EE#301

Tut#1

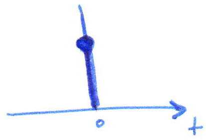
محمد بن صالح آل الشيخ
fac.KSU.edu.sa/MalShaykh

Signals

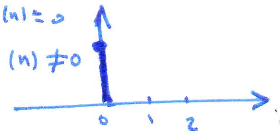
continuous → CT $x(t)$ | Discrete → DT $x(n)$

Impulse:-

$$\delta(t) = \begin{cases} \infty & (t) = 0 \\ \text{zero} & (t) \neq 0 \end{cases}$$

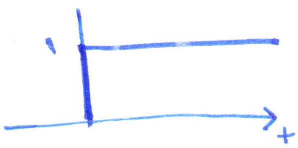


$$u(n) = \begin{cases} 1 & (n) = 0 \\ 0 & (n) \neq 0 \end{cases}$$

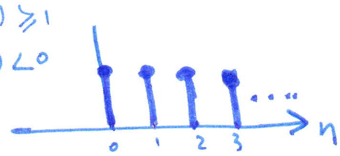


unit step:-

$$u(t) = \begin{cases} 1 & (t) \geq 0 \\ 0 & (t) < 0 \end{cases}$$

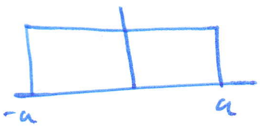


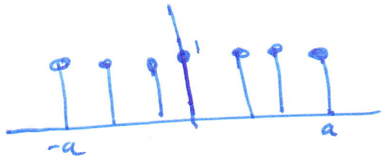
$$u(n) = \begin{cases} 1 & (n) \geq 1 \\ 0 & (n) < 0 \end{cases}$$



Rectangular pulse:

$$P_a(t) = \begin{cases} 1 & -a \leq (t) \leq a \\ 0 & \text{o.w} \end{cases}$$





Basic Operations:-

- $f(t-T)$:- Shift Right
- $f(t+T)$:- Shift left
- $f(-t)$: Time reversal
- $f(\alpha t)$: multiply by $\frac{1}{|\alpha|}$ (if $\alpha > 1$; compressing, $\alpha < 1$; Stretching)

(Note:- T is +ve)

General Form:-

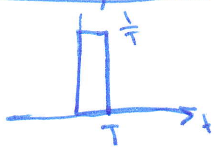
$$\boxed{y(t) = \underbrace{a}_{③}\, x(\underbrace{\alpha}_{①}(t + \underbrace{T}_{②}))}$$

① reverse, scale of time axis

2.5) Mathematical Description of Impulse function:-

- Area of $\delta(t)$ is 1

$$\therefore \int_{-\infty}^{\infty} \delta(t)\, dt = 1$$



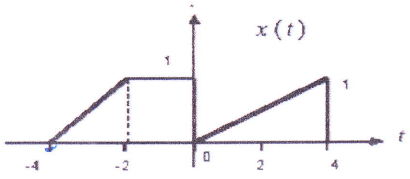
$$\delta(t) = \delta(-t)$$

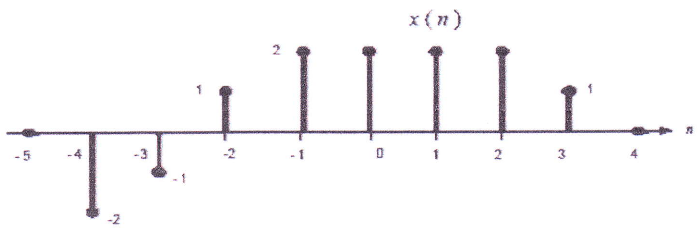
- $$\boxed{\delta(t) = \frac{d u(t)}{dt}}$$

- $$f(t) \cdot \delta(t - t_0) = f(t_0) \cdot \delta(t - t_0)$$

$$\therefore \int_{-\infty}^{\infty} f(t)\, \delta(t - t_0)\, dt = f(t_0)$$

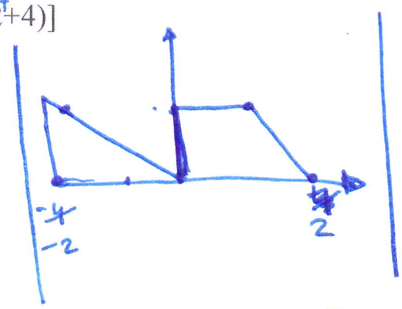
Q1) Given the continuous-time (CT) and discrete-time (DT) signals x(t) and x(n) shown below, sketch and label carefully the signals in (a),(b), and (c). [Note: u(t), u(n) denote the CT and DT unit step signals respectively]

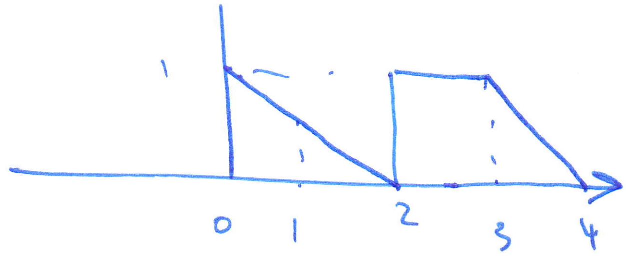




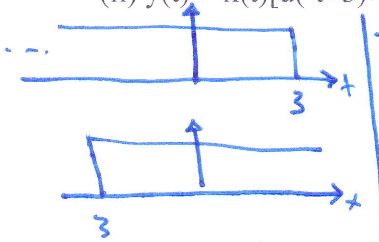
(a) (i) y(t) = x[(-2t+4)]

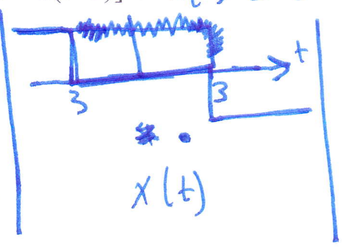
$y(t) = x[-2(t-2)]$

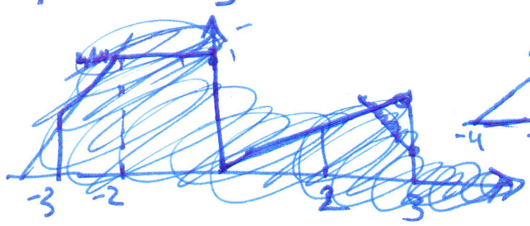


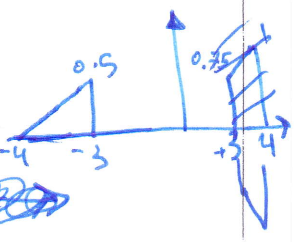


(ii) y(t) = x(t)[u(-t+3) – u(t+3)] $= x(t)\,[u(-(t-3)) - u(t+3)]$



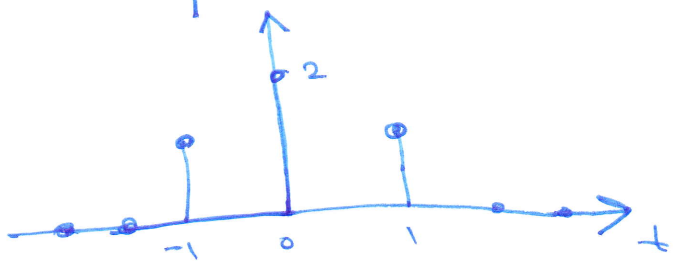






(b) y[n] = x[2n^2 +1]

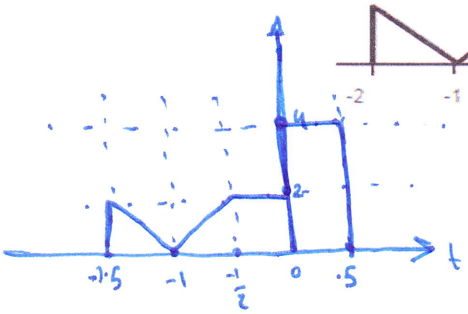
n	$x(2n^2+1)$
-2	x(9)
-1	x(3)
0	x(1)
1	x(3)

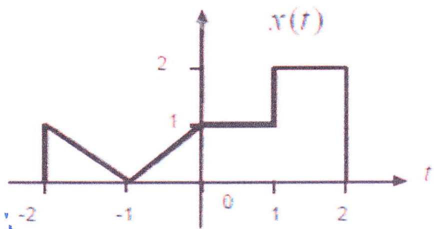


(c) For the following x(t) and x[n] find:-

(i) y(t)=2x(2t+1) $= 2x[2(t+0.5)]$

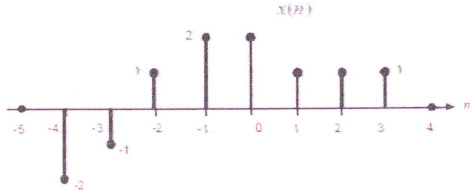
	-2	-1	0	1	2
t	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1
	-1.5	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$
x(t)	$\begin{bmatrix}1\\0\end{bmatrix}$	$[0]$	$[1]$	$\begin{bmatrix}2\\1\end{bmatrix}$	$\begin{bmatrix}0\\2\end{bmatrix}$
2x(t)	$\begin{bmatrix}2\\0\end{bmatrix}$	$[0]$	$[2]$	$\begin{bmatrix}4\\2\end{bmatrix}$	$\begin{bmatrix}0\\4\end{bmatrix}$

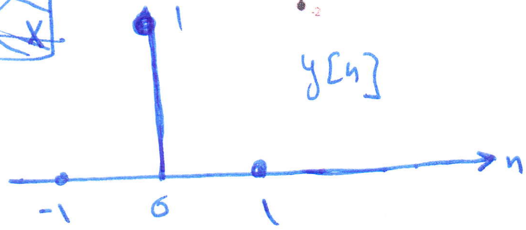




(ii) y[n] = x[3n+1] u[n] = $x\left[3\left(n+\frac{1}{3}\right)\right] u(n)$

n	0	1	2	3
	0	$\frac{1}{3}$	$\frac{2}{3}$	1
	X	0	X	X
$x[n]$	–	1	–	–





Q2) Evaluate the following

- $\sin(\tau)\ \delta(\tau+3)\ \cancel{d\tau} =$ $\sin(-3)\ \delta(\tau+3)$

- $\int_{0}^{\infty} \cos(t-\tau)\ \sin(\tau)\ \delta(\tau+3)\ d\tau =$

$\int \cos(t+3)\ \sin(-3)\ \delta(\tau+3)\ d\tau = \text{Zero}$

$\cancel{= \cos(t+3)\ \sin(-3)}$

- $\int_{-\infty}^{\infty} e^{-t}\ \delta(2t-1)\ dt =$

let $k = 2t - 1$
$dk = 2\,dt$
$\boxed{dt = \frac{1}{2} dk}$
$t = \frac{1}{2}(k+1)$

$\therefore \int_{-\infty}^{\infty} e^{-\frac{1}{2}(k+1)} \delta(k)\ \frac{1}{2} dk$

$= \frac{1}{2} \int e^{-\frac{1}{2}(0+1)} \delta(k)\, dk = \frac{1}{2} e^{-0.5}$

- $\int_{0}^{\infty} \delta(-t+4) \sin\left(\frac{\pi t}{8}\right) dt =$

$\int_{0}^{\infty} \delta(-(t-4)) \sin\left(\frac{\pi t}{8}\right) dt = \sin\left(\frac{\pi * 4}{8}\right)$

Q1) Classify if periodic and find the fundamental period:-

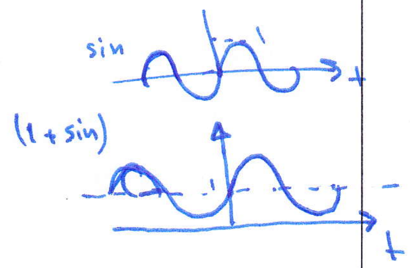
i) x(t) = cos (2.32πt)

∴ Periodic ; $T_0 = \frac{2\pi}{\omega} = \frac{2\pi}{2.32\pi} =$

ii) x(t) = [cos(2t + 2π/3)]^2

$\cos^2(x) = \frac{1}{2}[1+\cos(2x)]$ | ∴ $\frac{1}{2}[\overset{DC}{1} + \cos(4t + \frac{4\pi}{3})]$

Periodic ⇒ $T_0 = \frac{2\pi}{4} = \frac{\pi}{2}$ | Note



iii) x(t) = cos(2πt) + sin(0.5πt) +sin(πt)

$T_1 = 1$ $T_2 = 4$ $T_3 = 2$ | $T = LCM(1,2,4) = 4$

⇒ $\frac{T_1}{T_2} = \frac{1}{4}$; $\frac{T_1}{T_3} = \frac{1}{2}$; $\frac{T_2}{T_3} = \frac{4}{2}$

∵ all rational ⇒ Sum is periodic

iv) x[n] = cos(2n)

∵ DT ; check ; $N = \frac{2\pi K}{2} = \pi K$; ∵ Not an integer

∴ Not periodic

v) x[n] = cos(2πn) + exp(jπn)

$N_1 = \frac{2\pi K}{2\pi} = 1$; Periodic | ∵ Sum of DT periodic ⇒ periodic

$N_2 = \frac{2\pi}{\pi} k = 2$; periodic | $N = \frac{N_1 N_2}{gcd(N_1,N_2)} = \frac{2 \cdot 1}{1} = 2$

Q2) Calculate the Energy and power of the following, and state whether it's power or energy signal:-

i) x(t) = e $^{-2t}$ u(t) $E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt = \lim \int_0^T e^{-4t} dt = \lim \left[\frac{e^{-4t}}{-4} \Big|_0^T \right]$

$= \lim_{T\to\infty} \left(\frac{e^{-4T}}{-4} - \frac{1}{-4} \right) = \left(0 + \frac{1}{4}\right) = \frac{1}{4}$.

∵ $0 < E < \infty$ for finite energy ⇒ E - signal ⇒ $P_{avg} = 0$

ii) x(t) = $A\, e^{j(\omega t+\alpha)}$ ⇒ Periodic ∴ P

$P_{avg} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} A^2 |e^{j(\omega t+\alpha)}|^2 dt = \lim \frac{1}{2T} \int_{-T}^{T} A^2 \cdot 1\, dt$

$= \lim_{T\to\infty} \frac{A^2}{2T} \left[t \Big|_{-T}^{T} \right] = \lim_{T\to\infty} A^2 \frac{2T}{2T} = A^2$

∵ Finite avg power ∴ P signal ⇒ $E = \infty$

iii) x[n] =(0.5)^n u[n]

$E = \lim_{N\to\infty} \sum_{n=-N}^{N} |x[n]|^2 = \lim_{N\to\infty} \sum_{n=0}^{N} (0.5)^{2n} = \lim \sum_{0}^{N} \left(\frac{1}{4}\right)^n = \frac{1-0}{1-\frac{1}{4}} = 4/3$

if $\alpha \neq 1$

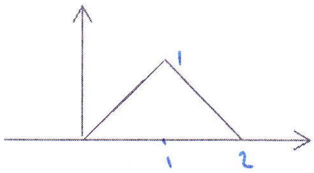
$\sum_{n=a}^{b} \alpha^n = \frac{\alpha^a - \alpha^{b+1}}{1-\alpha}$

$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}$ only for $|\alpha| < 1$

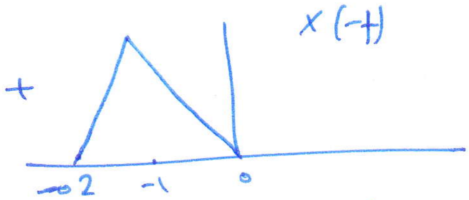
∵ finite Energy ⇒ ~~Power~~ Energy Signal ⇒ $P_{avg} = 0$

Q3) Determine and sketch the even and odd part of the following signals:-

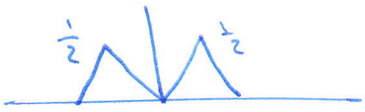
i) x(t)



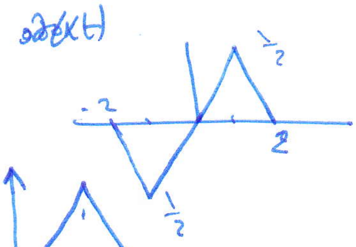
\+ x(-t)



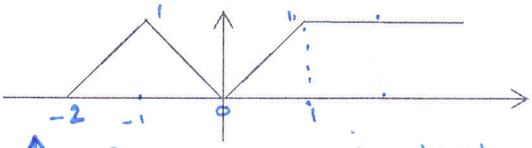
Ev(x(t))



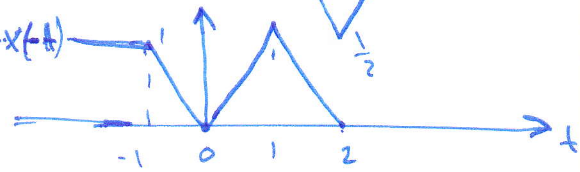
odd(x(t))



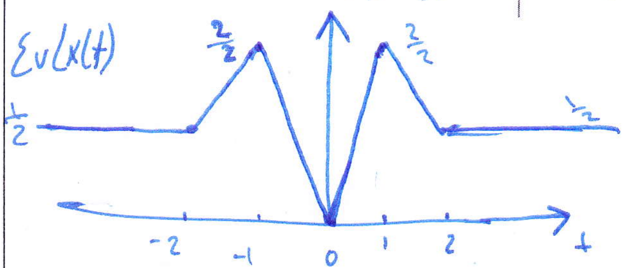
ii) x(t)



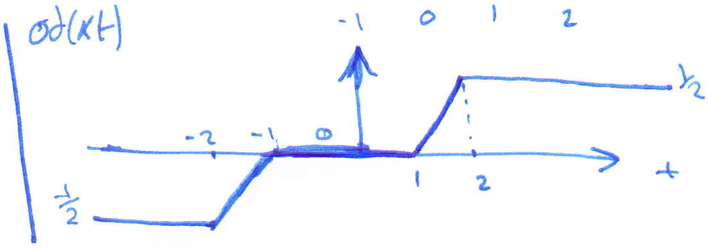
x(-t)



Ev(x(t))

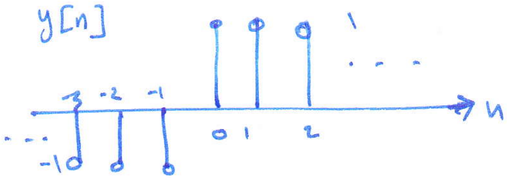


odd(x(t))

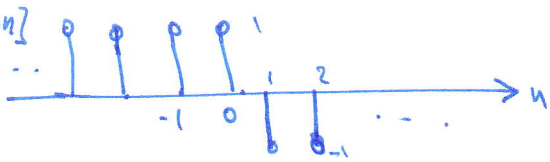


iii) y[n] = u[n] + u[-n+1] = u[n] + u[-(n+1)]

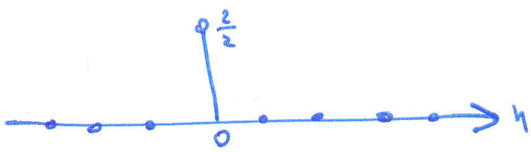
y[n]



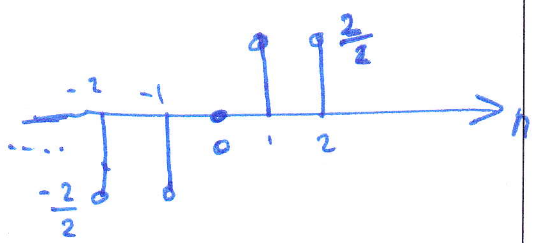
y[-n]



Ev(y[n])



odd(y[n])



Q1) State YES/NO to indicate whether or not, the systems in (a)-(c) in the table below possess / do not possess the properties shown in the table. [Note: x(t) , x(n) are system inputs while y(t) , y(n) are the corresponding outputs].

	Memoryless	Causal	Stable	Time Invariant	Linear	Invertible
a) y[n]=cos(x[n])	✓	✓	✓	✓	✗	✗
b) y(t)=x(t/3)	✗	✗	✓	✗	✓	✓
c) $y(t)=\int_{-\infty}^{2t} x(T)dT$	✗	✗	✗	✗	✓	✗

$\boxed{a)\ y[n] = \cos(x[n])}$

⊛ $y[1] = \cos(x[1])$, $y[-1] = \cos(x[-1])$ $\Rightarrow$ ∴ Memoryless & causal

⊛ if $|x[n]| \leq B \Rightarrow |y[n]| \leq |\cos(|x[n]|)| \leq |\cos(B)| \leq 1$ ∴ Stable

⊛ $x_1[n] \rightarrow y_1[n] = \cos(x_1[n])$

$x_2[n] = x_1[n-n_o] \rightarrow y_2[n] = \cos(x_2[n]) = \cos(x_1[n-n_o])$

$\Rightarrow y_1[n-n_o] = \cos(x_1[n-n_o]) = y_2[n]$ ∴ TI

⊛ $x_1[n] \rightarrow y_1[n] = \cos(x_1[n])$ $\quad x_2[n] \rightarrow y_2[n] = \cos(x_2[n])$

$x_3[n] = a x_1[n] + b x_2[n] \rightarrow y_3[n] = \cos(x_3[n])$

$= \cos(a x_1[n] + b x_2[n]) \neq a y_1[n] + b y_2[n]$

∴ not Linear

⊛ Not invertible (note: if $w[n] = \cos^{-1}(y[n]) = \pm x[n]$; we can't know the sign)

$\boxed{b)\ y(t) = x(\tfrac{t}{3})}$

⊛ $y(1) = x(\frac{1}{3})$ ∴ Present depends on past ∴ has Memory

⊛ $y(-1) = x(\frac{-1}{3})$ " " " future ∴ Non causal

⊛ if $|x(t)| \leq B \Rightarrow |y(t)| \leq |x(\frac{t}{3})| \leq B$ ∴ Stable

⊛ $x_1(t) \rightarrow y_1(t) = x_1(\frac{t}{3})$

$x_2(t) = x_1(t-t_o) \rightarrow y_2(t) = x_2(\frac{t}{3}) = x_1(\frac{t}{3} - t_o)$

$\Rightarrow y_1(t-t_o) = x(\frac{t-t_o}{3}) \neq y_2(t)$ ∴ TV

⊛ $x_1(t) \rightarrow y_1(t) = x_1(\frac{t}{3})$ $\quad x_2(t) \rightarrow y_2(t) = x_2(\frac{t}{3})$

$x_3(t) = a x_1(t) + b x_2(t) \rightarrow y_3(t) = x_3(\frac{t}{3}) = a x_1(\frac{t}{3}) + b x_2(\frac{t}{3}) = a y_1(t) + b y_2(t)$

∴ Linear

⊛ Let Inverse system $w(t) = y(3t) = x(\frac{3t}{3}) = x(t)$ ∴ Invertible

c) $y(t) = \int_{-\infty}^{2t} x(\tau)\, d\tau$

⊛ ~~$y(t)$~~ notice integral boundries

$\Rightarrow$ <u>has memory</u> $\Rightarrow$ <u>Non Causal</u> $\left(y(1) = \int_{-\infty}^{2(1)} x(\tau)\, d\tau \right)$

⊛ assume $x(\tau) = 1$; $y(t) = \int 1\, d\tau = t\Big|_{-\infty}^{2t} = 2t + \infty = \infty$ $\therefore$ <u>unStable</u>

⊛ $x_1(t) \rightarrow y_1(t) = \int_{-\infty}^{2t} x_1(\tau)\, d\tau$

$x_2(t) = x_1(t - t_0) \rightarrow y_2(t) = \int_{-\infty}^{2t} x_2(\tau)\, d\tau = \int_{-\infty}^{2t} x_1(\tau - t_0)\, d\tau$

$\Rightarrow y_1(t - t_0) = \int_{-\infty}^{2(t-t_0)} x_1(\tau)\, d\tau \neq y_2(t)$ $\therefore$ <u>TV</u>

⊛ $x_1(t) \rightarrow \ldots$ $x_2(t) \rightarrow \ldots$

$x_3(t) = a x_1(t) + b x_2(t) \rightarrow y_3(t) = \int_{-\infty}^{2t} x_3(\tau)\, d\tau = \int_{-\infty}^{2t} [a x_1(\tau) + b x_2(\tau)]\, d\tau$

$= \int_{-\infty}^{2t} a x_1(\tau)\, d\tau + \int_{-\infty}^{2t} b x_2(\tau)\, d\tau$

$\therefore$ <u>Linear</u> $= a y_1(t) + b y_2(t)$

⊛ Non Invertible [notice that it's bounded]

$x(t) \rightarrow \boxed{y(t) = \int x(\tau)\, d\tau} \rightarrow \boxed{w(t) = \frac{d}{dt}} \rightarrow x(t)$

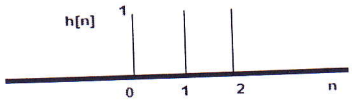
However

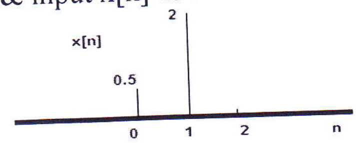
$x(t) \rightarrow \boxed{\int_{-\infty}^{2t}} \rightarrow \boxed{d/dt} \rightarrow ??$

(if $x(t) = 1 \Rightarrow \frac{d}{dt} y(t) = \frac{d}{dt} \int_{-\infty}^{2t} 1\, d\tau = \frac{d}{dt}(2t + \infty) =$ undefined (because $\frac{d\infty}{dt} =$ undefined))

Note: $\frac{d}{dt} \int f(t)\, dt = f(t)$;

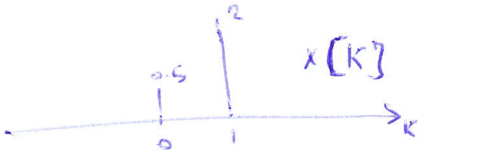
Q1) Consider an LTI system with impulse response h[n] & input x[n] as illustrated. Find the convolution sum.

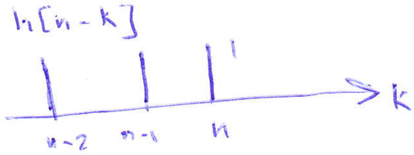




Sol 1

$y[n]=x[n]*h[n]=\sum_{k=-\infty}^{\infty}x[k]\,h[n-k]$





$n<0 \Rightarrow y[n]=0$

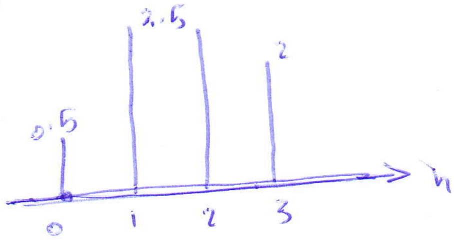
$n=0 \Rightarrow y[0]=1*\frac{1}{2}=\frac{1}{2}$

$n=1 \Rightarrow y[1]=\frac{1}{2}+2=2.5$

$n=2 \Rightarrow y[2]=\frac{1}{2}+2=2.5$

$n=3 \Rightarrow y[3]=1*2=2$

$\therefore y[n]$

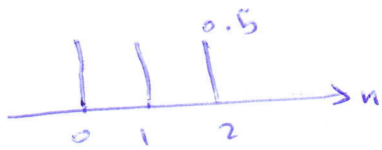


Sol 2

$y[n]=x[n]*h[n]=\sum_{k=-\infty}^{\infty}x[k]\,h[n-k]$

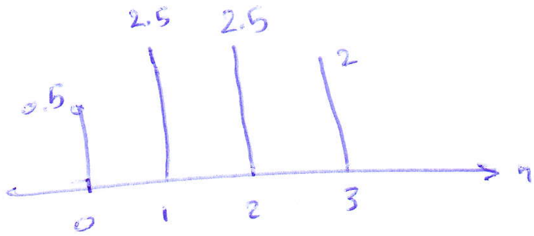
$=x[0]\,h[n]+x[1]\,h[n-1]$

$=0.5\,h[n]+2\,h[n-1]$



+

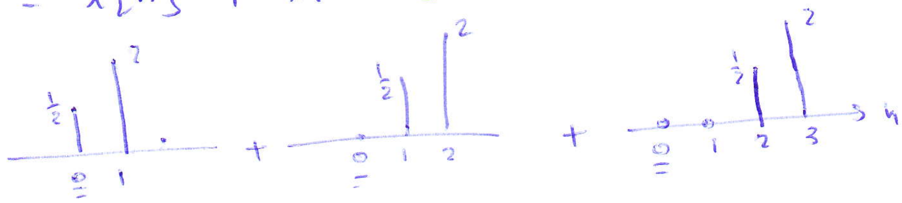
$\Rightarrow y[n]$



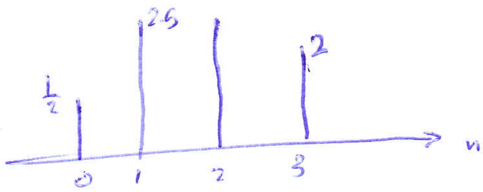
Sol 3)

$$y[n] = x[n] * (\delta(n) + \delta(n-1) + \delta(n-2))$$

$$= x[n] + x[n-1] + x[n-2]$$

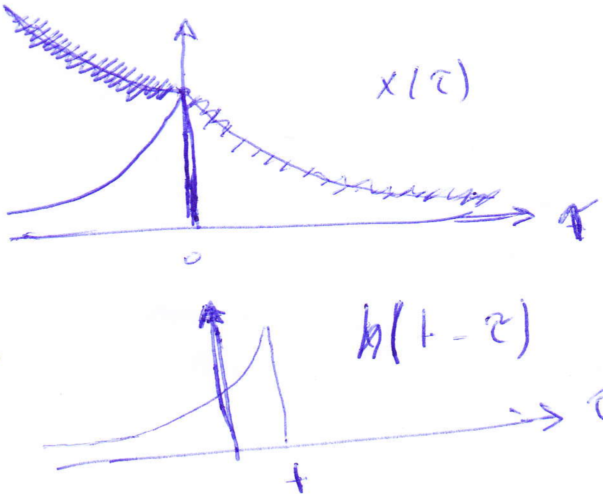


$\Longrightarrow$ $y[n]$



Q2) For a LTI system given that h(t) = e^{-2t} u(t) and x(t) = e^{2t} u(-t), determine the output y(t).

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

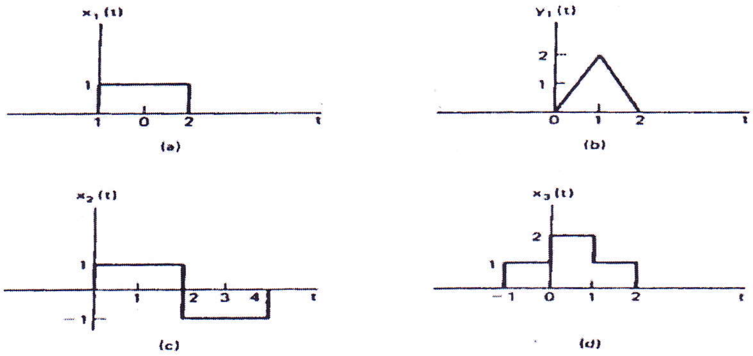


$$= \begin{cases} \int_{-\infty}^{t} e^{-2(t-\tau)} e^{2\tau} d\tau & t < 0 \\ \int_{-\infty}^{0} e^{-2(t-\tau)} e^{2\tau} d\tau & t \geq 0 \end{cases}$$

$$= \begin{cases} e^{-2t} \int_{-\infty}^{t} e^{4\tau} d\tau & t < 0 \\ e^{-2t} \int_{-\infty}^{0} e^{4\tau} d\tau & t \geq 0 \end{cases}$$

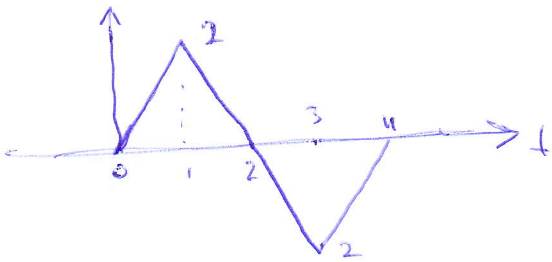
$$= \begin{cases} e^{-2t} \left[\frac{e^{4\tau}}{4} \Big|_{-\infty}^{t} \right] = \frac{e^{-2t}}{4}(e^{4t} - 0) = \frac{e^{2t}}{4} & t < 0 \\ e^{-2t} \left[\frac{e^{4\tau}}{4} \Big|_{-\infty}^{0} \right] = \frac{e^{-2t}}{4}(1 - 0) = \frac{e^{-2t}}{4} & t \geq 0 \end{cases}$$

Q3) Consider an LTI system whose response to the input $x_1(t)$ is the signal $y_1(t)$. Determine and sketch the response of system to $x_2(t)$ $x_3(t)$

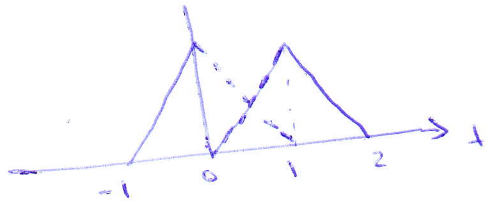


LTI

$x_2(t) = x_1(t) - x_1(t-2) \longrightarrow y_2(t) = y_1(t) - y_2(t-2)$



$x_3(t) = x_1(t) + x_1(t+1) \longrightarrow y_3(t) = y_1(t) + y(t+1)$



$y_3(t)$