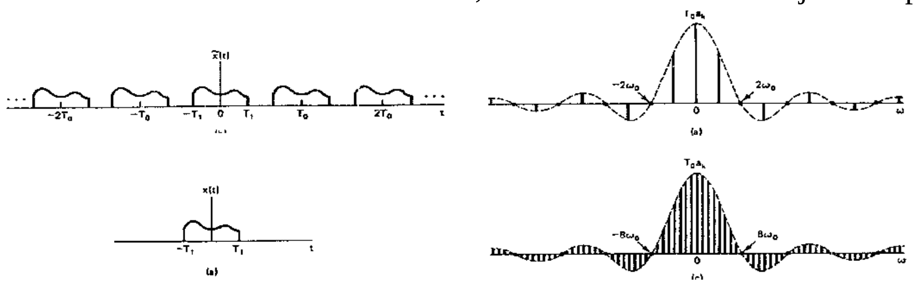
- **<u>Fourier Transform</u>:-**

In the following figure, as we increase the period T_o of the rectangular pulse, we notice that the FS coefficients (a_k) becomes denser. As $T_o \rightarrow \infty$, the signal becomes aperiodic and the transform x(jw) "or the spectrum" becomes a continuous function. Hence, we notice that the FS is just sampling of FT.



<u>CTFT:-</u>

$\rightarrow$ $\boxed{x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\, d\omega}$; $\boxed{X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\, dt}$

<u>DTFT:-</u>

$\rightarrow$ $\boxed{x[n] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(e^{j\omega})e^{j\omega n}\, d\omega}$; $\boxed{X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}}$

*<u>*The FT properties & FT pairs Tables could be found in the uploaded "Fourier tables" pdf file.</u>*

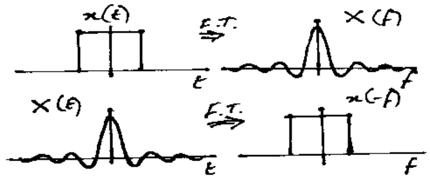
<u>Note:-</u> For a periodic signal x(t) having FS coefficients ak. The FT is $\boxed{X(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} ak\ \delta(\omega - k\omega o)}$

<u>Important FT pairs:-</u>

Signal	Fourier transform
$\delta(t)$	1
$u(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$
$\delta(t - t_0)$	$e^{-j\omega t_0}$
$e^{-at}u(t), \Re e\{a\} > 0$	$\frac{1}{a + j\omega}$
$\frac{\sin Wt}{\pi t}$	$X(j\omega) = \begin{cases} 1, & \lvert\omega\rvert < W \\ 0, & \lvert\omega\rvert > W \end{cases}$

Signal	Fourier transform
$x(t) = 1$	$2\pi\delta(\omega)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
$\cos\omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
$\sin\omega_0 t$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$

Note:- $sinc(x) = \frac{\sin(\pi x)}{\pi x} = \begin{vmatrix} 1 & x = 0 \\ 0 & \pm 1, \pm 2, \ldots \end{vmatrix}$



Q1)

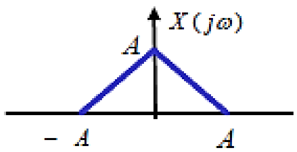
Find the frequency response for an LTI system described by the difference equation: $y(n) = 2x(n) + (1/3)y(n-1)$.

Q2)

(a) (i) Determine the frequency response $H(j\omega)$ for an LTI system with impulse response $h(t) = e^{-0.5t}u(t)$.

(ii) Sketch the magnitude and phase spectrum of the system.

(b) (i) Determine and sketch the FT $Y(j\omega)$ for the signal $y(t) = x(t)\sin 2\pi t$, given $X(j\omega)$ as shown below.



(ii) Determine the continuous-time signal $x(t)$ having the FT $X(j\omega) = \begin{cases} 2 & |\omega| < 5 \\ 0 & \text{otherwise} \end{cases}$.

(iii) Determine the continuous-time signal $x(t)$ having the FT $X(j\omega) = 0.5P_a(\omega - \omega_0) + 0.5P_a(\omega + \omega_0)$; where $P_a(\omega)$ denotes a rectangular pulse in frequency domain.

(c) Given the Fourier Transform pairs $x(t) = e^{-|t|} \xleftrightarrow{FT} X(j\omega) = \dfrac{2}{1+\omega^2}$,

(i) Use appropriate Fourier transform properties to find the Fourier transform of $y(t) = te^{-|t|}$.

(ii) Using the result in part (i), and along with the duality property, determine the Fourier transform of $y(t) = \dfrac{4t}{(1+t^2)^2}$.

Q3)

a) (i) Find the FT for the DT signal $x(n) = 2\delta(n) + 2\delta(n-1) - 2\delta(n-2) - 2\delta(n-3)$.

(ii) Let $X(e^{j\omega})$ be the FT of the DT signal $x(n)$, find the FT $Y(e^{j\omega})$ for the signal $y(n) = x(2-n)$, in terms of $X(e^{j\omega})$.

b) Find the discrete-time Fourier transform (DTFT) for the signal $x(n) = a^n u(n-2)$ where $|a| < 1$.

c) Determine the discrete-time signal $x(n)$ having the FT $X(e^{j\omega}) = \cos^2 \omega$.