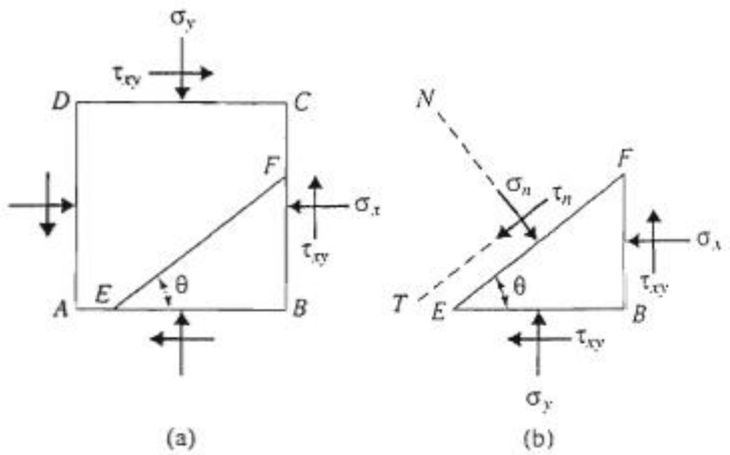
Normal and Shear Stresses on a Plane:

1) Determine σ_n, τ_n by using Eqns:



(a) A soil element with normal and shear stresses acting on it; (b) free body diagram of *EFB* as shown in (a)

$$\sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_n = \frac{\sigma_y - \sigma_x}{2} \sin 2\theta - \tau_{xy} \cos 2\theta$$

$$if\ \tau_n = 0, \text{ we get}$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - (\tan\theta)^2} = \frac{2\tau_{xy}}{\sigma_y - \sigma_x}$$

Major principal stress:

$$\sigma_n = \sigma_1 = \frac{\sigma_y + \sigma_x}{2} + \sqrt{\left[\frac{(\sigma_y - \sigma_x)}{2}\right]^2 + \tau_{xy}^2}$$

Minor principal stress:

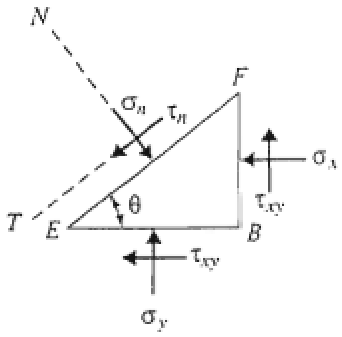
$$\sigma_n = \sigma_3 = \frac{\sigma_y + \sigma_x}{2} - \sqrt{\left[\frac{(\sigma_y - \sigma_x)}{2}\right]^2 + \tau_{xy}^2}$$

Note:

$\sigma_n = +ve\ if\ compression$

$\tau_n = +ve\ if\ it\ rotates\ element\ ccw$

2) Determine σ_n, τ_n by using Mohr's circle:



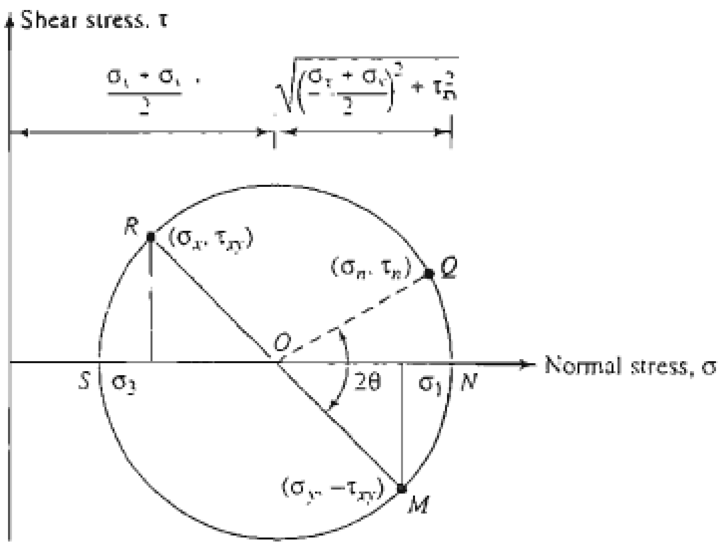
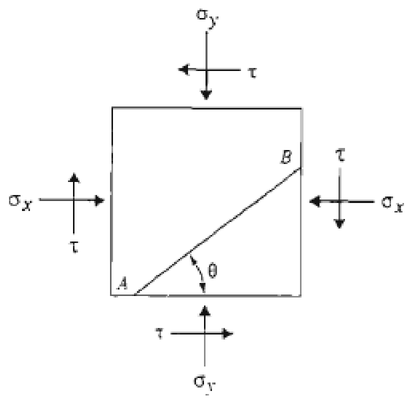


Figure 9.2 Principles of the Mohr's circle

5.10

A soil element is shown in figure below for the following stresses conditions; determine the maximum and minimum principal stresses. Also determine the normal and shear stresses on plane AB.

- $\sigma_x = 420\ kN/m^2$
- $\sigma_y = 140\ kN/m^2$
- $\tau = 128\ kN/m^2$
- $\theta = 20°$



Sol.

1) by using Eqns:

Principal stresses:

Major principal stress:

$$\sigma_n = \sigma_1 = \frac{\sigma_y + \sigma_x}{2} + \sqrt{\left[\frac{(\sigma_y - \sigma_x)}{2}\right]^2 + \tau_{xy}^2}$$

Minor principal stress:

$$\sigma_n = \sigma_3 = \frac{\sigma_y + \sigma_x}{2} - \sqrt{\left[\frac{(\sigma_y - \sigma_x)}{2}\right]^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{140+420}{2} + \sqrt{\left[\frac{140-420}{2}\right]^2 + (-128)^2} = 469.7 kN/m^2$$

$$\sigma_3 = \frac{140+420}{2} - \sqrt{\left[\frac{140-420}{2}\right]^2 + (-128)^2} = 90.3 kN/m^2$$

Stresses on plane AB:

$$\sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_n = \frac{140+420}{2} + \frac{140-420}{2}\cos(2\times 20) + (-128)\sin(2\times 20) = 90.47 kN/m^2$$

$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$\tau_n = \frac{140-420}{2}\sin(2\times 20) - (-128)\cos(2\times 20) = 8.06 kN/m^2$$

2) by using Mohr's circle:

Radius=$\sqrt{\left[\frac{\sigma_y - \sigma_x}{2}\right]^2 + {\tau_{xy}}^2}$

R=$\sqrt{\left[\frac{140-420}{2}\right]^2 + (-128)^2} = 189.7$

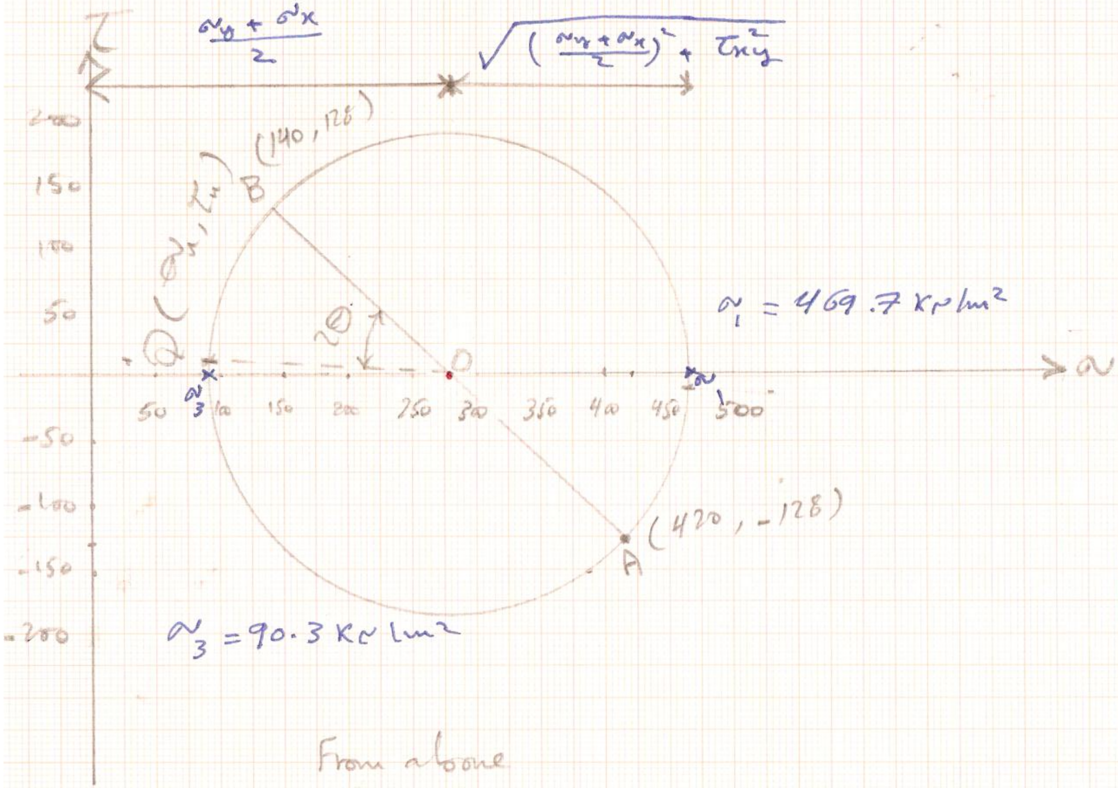
$$\overline{OO'} = \frac{\sigma_y + \sigma_x}{2} = \frac{140 + 420}{2} = 280$$

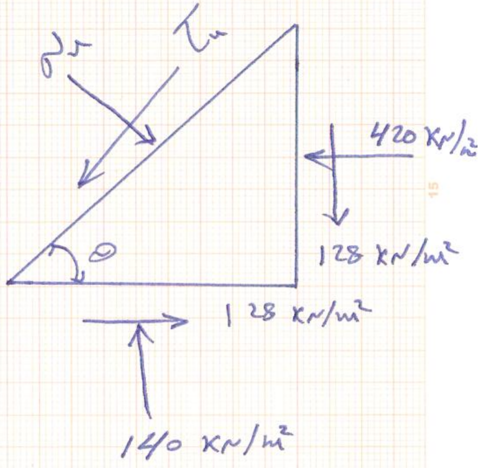
(The Mohr's Circle is plotted on the next page)

$\sigma_x = 420 \ kN/m^2$, $\tau_x = -128 \ kN/m^2$

$\sigma_y = 140 \ kN/m^2$, $\tau_y = 128 \ kN/m^2$

$\theta = 20^\circ$, $2\theta = 40^\circ$





$\sigma_1 = 469.7 \ kN/m^2$

$\sigma_3 = 90.3 \ kN/m^2$

From above

$\sigma_n = 90.48 \ kN/m^2$

$\tau_n = 8.08 \ kN/m^2$

scale 1 cm = 50