- **Z - Transform:-**

→ $\boxed{X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}}$ ; $\boxed{x[n] = \frac{1}{2\pi j} \oint X(z) z^{n-1}\, dz}$

where z = $re^{j\omega}$ (r is the magnitude and w is the angle)

Laplace was the generalization of CTFT. Similarly, ZT is the generalization of **DTFT**. It allows us to analyze unstable system or to assess its stability and causality. Furthermore, using the pole-zero compensation we could redesign an unstable system to a stable one. The algebraic representation of X(z) is not enough to determine x(t), we must know ROC as well.

**Region of convergence (ROC):-** The range of z values for which the LT integral converges.
*Note:- ROC doesn't contain any poles and it is a continuous ring in the z- plane centered about the origin.*

*Note:-* If you substitute z with $re^{j\omega}$ , you could notice that we can think of ZT as FT of signal weighted by an exponential, that is ZT = DTFT{x[n] $r^{-n}$}.

**Analysis of LTI systems Using ZT:-** a system with rational function is

- Causal if → ROC is to the exterior of a circle outside the outmost pole, and the order of the numerator of the transfer function does not exceed that of the denominator.
- Stable if → ROC includes the unity circle |z|=1 .

Notes:-
* for a right sided sequence x[n] & |z| = $r_o$ is in ROC → all values for which |z| > $r_o$ are in the ROC.
* for a Left sided sequence x[n] & |z| = $r_o$ is in ROC → all values for which |z| < $r_o$ are in the ROC.
* for a two sided sequence x[n] & & |z| = $r_o$ is in ROC→ ROC is a ring in the z-plane including the circle & |z| = $r_o$.
* for a finite duration function x[n] → ROC is the entire z-plane except possibly z=0 and/or z=∞.

**Unilateral Z -Transform (uZT):-** $\boxed{X(z) = \sum_{n=0}^{\infty} x[n]\, z^{-n}}$
useful for analyzing causal systems. ROC in uZT is always the exterior of a circle.

**The ZT properties & ZT pairs Tables could be found in the uploaded "FT, LT, ZT tables" pdf file.*

Q1) Find the Z-transform of $x[n] = a^n u[n]$.

Q2) Given $X(z) = 4z^2 + 2 + 3z^{-1}$, where $0<|z|<\infty$. Determine x[n]

Q3) Given the system function $H(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{1-2z^{-1}}$

i) Find the poles and zeroes.
ii) Determine the impulse response for each of the following cases

A) System is stable. B) System is causal (if possible). C) System in neither stable nor causal

Q4) The difference equation of a causal, LTI system, initially at rest is:- $y[n] + 3\,y[n-1] = x[n]$
Given $H(z) = 1/(1+3z^{-1})$. Determine the output if the input is a*u[n].