- **Laplace Transform:-**

→ $\boxed{X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt}$; $\boxed{x(t) = \frac{1}{2\pi}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}\,d\omega}$ where $s = \sigma + j\omega$

The generalization of the exponential's power allows us to analyze unstable system or to assess its stability and causality. Furthermore, using the pole-zero compensation we could redesign an unstable system to a stable one. The algebraic representation of X(s) is not enough to determine x(t), we must know ROC as well.

Region of convergence (ROC):- The range of s values for which the LT integral converges.
Note:- ROC doesn't contain any poles and it is a continuous plane.

Note:- If you substitute s with $\sigma + j\omega$ then separate the exponential, you could notice that we can think of LT as FT of signal weighted by an exponential, that is LT = FT{x(t) $e^{-\sigma t}$}.

➢ **Partial fraction:-** Let H(s) = N(s)/D(s)

*The roots of the polynomial numerator N(s) are referred to as Zeroes (in plot use o)
*The roots of the polynomial denominator D(s) are referred to as Poles(in plot use x)
[Note:- The total numbers of zeroe (in finite s plan & infinity) = number of poles]

- *General case:-* $H(s) = \frac{N(s)}{D(s)} = \frac{N(s)}{(s-p1)(s-p2)....(s-pn)} = \frac{A1}{(s-p1)} + \frac{A2}{(s-p2)} + \cdots + \frac{An}{(s-Pn)}$

→ $\boxed{An = [(s-Pn)H(s)]|_{s=Pn}}$

- *Case of repeated poles:-* $H(s) = \frac{N(s)}{D1(s)\,(s-Pi)^r} = \frac{A1}{(s-p1)} + \frac{A2}{(s-p2)} + \cdots + \frac{Ar}{(s-Pi)^r} + K(s)$; *where K(s)=N(s)/D₁(s)*

→ $\boxed{A_{r-m} = \frac{1}{m!}\{\frac{d^m}{ds^m}[(s-Pi)^r H(s)]\}\,|_{s=Pi}}$

Analysis of LTI systems Using LT:- a system is

- Causal if → ROC is to the right of the rightmost pole. (convers is not necessary true)
- Non-causal if → ROC is to the left of the leftmost pole. (convers is not necessary true)
- Stable if → ROC includes the jw-axis.

Notes:-
* for a right sided function x(t) & Re{s} = a is in ROC → all values for which Re{s}>a is in the ROC.
* for a left sided function x(t) & Re{s} = a is in ROC → all values for which Re{s}<a is in the ROC.
* for a two sided function x(t) & Re{s} = a is in ROC → ROC is strip in the s-plane.
* for a finite duration function x(t) → ROC is the entire s-plane.

Unilateral Laplace Transform (uLT):- $\boxed{X(s) = \int_{0+}^{\infty} x(t)e^{-st}\,dt}$

useful for analyzing causal systems. ROC in uLT must be to the right of the rightmost pole.

→ Solving Differential Equations using uLT:-

Apply the uLT property $\boxed{LT\left[\frac{d^n x(t)}{dt^n}\right] = s^n X(s) - \sum_{p=0}^{n-1} s^{n-p-1}\frac{d^p x(t)}{dt^p}\,|_{t=0+}}$ [i.e:- $x''(t) = s^2X(s) - sX(0^+) - X'(0^+)$]

Note:- The solution of a DE eq. consists of to parts:- The zero state response "$y_{zs}(t)$" + the zero input response "$y_{zi}(t)$"
"$y_{zs}(t)$":- the response when the Initial conditions are zero.
"$y_{zi}(t)$":- the response when the system inputs are zero.

The LT properties & LT pairs Tables could be found in the uploaded "FT, LT, ZT tables" pdf file.

Q1) Given that $H(s) = (s+5)/(s^2 -2s -3)$

Determine the impulse response for each of the following cases

A) System is stable. B) System is causal. C) System in neither stable nor causal

Q2) The output $y(t)$ of a causal LTI system is related to the input $x(t)$ by the differential equation

$$y''(t) + 2y'(t) - 3y(t) = 2x(t)$$

a) Determine the transfer function given that the system is at rest.
b) Sketch the pole-zero plot and check the stability.
c) If the system is unstable, design a stable and causal version of the system having same order and amplitude distortion characteristics and draw the pole-zero plot of the designed system.
d) Find the frequency response and the impulse response of the designed system.

Self-Study:-

Book's questions no. [**9.40**, **9.33**, **9.22** (using pairs & properties not the integral), 9.21, 9.9, 9.6, 9.5, 9.29]

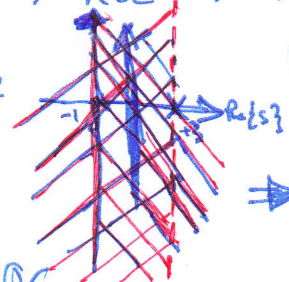
Q1 $H(s) = \frac{(s+5)}{s^2-2s-3} = \frac{s+5}{(s+1)(s-3)} = \frac{A}{s-3} + \frac{B}{s+1}$

$A = [(s-3)\,H(s)]\big|_{s=3} = \frac{8}{4} = 2$

$B = [(s+1)\,H(s)]\big|_{s=-1} = \frac{4}{-4} = -1$ $\quad \therefore H(s) = \frac{2}{s-3} - \frac{1}{s+1}$

a) if stable; ROC must include jw-axis

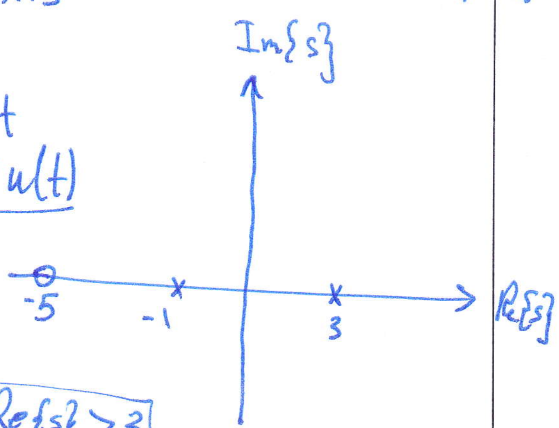
hence



$\boxed{-1 < \text{ROC} < 3}$ Re{s}

$\Rightarrow \underline{h(t) = -2e^{3t}u(-t) - e^{-t}u(t)}$

pole-zero plot



b) Causal; ROC must be to the right of the Right most pole $\Rightarrow$ $\boxed{\text{ROC} \Rightarrow Re\{s\} > 3}$

$\therefore \underline{h(t) = 2e^{3t}u(t) - e^{-t}u(t)}$

c) Neither; ROC $\Rightarrow$ $\boxed{Re\{s\} < -1}$ $\Rightarrow \underline{h(t) = -2e^{3t}u(-t) + e^{-t}u(-t)}$

Q2 a) Using uLT differentiation property

$$s^2Y(s) - s\cancel{y(0^+)}^{\,0} - \cancel{y'(0^+)}^{\,0} + 2[sY(s) - \cancel{y(0^+)}^{\,0}] - 3Y(s) = 2X(s)$$

∵ System initially at rest ⇒ I.C = 0

⇒ $s^2Y(s) + 2sY(s) - 3Y(s) = 2X(s)$

$Y(s)[s^2 + 2s - 3] = 2X(s)$

$$\therefore H(s) = \frac{Y(s)}{X(s)} = \frac{2}{s^2+2s-3} = \frac{2}{(s-1)(s+3)}$$

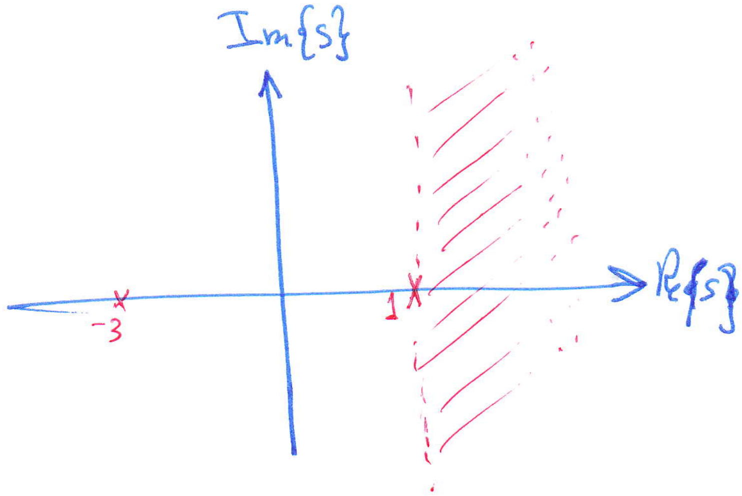
b) ∵ Causal

∴ ROC must be to the right

(Note:- 2 zeroes at infinity)

∵ doesn't include jw-axis

∴ unStable

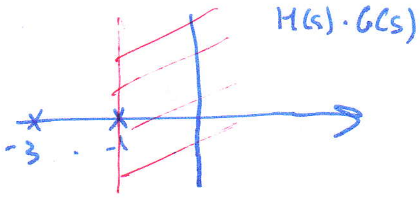


c) $\rightarrow \boxed{H(s)} \rightarrow \boxed{C(s)} \rightarrow$

The compensator must cancel the pole at 1 so that ROC includes the jw-axis

$$\Rightarrow \boxed{C(s) = \frac{(s-1)}{(s+1)}}$$

to keep the magnitude unchanged



d) Total response:- $T(s) = C(s)\,H(s) = \dfrac{2}{(s+1)(s+3)}$

$T(j\omega) = T(s)\big|_{s=j\omega}$

$$T(t) = \mathcal{L}^{-1}\{T(s)\} = \mathcal{L}^{-1}\left\{\frac{+1}{s+1} + \frac{-1}{s+3}\right\}$$

P.F

(Note:- ROC to the right)

$$= e^{-t}u(t) - e^{-3t}u(t)$$