Q1) Find the Z-transform of $x[n] = a^n u[n]$.

Q2) Given $X(z) = 4z^2 + 2 + 3z^{-1}$, where $0<|z|<\infty$. Determine $x[n]$

Q3) Given that $$H(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{1-2z^{-1}}$$

i) Find the poles and zeroes.

ii) Determine the impulse response for each of the following cases

A) System is stable. B) System is causal (if possible). C) System in neither stable nor causal

Q4) The difference equation of a causal, LTI system, initially at rest is:- $y[n] + 3y[n-1] = x[n]$

Given $H(z) = 1/(1+3z^{-1})$. Determine the output if the input is $a*x[n]$.

Q1

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{0}^{\infty} a^n z^{-n}$$

$$= \sum_{0}^{\infty} (az^{-1})^n$$

$$= \frac{(az^{-1})^0 - (az^{-1})^{\infty+1}}{1 - az^{-1}}$$

$$= \frac{1-0}{1-az^{-1}} \qquad |az^{-1}|<1$$

$$= \frac{1}{1-az^{-1}} \qquad |z^{-1}|<|a^{-1}|$$

$$\underline{|z|>|a|}$$

Q2

$$X(z) = 4z^2 + 2 + 3z^{-1}$$

* From:- $\delta[n] \xleftrightarrow{z} 1$

and $x[n-n_0] \xleftrightarrow{z} z^{-n_0} X(z)$

$\therefore x[n] = z^{-1}\{X(z)\}$

$$= 4\delta[n+2] + 2\delta[n] + 3\delta[n-1]$$

Q3 $H(z) = \dfrac{1}{1-\frac{1}{2}z^{-1}} + \dfrac{1}{1-2z^{-1}}$

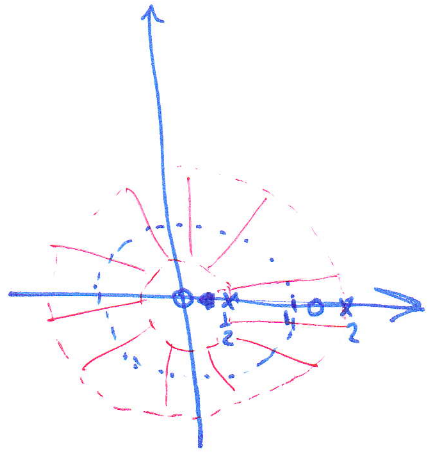
i) $H(z) = \dfrac{(1-2z^{-1}) + (1-\frac{1}{2}z^{-1})}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})} = \dfrac{2z^2 - \frac{5}{2}z}{z^2 - \frac{5}{2}z + 1}$

$\therefore$ two zeroes @ $z=0$ & $z=\frac{5}{4}$

" poles @ $z=\frac{1}{2}$ & $z=2$

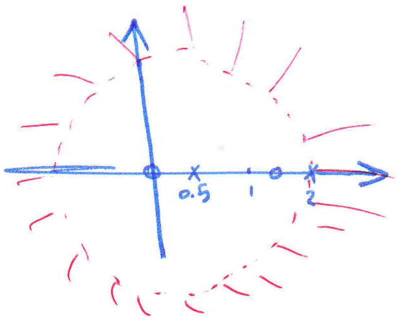
ii) A] if stable $\Rightarrow$ ROC must include the unity circle

$\therefore$ ROC $\frac{1}{2} < |z| < 2$



$\therefore h[n] = (\frac{1}{2})^n u[n] - 2^n u[-n-1]$

B] $\because$ The order of the numerator equals the denomenator's $\Rightarrow$ it's possible to be causal

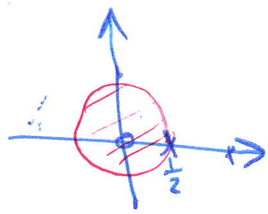


if causal $\Rightarrow$ ROC is outside the outmost pole $|z| > 2$

$\therefore h[n] = (\frac{1}{2})^n u[n] + 2^n u[n]$

c] Niether $\Rightarrow$ ROC $|z| < \frac{1}{2}$

& $h[n] = -(\frac{1}{2})^n u[-n-1] - 2^n u[-n-1]$



Q4 $Y(z) = H(z)\,X(z)$

* from :- $u[n] \overset{z}{\longleftrightarrow} \dfrac{1}{1-z^{-1}}$

$Y(z) = \dfrac{1}{1+3z^{-1}} \cdot \dfrac{\alpha}{1-z^{-1}} = \dfrac{A}{1+3z^{-1}} + \dfrac{B}{1-z^{-1}}$

$\therefore A = \left[(1+3z^{-1})\,Y(z)\right]_{z=-3} = \dfrac{\alpha}{1-\frac{1}{-3}} = \frac{3}{4}\alpha$

$B = \left[(1-z^{-1})\,Y(z)\right]_{z=1} = \dfrac{\alpha}{1+3} = \frac{1}{4}\alpha$

$\therefore Y(z) = \dfrac{\frac{3}{4}\alpha}{1+3z^{-1}} + \dfrac{\frac{1}{4}\alpha}{1-z^{-1}}$

$z^{-1}\updownarrow$

$y[n] = \frac{3}{4}\alpha\,(-3)^n u[n] + \frac{1}{4}\alpha\; u[n]$

(Note:- $\because$ causal $\therefore$ ROC $|z| > |-3|$)