2} = \frac{t_{field}}{(4)^2}$$

$$t_{field} = 1{,}126{,}400min = 782days$$

b-

$$c_v = \frac{T_v H^2_{dr}}{t_v}$$

At U= 50% T_{50}=0.197 (from table 6.2)

$$c_v = \frac{T_{50} H^2_{dr\ (lab)}}{t_{50(lab)}}$$

$$c_v = \frac{0.197 \times \left[\frac{0.025}{2}\right]^2}{11min} = 2.8 \times 10^{-6} m^2/min$$

At U=70% T_{70}=0.403 (from table 6.2)

$$c_v = \frac{T_{70} H^2_{dr\ (lab)}}{t_{70(lab)}}$$

$$2.8 \times 10^{-6} = \frac{0.403 \times \left[\frac{0.025}{2}\right]^2}{t_{70(lab)}}$$

$$t_{70(lab)} = 22.5min$$

And from:

$$\frac{t_{lab}}{H^2_{dr(lab)}} = \frac{t_{field}}{H^2_{dr(field)}}$$

$$\frac{22.5\ min}{\left[\frac{0.025m}{2}\right]^2} = \frac{t_{field}}{(4)^2}$$

Then: $t_{field} = 2,304,000min = 1600days$

6.10

For a laboratory consolidation test on a clay sample (drained on both sides), the following results were obtained.

- *Thickness of the clay soil=25mm*
- $\sigma'_1 = 50kN/m^2$ $\qquad e_1 = 0.92$
- $\sigma'_2 = 100kN/m^2$ $\qquad e_2 = 0.8$
- *Time for 50% consolidation (t_{50})=2.2 min*

Determine the coefficient of permeability of the clay for the loading range.

Sol.

$$m_v = \frac{a_v}{1+e_{av}} = \frac{\frac{\Delta e}{\Delta \sigma'}}{1+e_{av}} = \frac{\frac{0.92-0.8}{100-50}}{1+\frac{0.92+0.8}{2}} = \frac{\frac{0.12}{50}}{1+0.86} = 0.00129\ m^2/kN$$

$$c_v = \frac{T_v H^2_{dr}}{t_{50}} = \frac{(0.197)\left(\frac{0.025}{2}\right)^2}{2.2\ min} = 1.399 \times 10^{-5}\ m^2/min$$

$$k = c_v m_v \gamma_w = (1.399 \times 10^{-5})(0.00129)(9.81) = \mathbf{1.77 \times 10^{-7}\ m/min}$$