- **Laplace Transform:-**

→ $\boxed{X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt}$; $\boxed{x(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}\,d\omega}$ where s = σ + jω

The generalization of the exponential's power allows us to analyze unstable system or to assess its stability and causality. Furthermore, using the pole-zero compensation we could redesign an unstable system to a stable one. The algebraic representation of X(s) is not enough to determine x(t), we must know ROC as well.

Region of convergence (ROC):- The range of s values for which the LT integral converges.
Note:- ROC doesn't contain any poles and it is a continuous plane.

Note:- If you substitute s with σ + jω then separate the exponential, you could notice that we can think of LT as FT of signal weighted by an exponential, that is LT = FT{x(t) $e^{-\sigma t}$}.

- **Partial fraction:-** Let H(s) = N(s)/D(s)

*The roots of the polynomial numerator N(s) are referred to as Zeroes (in plot use o)
*The roots of the polynomial denominator D(s) are referred to as Poles(in plot use x)
[*Note:- The total numbers of zeroes (in finite s-plane & infinity) = number of poles*]

- *General case:-* $H(s) = \frac{N(s)}{D(s)} = \frac{N(s)}{(s-p1)(s-p2)....(s-pn)} = \frac{A1}{(s-p1)} + \frac{A2}{(s-p2)} + \cdots + \frac{An}{(s-Pn)}$

→ $\boxed{An = [(s-Pn)H(s)]|_{s=Pn}}$

- Case of repeated poles:- $H(s) = \frac{N(s)}{D1(s)\,(s-Pi)^r} = \frac{A1}{(s-p1)} + \frac{A2}{(s-p2)} + \cdots + \frac{Ar}{(s-Pi)^r} + K(s)$; *where K(s)=N(s)/$D_1$(s)*

→ $\boxed{A_{r-m} = \frac{1}{m!}\{\frac{d^m}{ds^m}[(s-Pi)^r H(s)]\}|_{s=Pi}}$

Analysis of LTI systems Using LT:- a system is

- Causal if → ROC is to the right of the rightmost pole. (convers is not necessary true)
- Non-causal if → ROC is to the left of the leftmost pole. (convers is not necessary true)
- Stable if → ROC includes the jw-axis.

Notes:-
* for a right sided function x(t) & Re{s} = a is in ROC → all values for which Re{s}>a are in the ROC.
* for a left sided function x(t) & Re{s} = a is in ROC → all values for which Re{s}<a are in the ROC.
* for a two sided function x(t) & Re{s} = a is in ROC → ROC is strip in the s-plane.
* for a finite duration function x(t) → ROC is the entire s-plane.

Unilateral Laplace Transform (uLT):- $\boxed{X(s) = \int_{0+}^{\infty} x(t)e^{-st}\,dt}$

useful for analyzing causal systems. ROC in uLT must be to the right of the rightmost pole.

→ Solving Differential Equations using uLT:-

Apply the uLT property $\boxed{LT\left[\frac{d^n x(t)}{dt^n}\right] = s^n X(s) - \sum_{p=0}^{n-1} s^{n-p-1}\frac{d^p x(t)}{dt^p}|_{t=0+}}$ [i.e:- x''(t) = s^2X(s) - sx(0^+) - x'(0^+)]

Note:- The solution of a DE eq. consists of to parts:- The zero state response "y_{zs}(t)" + the zero input response "y_{zi}(t)"
"y_{zs}(t)":- the response when the Initial conditions are zero.
"y_{zi}(t)":- the response when the system inputs are zero.

The LT properties & LT pairs Tables could be found in the uploaded "FT, LT, ZT tables" pdf file.

Q1) Given that $H(s) = (s+5)/(s^2 - 2s - 3)$
Determine the impulse response for each of the following cases

A) System is stable. B) System is causal. C) System in neither stable nor causal

Q2) The output $y(t)$ of a causal LTI system is related to the input $x(t)$ by the differential equation

$$y''(t) + 2y'(t) - 3y(t) = 2x(t)$$

a) Determine the transfer function given that the system is at rest.
b) Sketch the pole-zero plot and check the stability.
c) If the system is unstable, design a stable and causal version of the system having same order and amplitude distortion characteristics and draw the pole-zero plot of the designed system.
d) Find the frequency response and the impulse response of the designed system.

Self-Study:-
Book's questions no. [**9.40**, **9.33**, **9.31, 9.22** (using pairs & properties not the integral), 9.21, 9.9, 9.6, 9.5, 9.29]